Empa
Materials Science and Technology

Flexural strengthening of reinforced concrete according to Swiss Code 166 (2024)

ETH Lecture 101-0167-01L

Fibre Composite Materials in Structural Engineering

Christoph Czaderski

29. October 2025

Program overview of the lectures and laboratory work

- Wednesday 29.10.2025, 15:45-17:30 (ETH Hönggerberg, HIL E10.1)
  - Lecture on Flexural Strengthening
  - Preparations for laboratory competition (Beam) and second written intermediate exam

- Wednesday 26.11.2025, 15:45-17:30 (Empa Dübendorf)
  - Meeting point at ETH Hönggerberg 15:30!! Transport organized by Empa.
  - Application of Externally Bonded FRP Reinforcement (Confinement) for laboratory competition
  - Video of the beam failure test
  - Empa structural laboratory tour (if time available)

- Wednesday 10.12.2025, 15:45 – ca. **18:00** (Empa Dübendorf)
  - Meeting point at ETH Hönggerberg 15:30!! Transport organized by Empa.
  - Laboratory experiments and awarding of lab competition
  - Second written interim exam

Overview of the lecture

- Information about the second written intermediate exam
- Introduction
- Bond between CFRP strip and concrete
  - Lap-shear test
  - Slip, Bond shear stress and Bond shear stress-Slip-Relation
  - Simplified modeling
- Debonding failure modes according to the old and the new SIA 166
- Summary, debonding failure modes treated in SIA 166
- Example
- Video of CFRP strip application on a beam, competition on prediction of failure load
- Several additional topics according to SIA 166

Content of the second exam on 10.12.2025, 15:45 - ca. **18:00**!!! at Empa Dübendorf

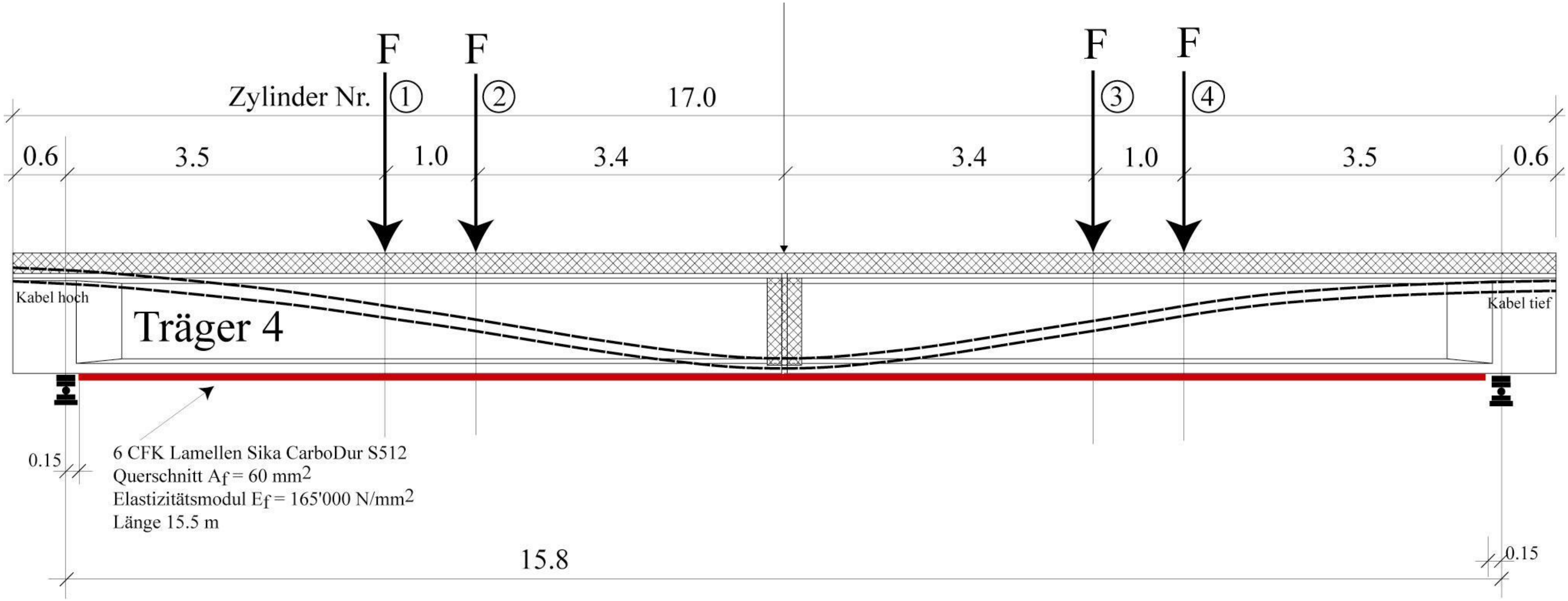
Topics:

- Flexural strengthening of RC according to SIA166
  Lecture from 29.10.2025, Lecturer Dr. C. Czaderski
- Column confinement of RC
  Lecture from 05.11.2025, Lecturer Prof. Dr. M. Motavalli
- Externally bonded FRP reinforcement for metallic structures
  Lecture from 12.11.2025, Lecturer Dr. H. Heydarinouri
- Design of FRP profiles and all FRP structures
  Lecture from 15.10.2025, Lecturer Prof. Dr. M. Shahverdi
- Conceptual questions on the topics which were presented in the mentioned lectures. Furthermore, some calculations have to be performed.

- Time: 60 Minutes
- No laptops, tablets, smart phones etc.
- Only a calculator
- One A4 – Summary (both sides or two pages one side)

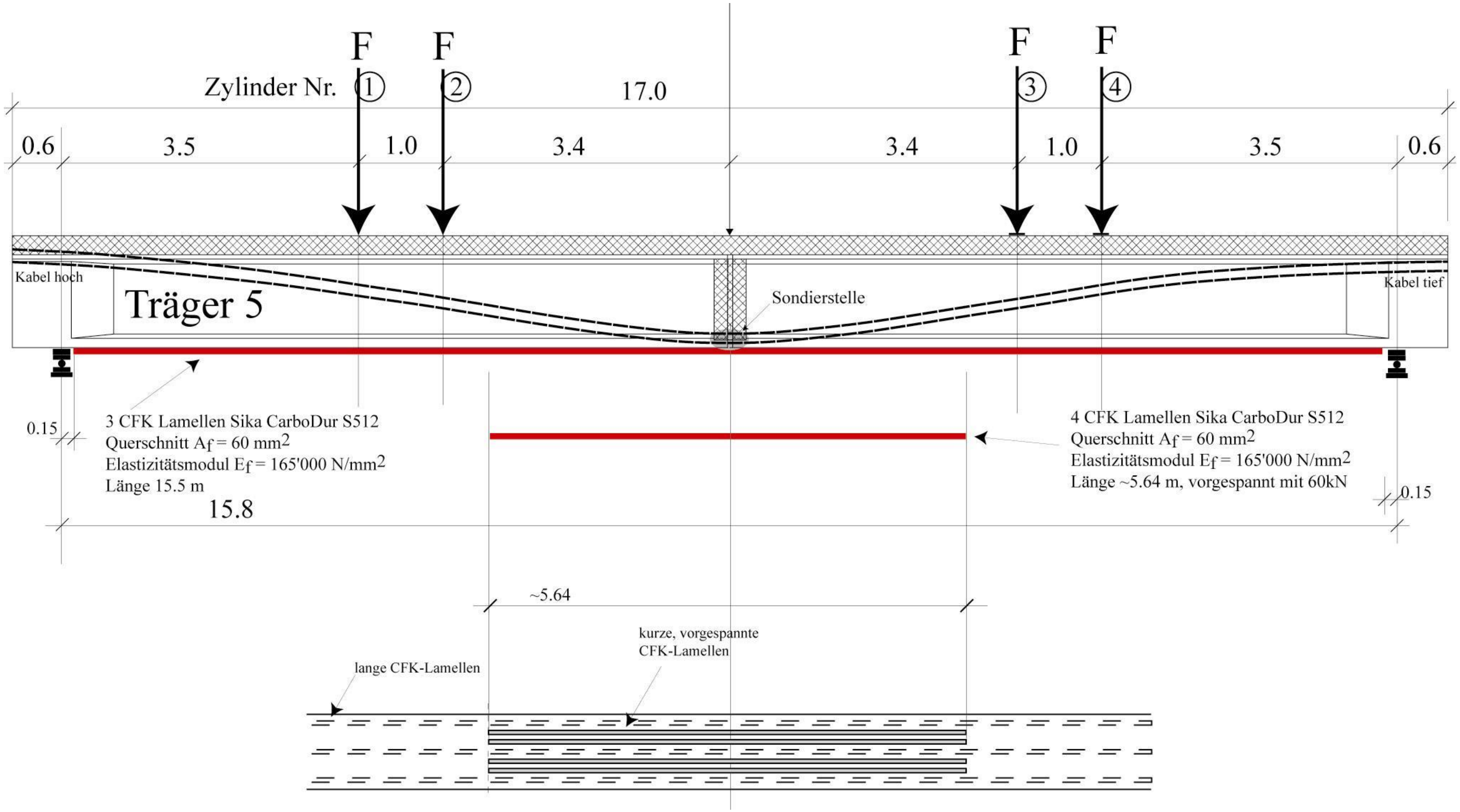
Introduction


EMPA
4
F = 421 KN


F
F
F
F
Zylinder Nr.
①
②
③
④
17.0
0.6
3.5
1.0
3.4
3.4
1.0
3.5
0.6
Kabel hoch
Träger 4
Kabel tief
0.15
6 CFK Lamellen Sika CarboDur S512
Querschnitt A_f = 60 mm^2
Elastizitätsmodul E_f = 165'000 N/mm^2
Länge 15.5 m
15.8
0.15


F = 421 KN


2 (4/11)
EMPA

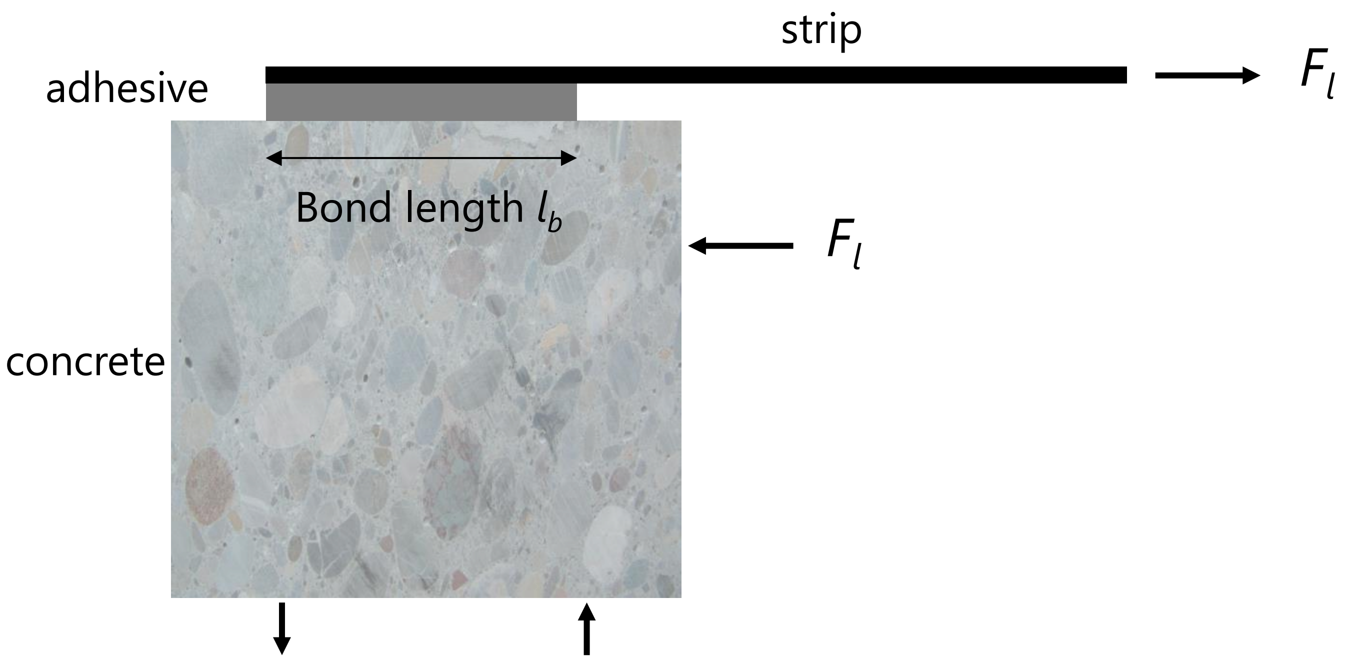


F
F
F
F
Zylinder Nr.
1
2
3
4
17.0
0.6
3.5
1.0
3.4
3.4
1.0
3.5
0.6
Kabel hoch
Träger 5
Sondierstelle
Kabel tief
0.15
3 CFK Lamellen Sika CarboDur S512
Querschnitt Af = 60 mm2
Elastizitätsmodul Ef = 165'000 N/mm2
Länge 15.5 m
15.8
4 CFK Lamellen Sika CarboDur S512
Querschnitt Af = 60 mm2
Elastizitätsmodul Ef = 165'000 N/mm2
Länge ~5.64 m, vorgespannt mit 60kN
0.15
~5.64
kurze, vorgespannte
CFK-Lamellen
lange CFK-Lamellen

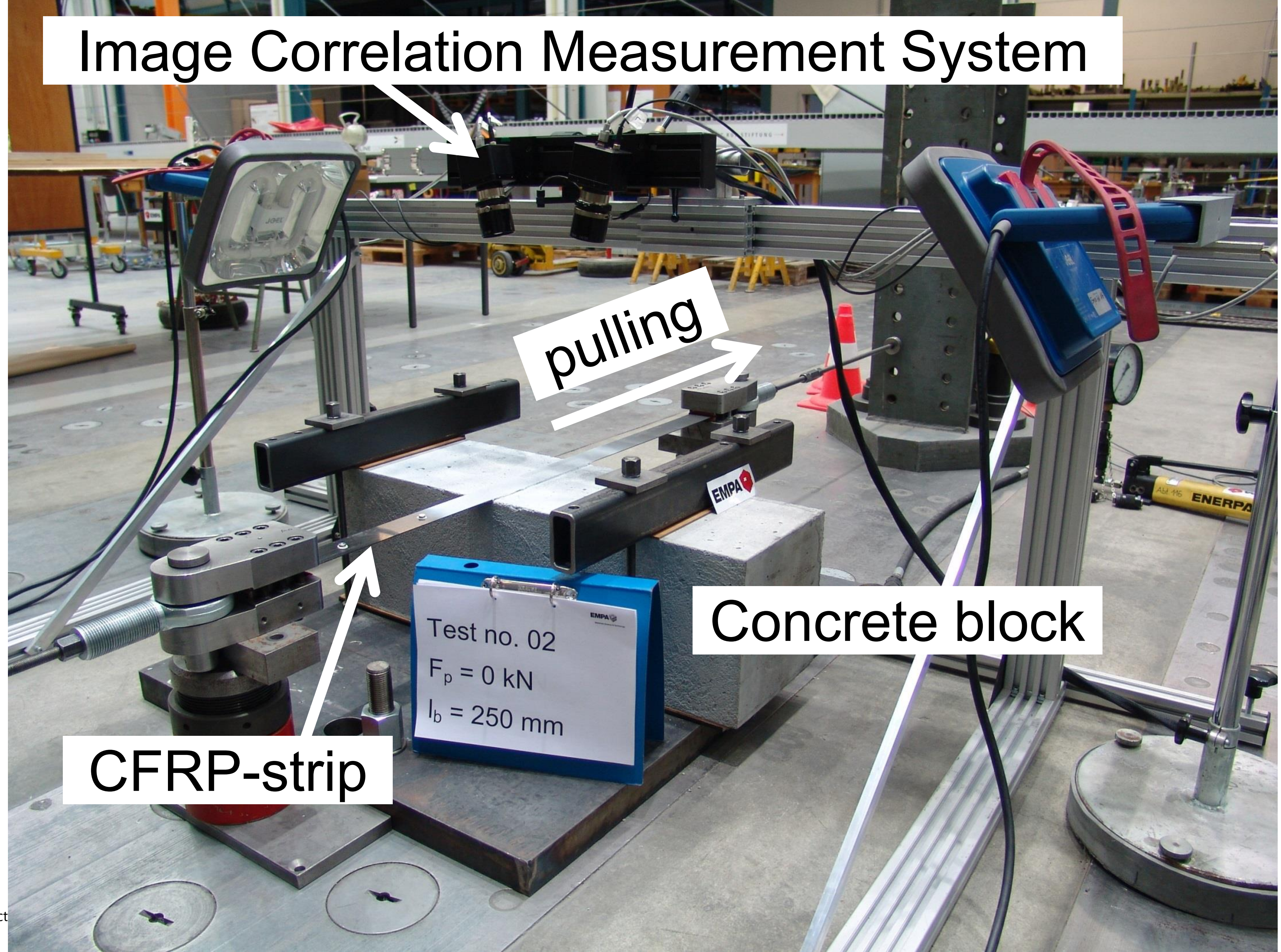

EMPA

Bond between CFRP strip and concrete

Lap-shear test



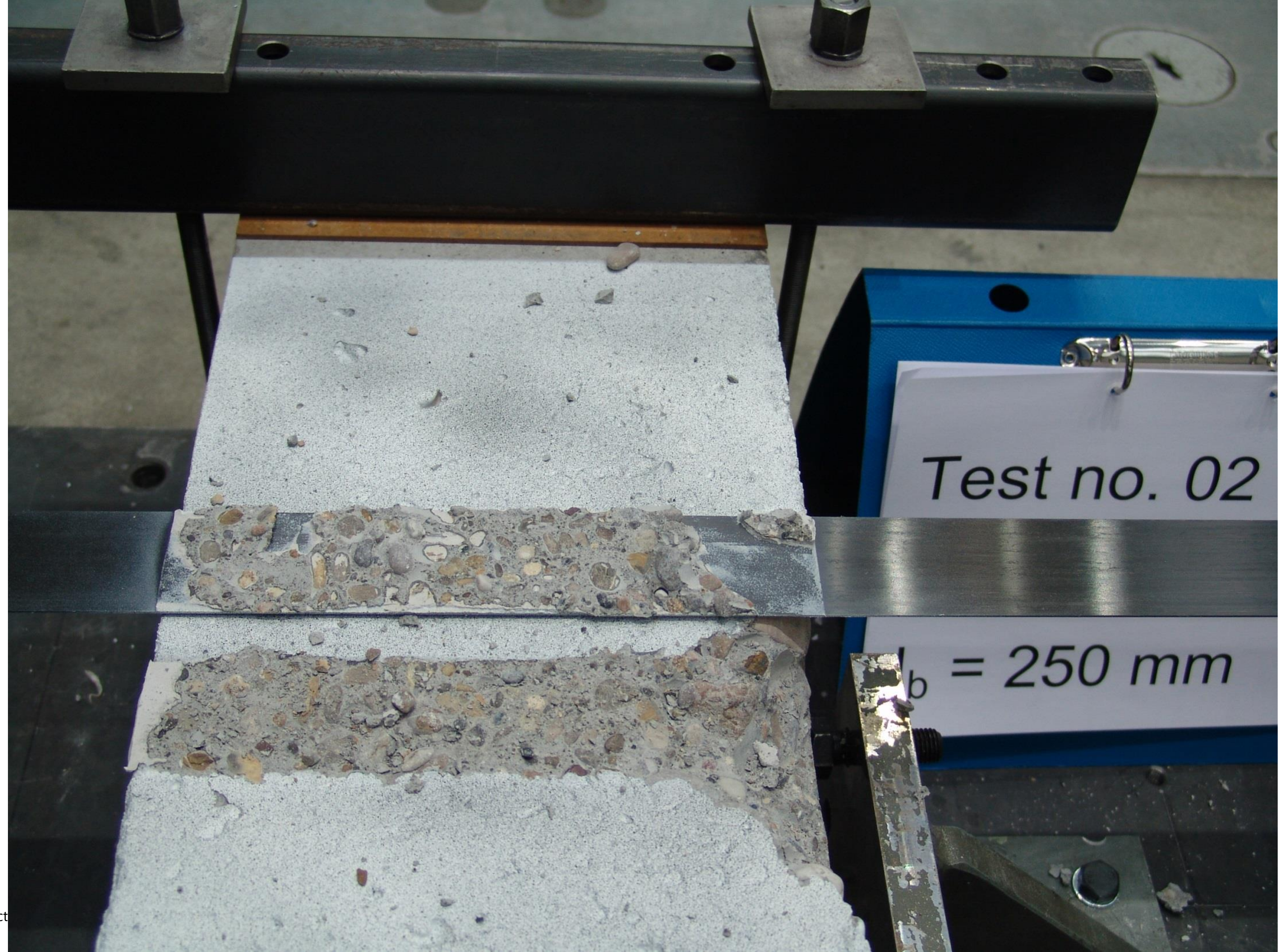
Empa
Materials Science and Technology

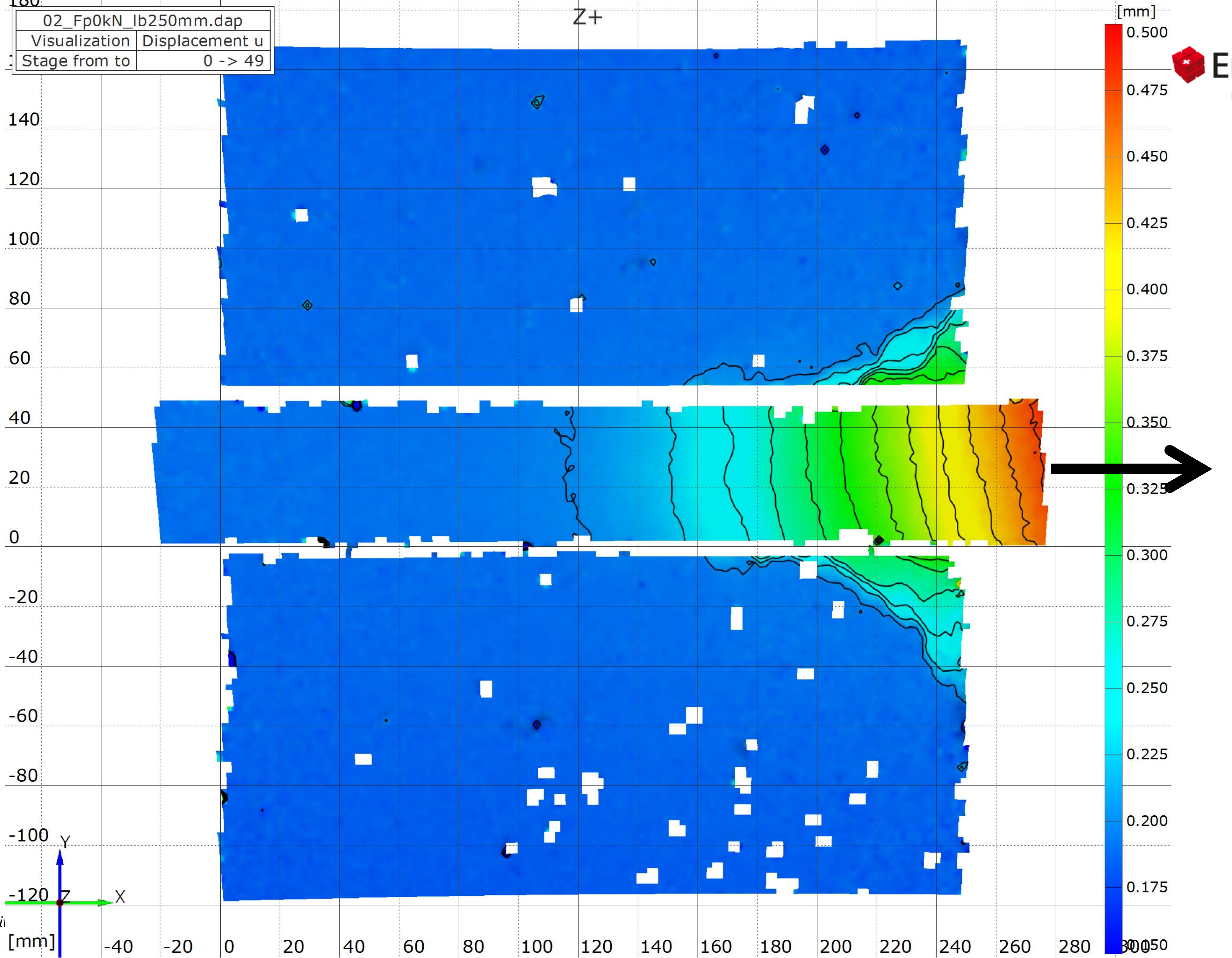


Empa
Materials Science and Technology

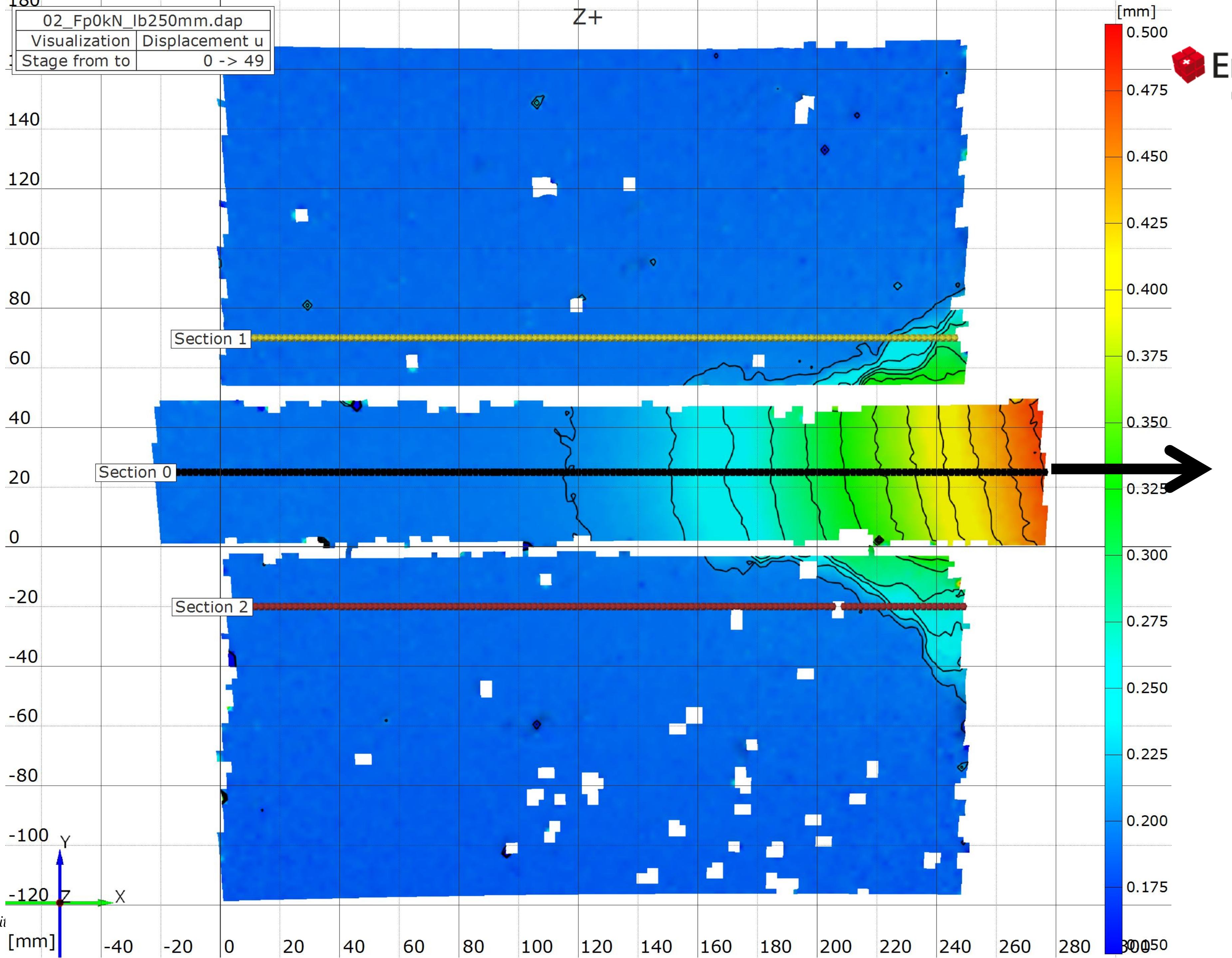
Empa
Materials Science and Technology

Empa
Materials Science and Technology

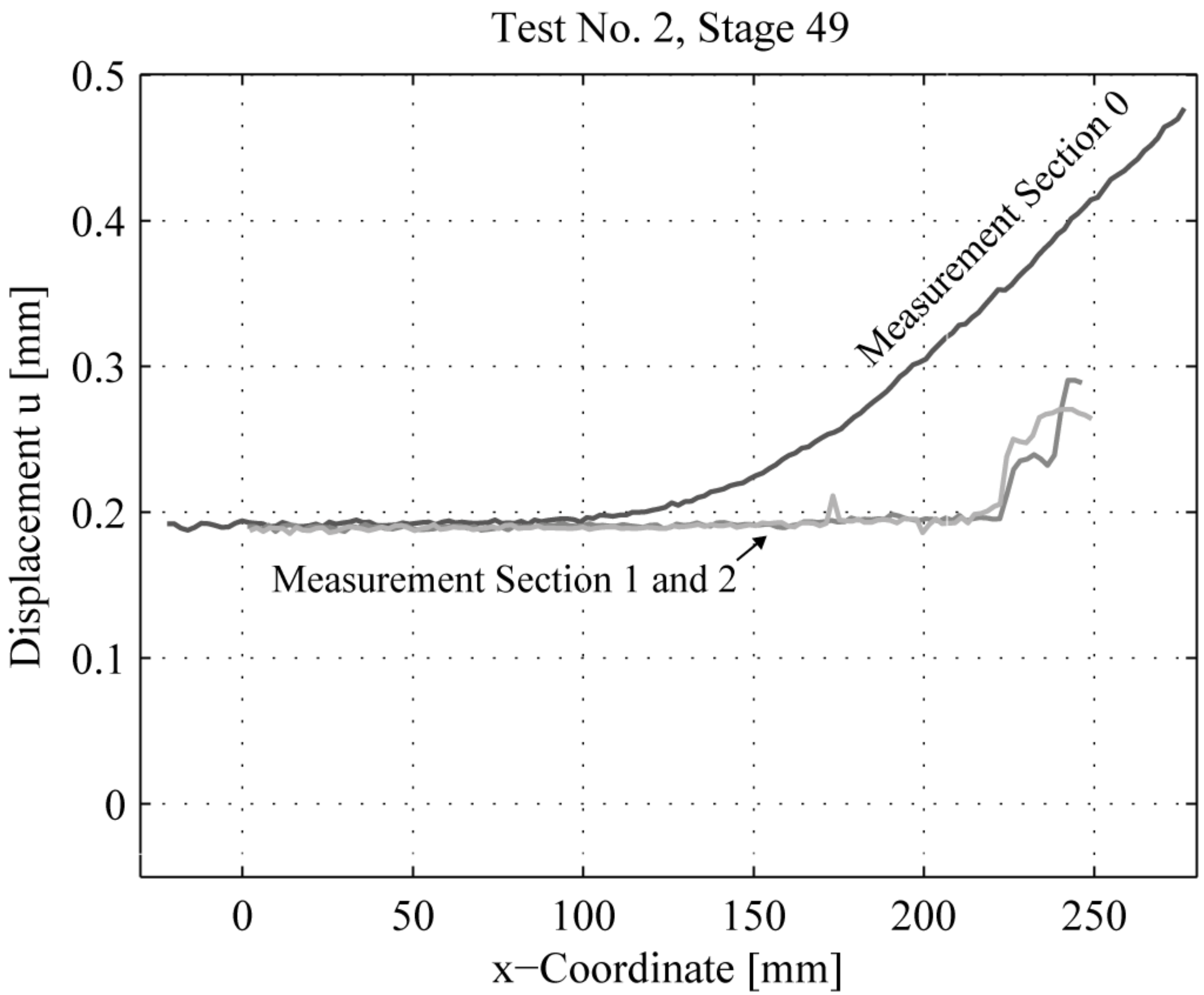




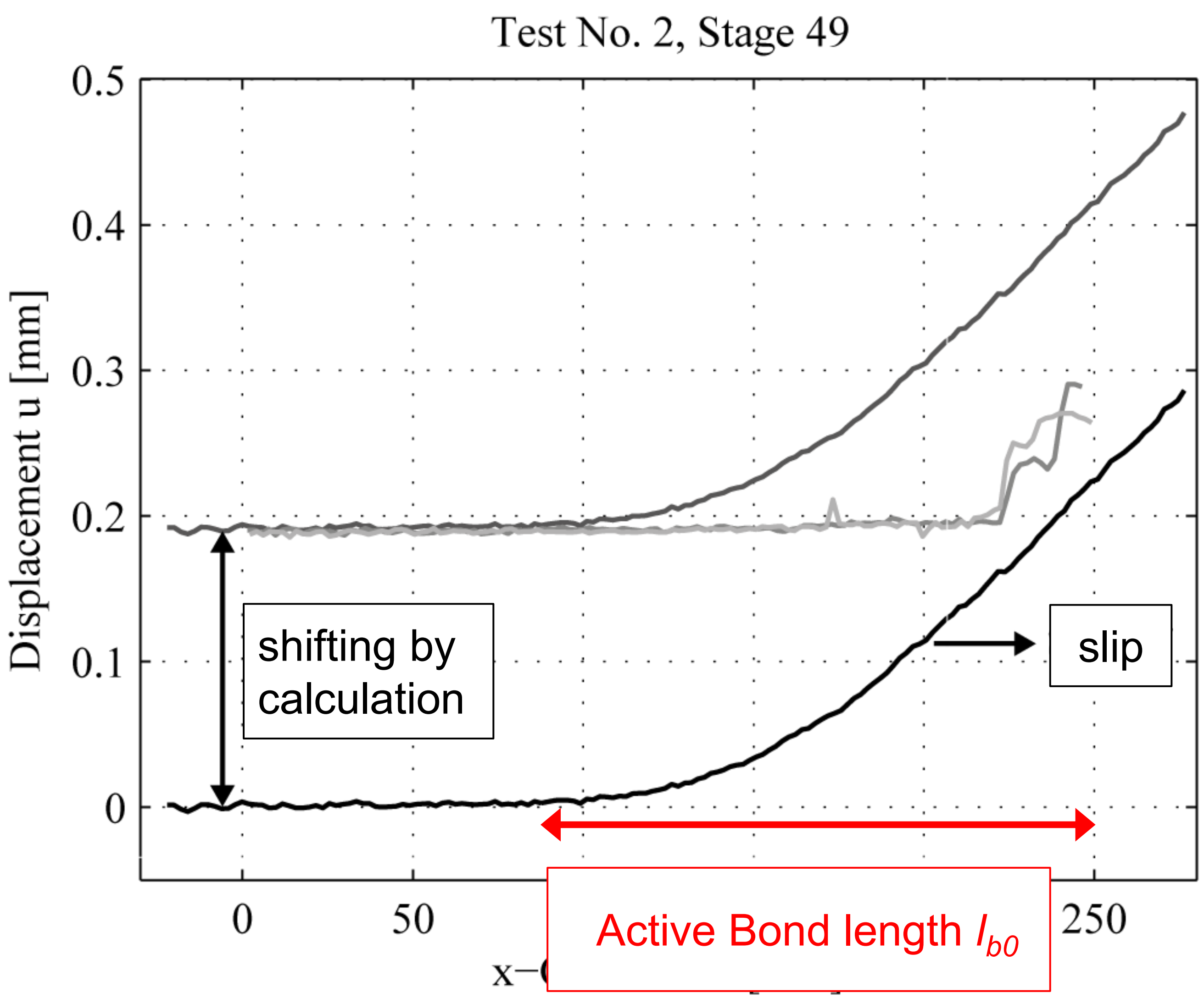
Empa
Materials Science and Technology



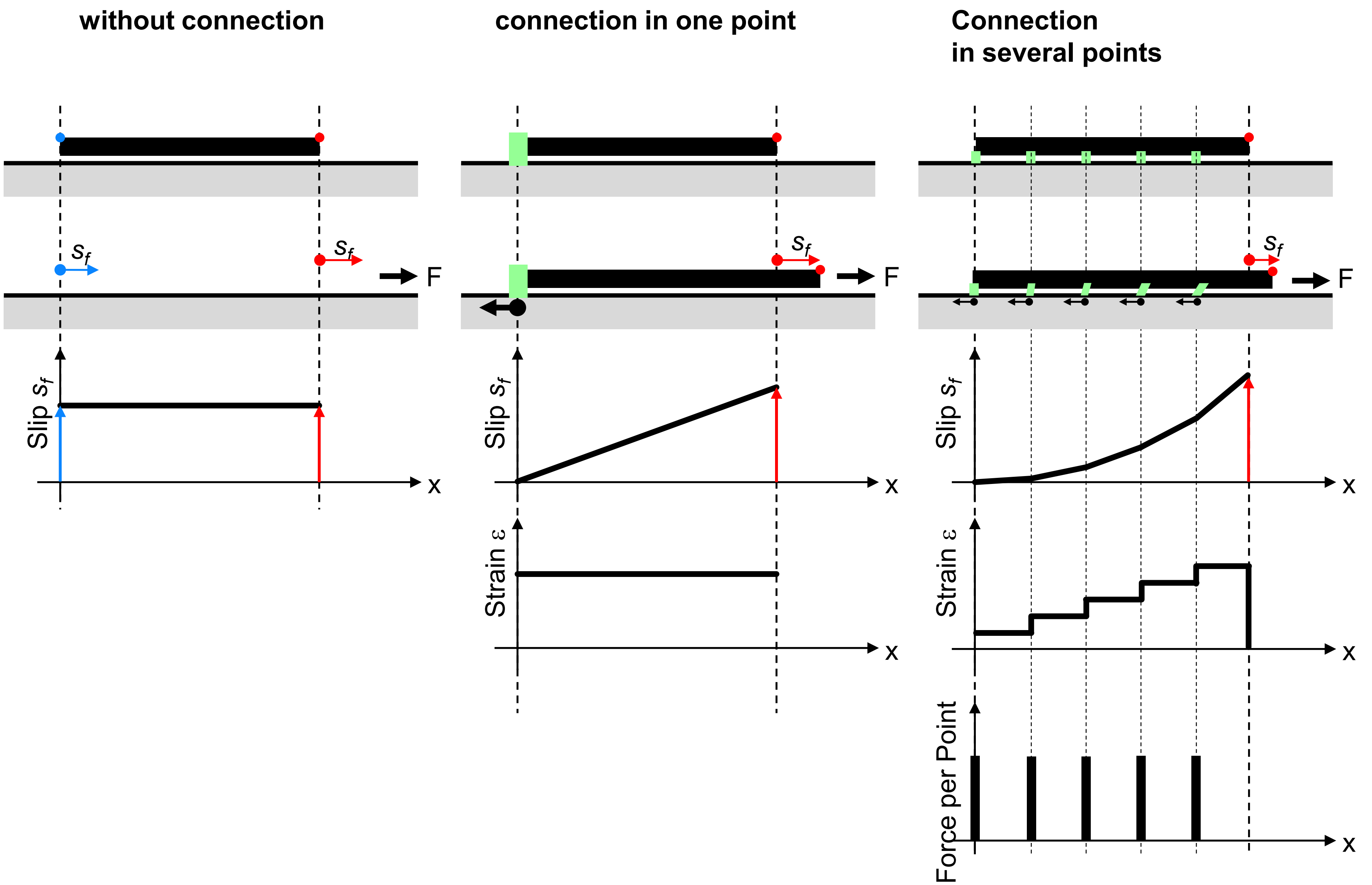
Empa
Materials Science and Technology



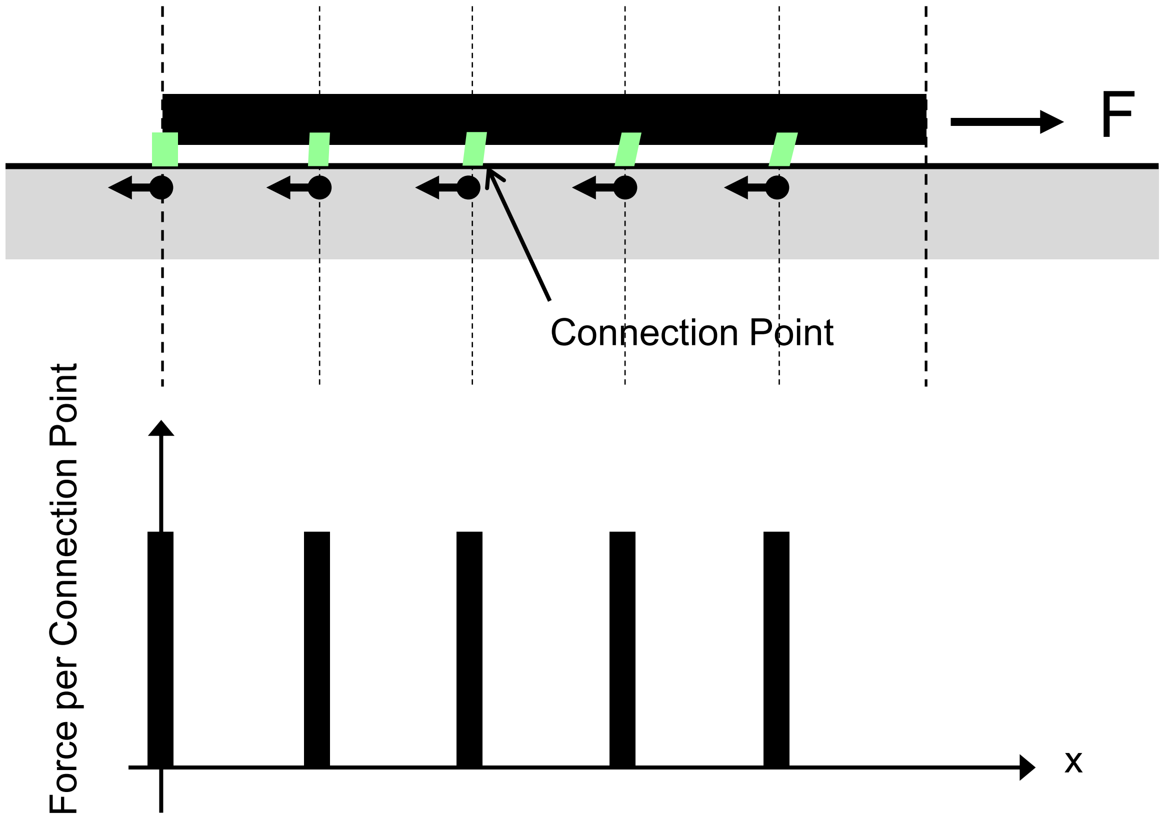
Test No. 2, Stage 49
Displacement u [mm]
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
x−Coordinate [mm]
Measurement Section 0
Measurement Section 1 and 2

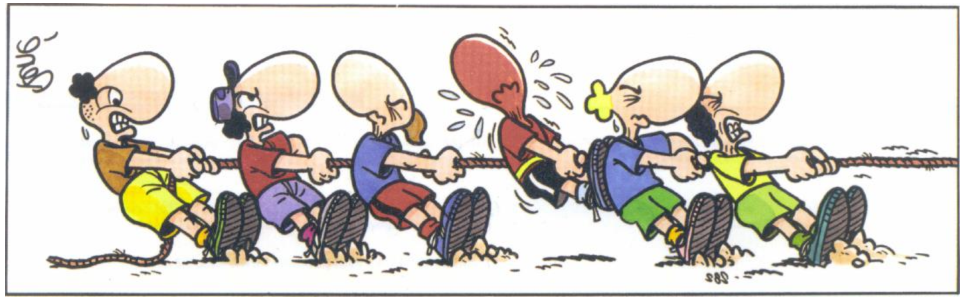


Active bond length: the length which is actively involved in the force transfer from the strip to the concrete.



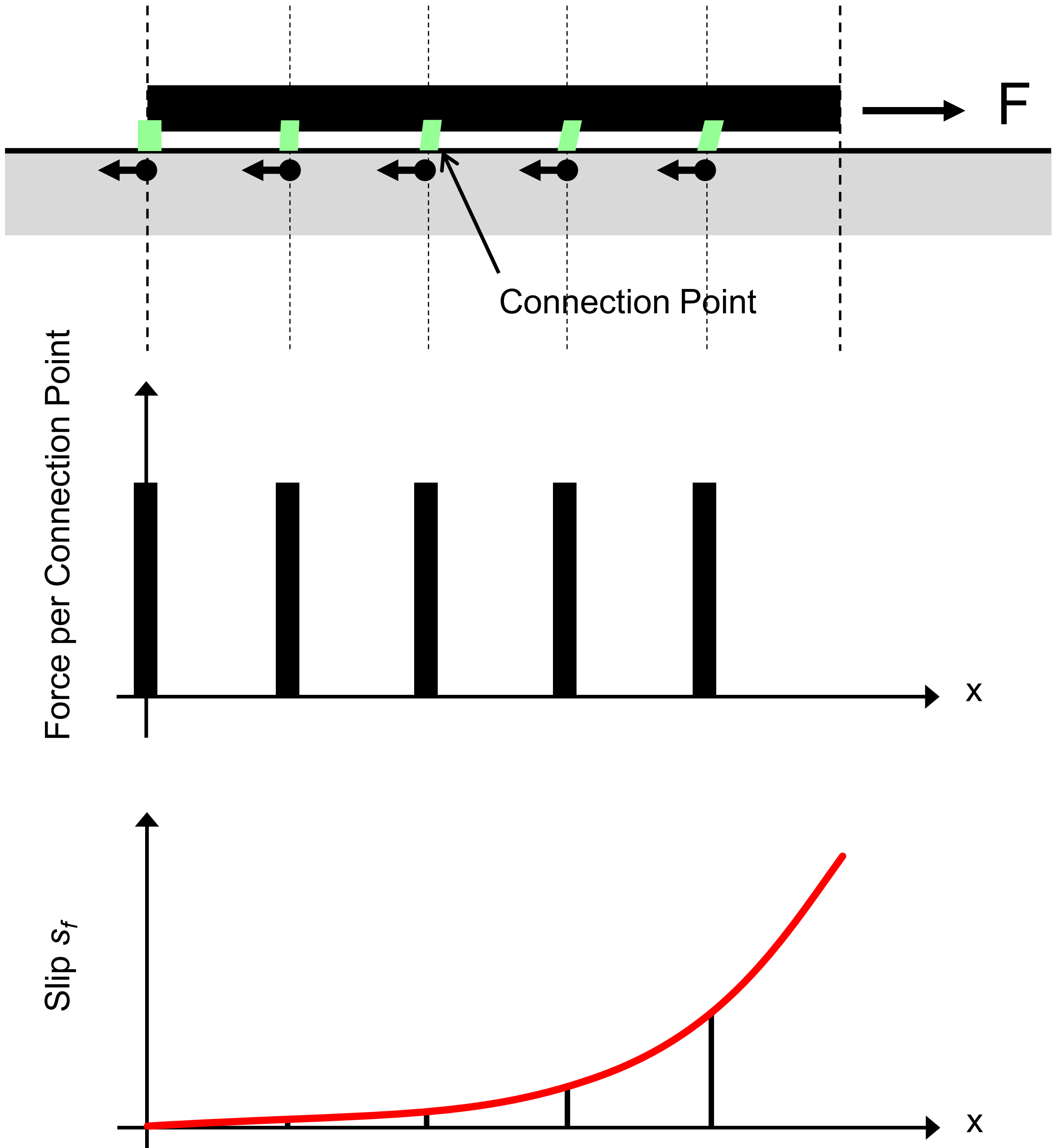
without connection
connection in one point
Connection
in several points
s_f
F
Slip s_f
x
Strain ε
Force per Point



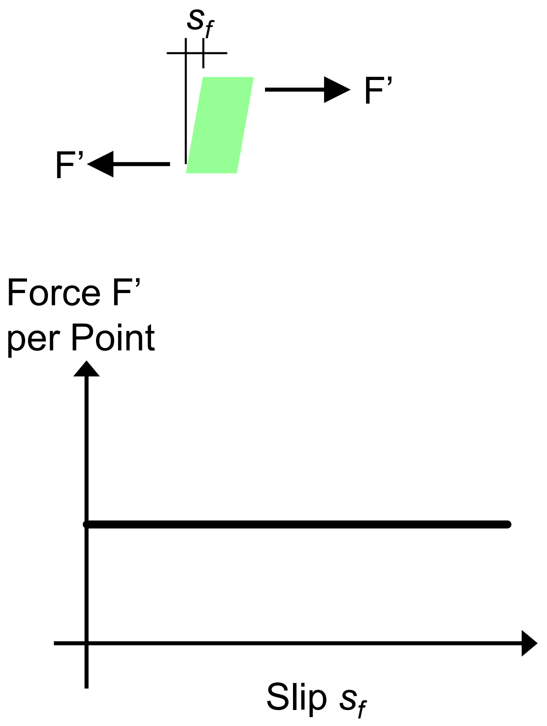


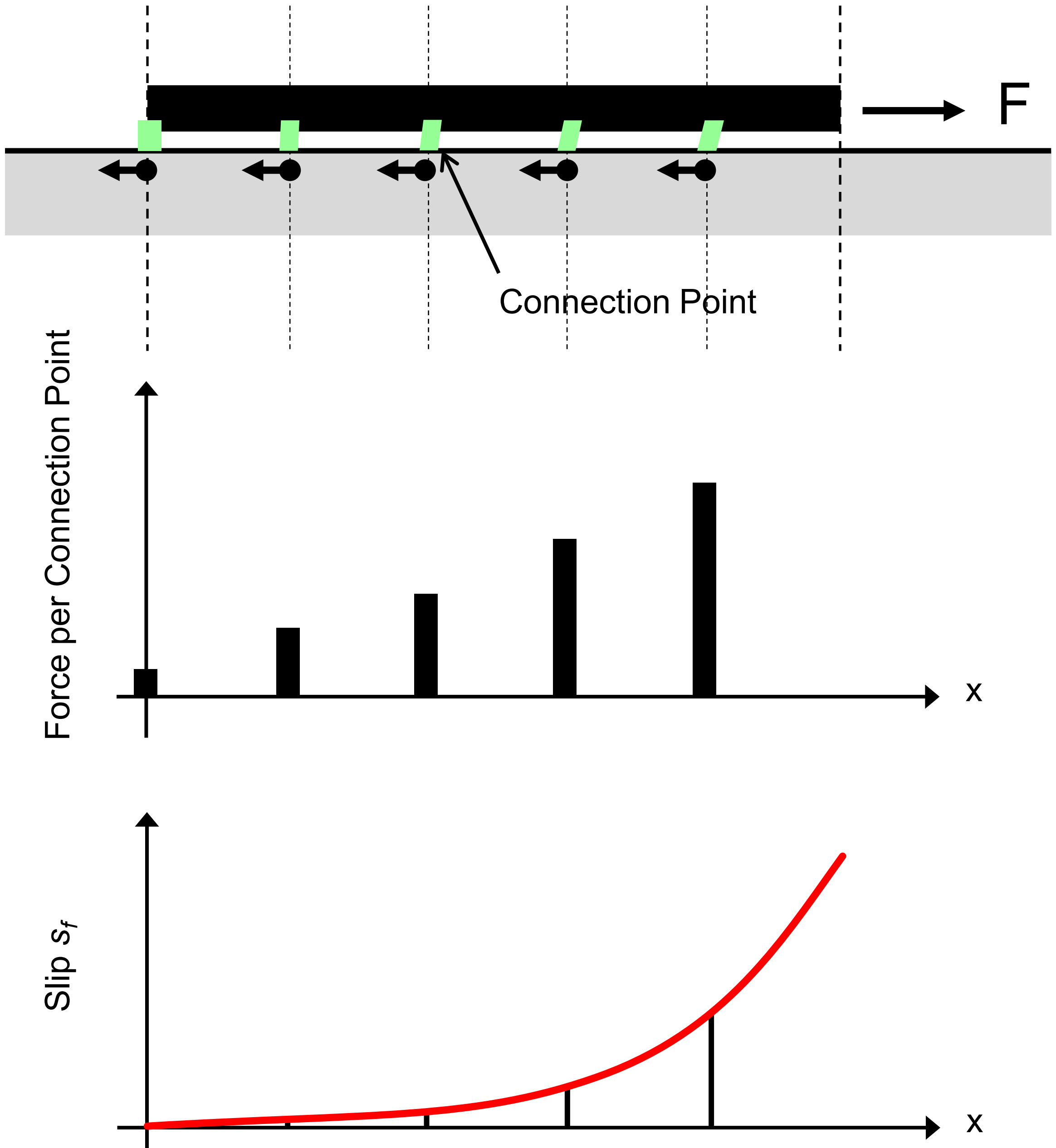
rope pulling

Picture taken from http://www.seilziehclub-gonten.ch/images/diverse/Comic1.JPG (displayed mirrored)

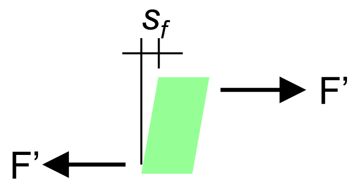


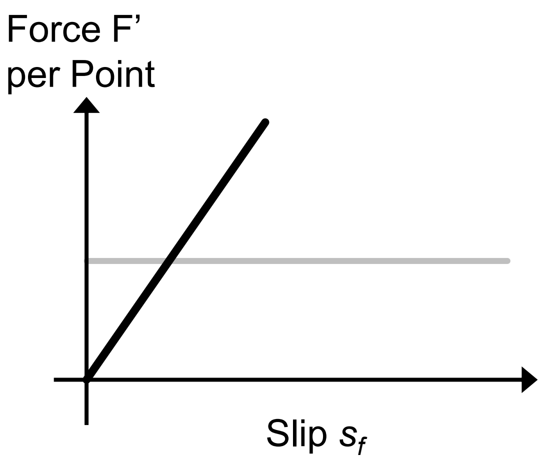
Behavior of the Connection Point:

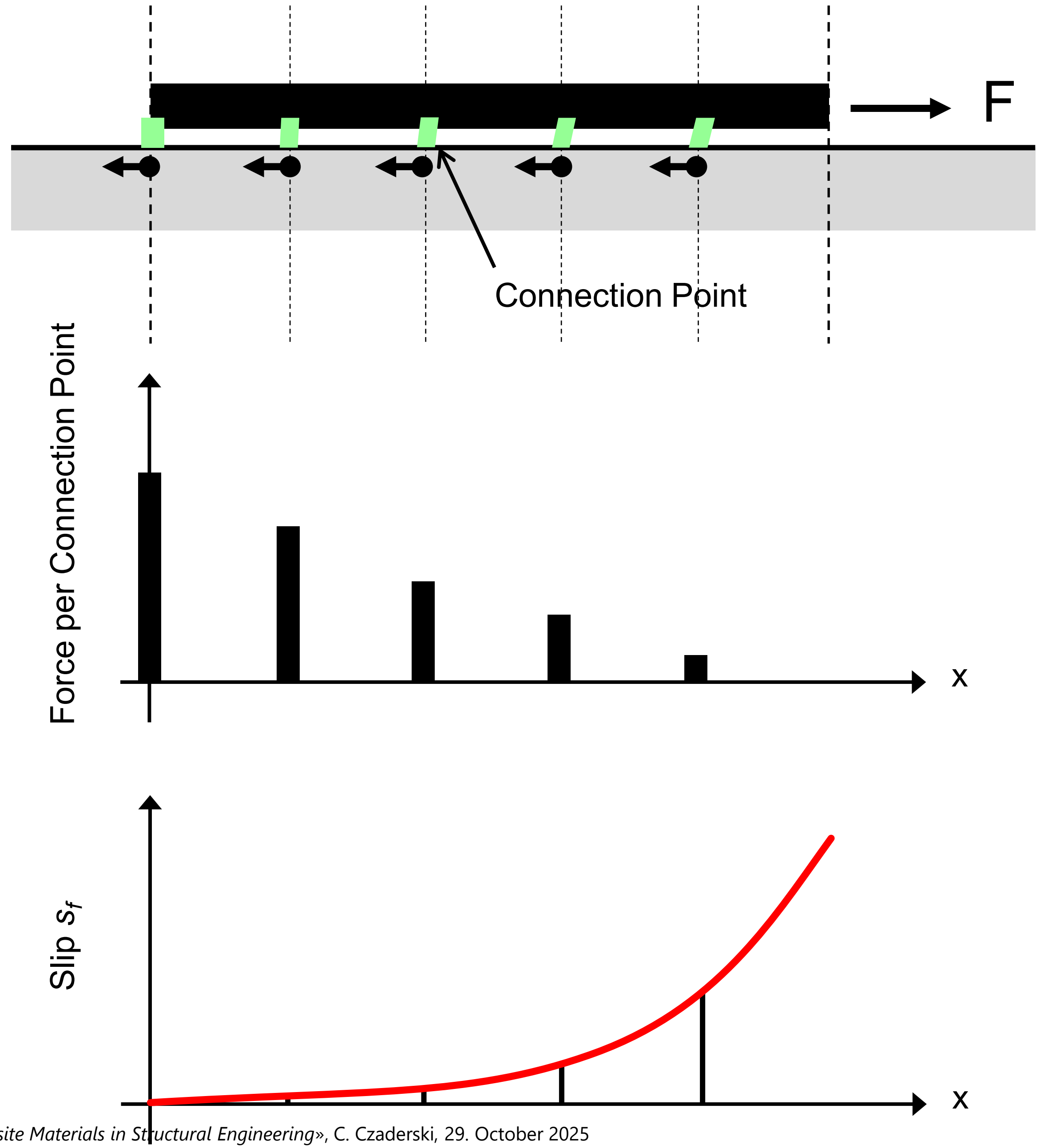




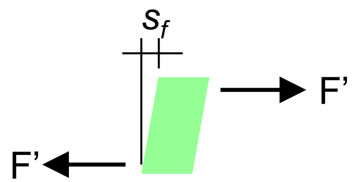
Behavior of the Connection Point:

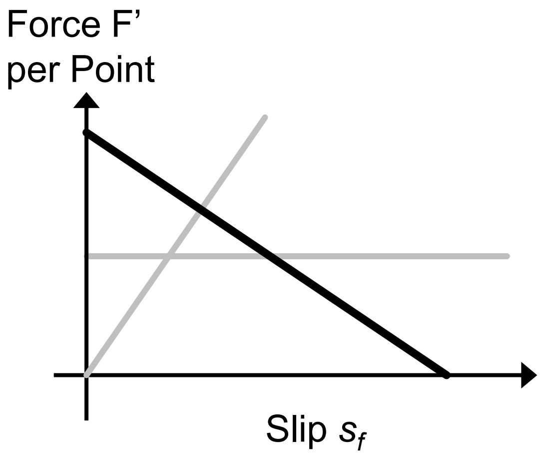


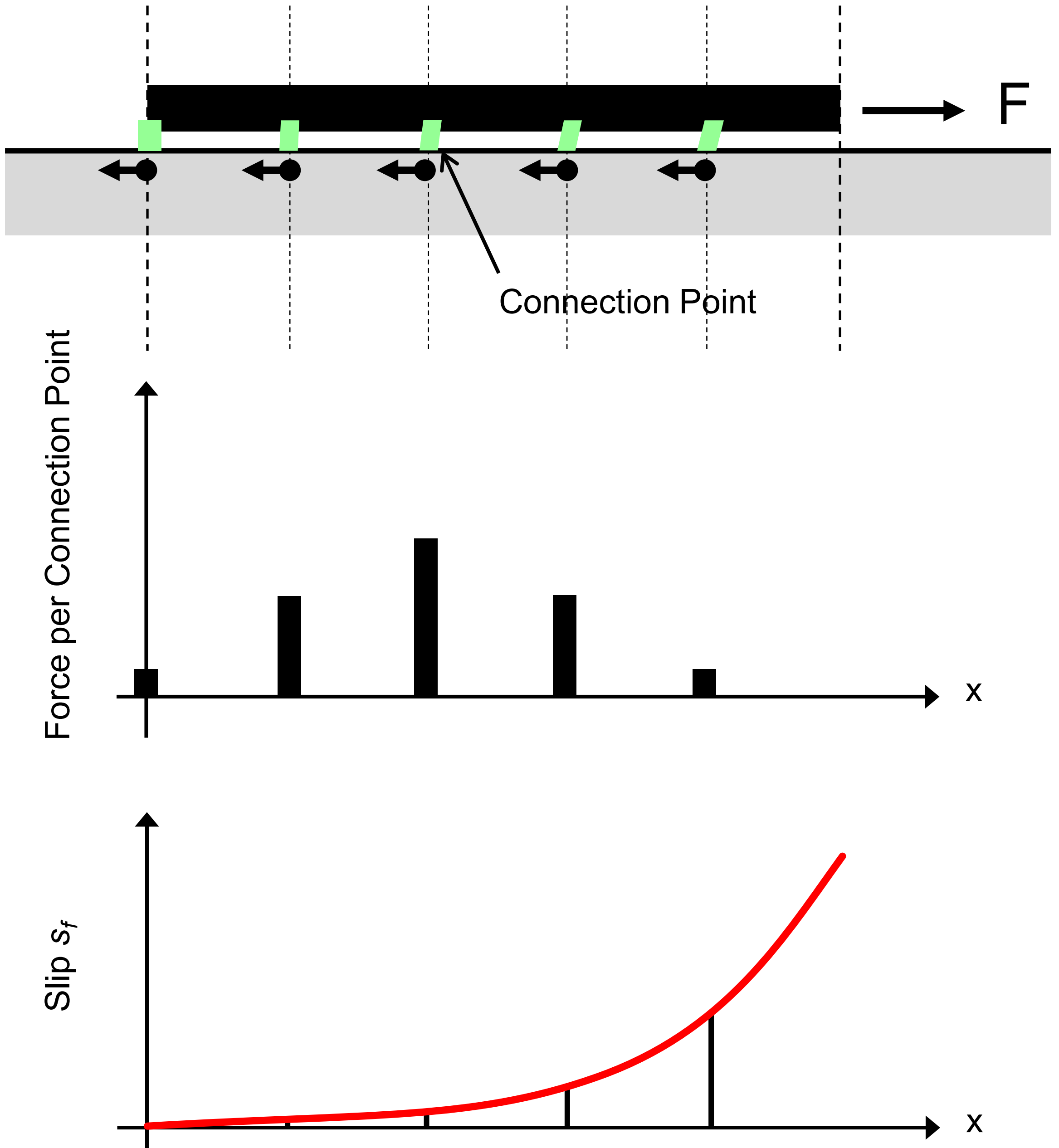




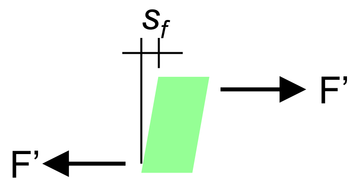
Behavior of the Connection Point:

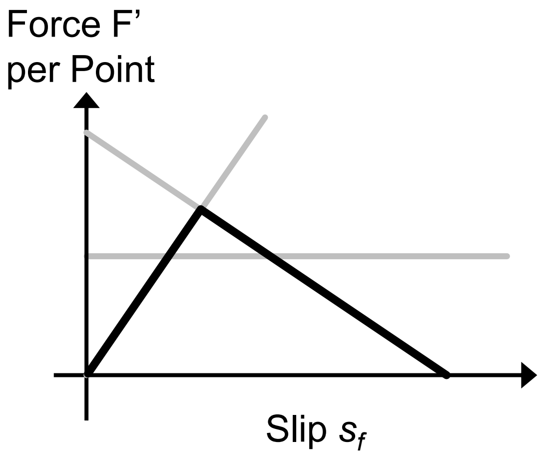


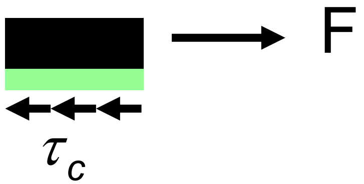




Behavior of the Connection Point:

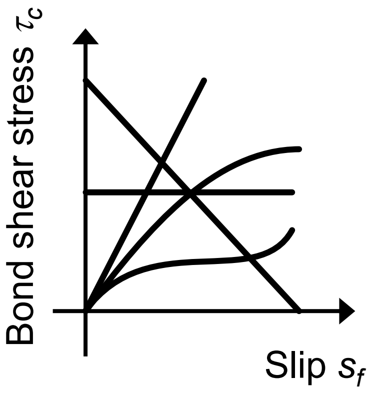






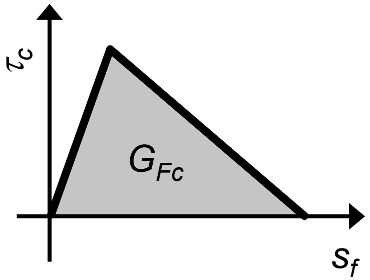
$$\tau_c = \frac{Force}{unit\ area}\ \frac{N}{mm2}$$

$\tau_c =$ bond shear stress



Bond shear stress – Slip – Relation

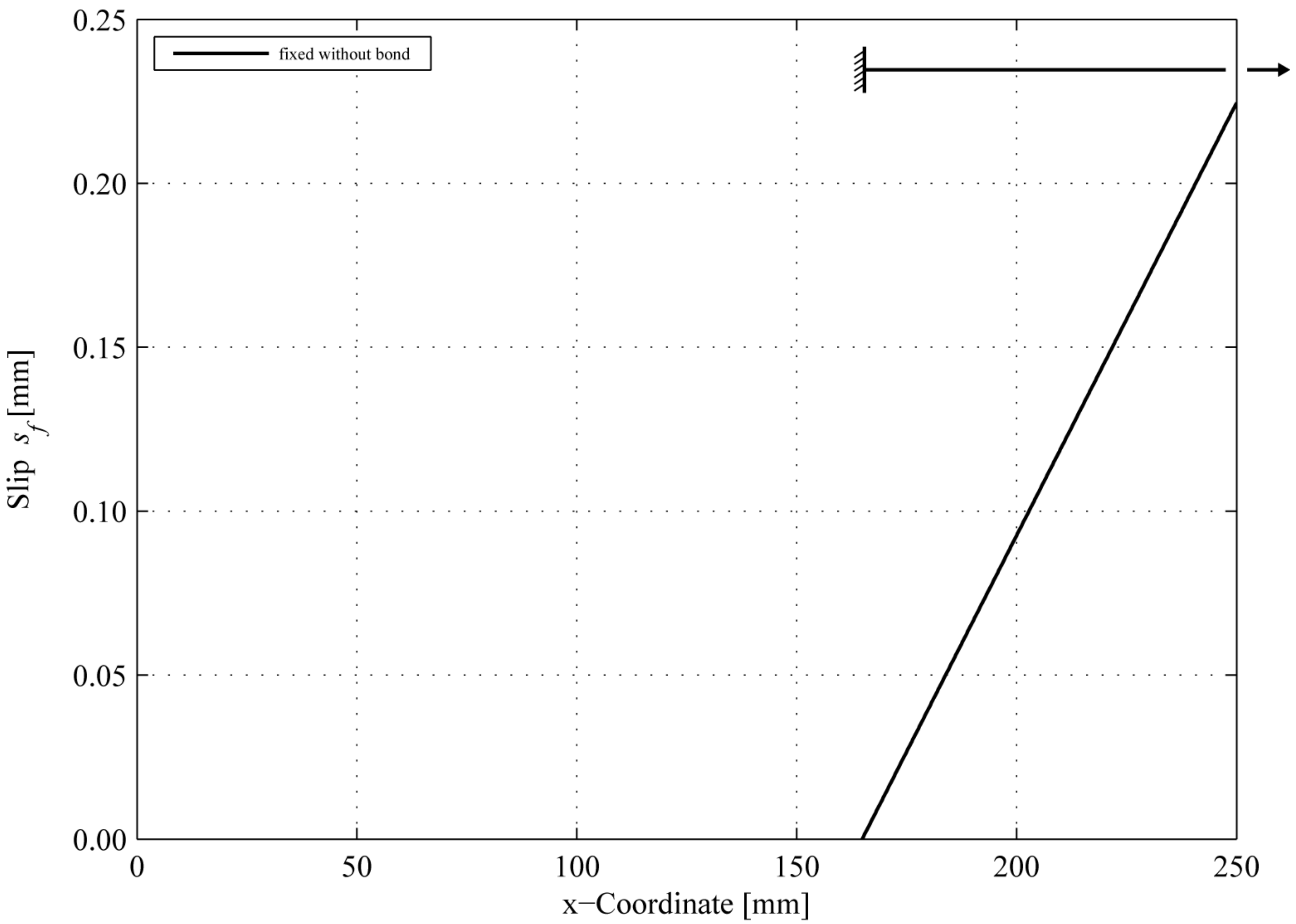
This relation characterizes the bond behavior.

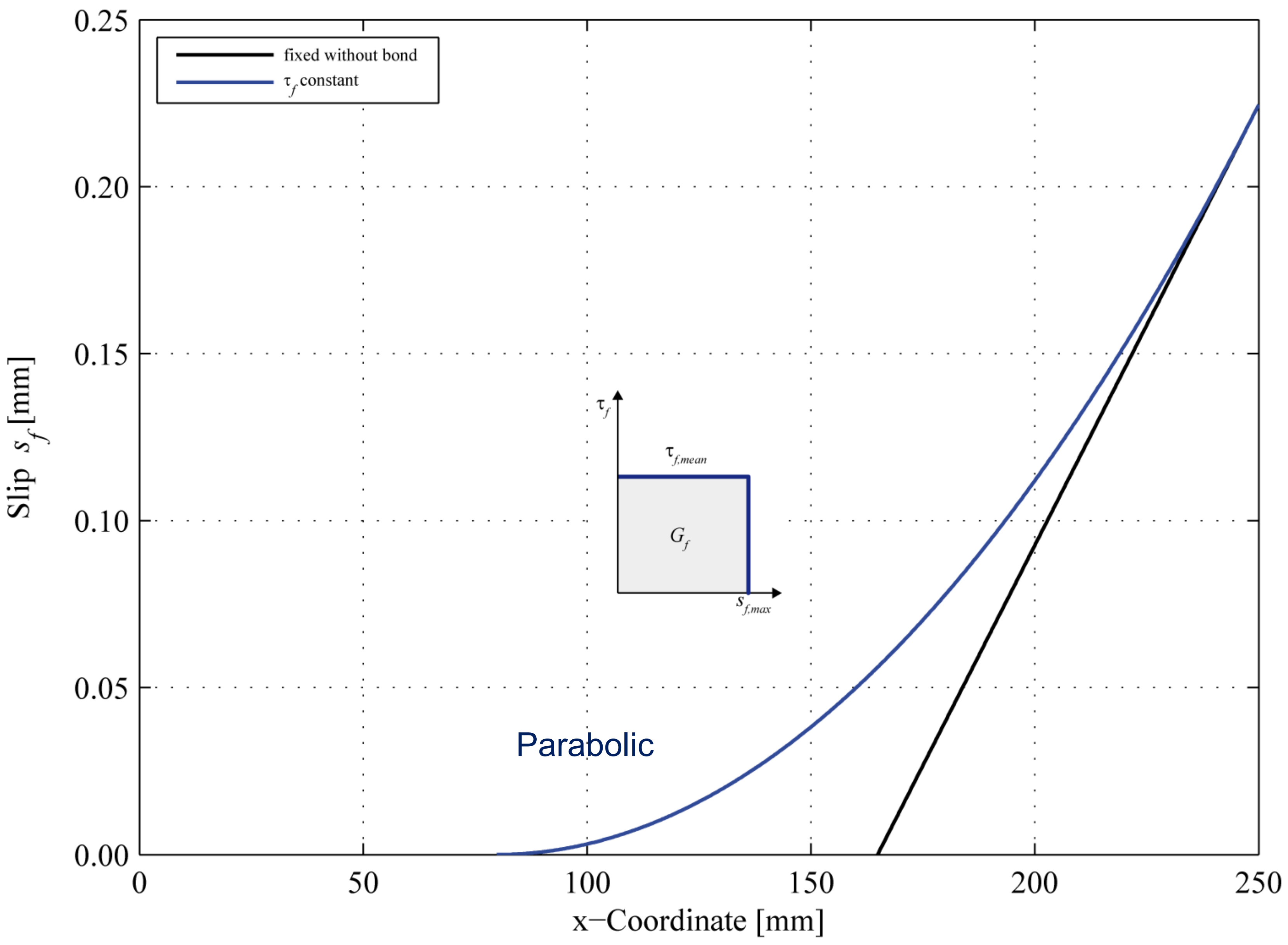


The area under the bond shear stress – slip – relation corresponds to the **specific fracture energy G_{Fc}** of externally bonded reinforcement (EBR) bonded **on concrete**.

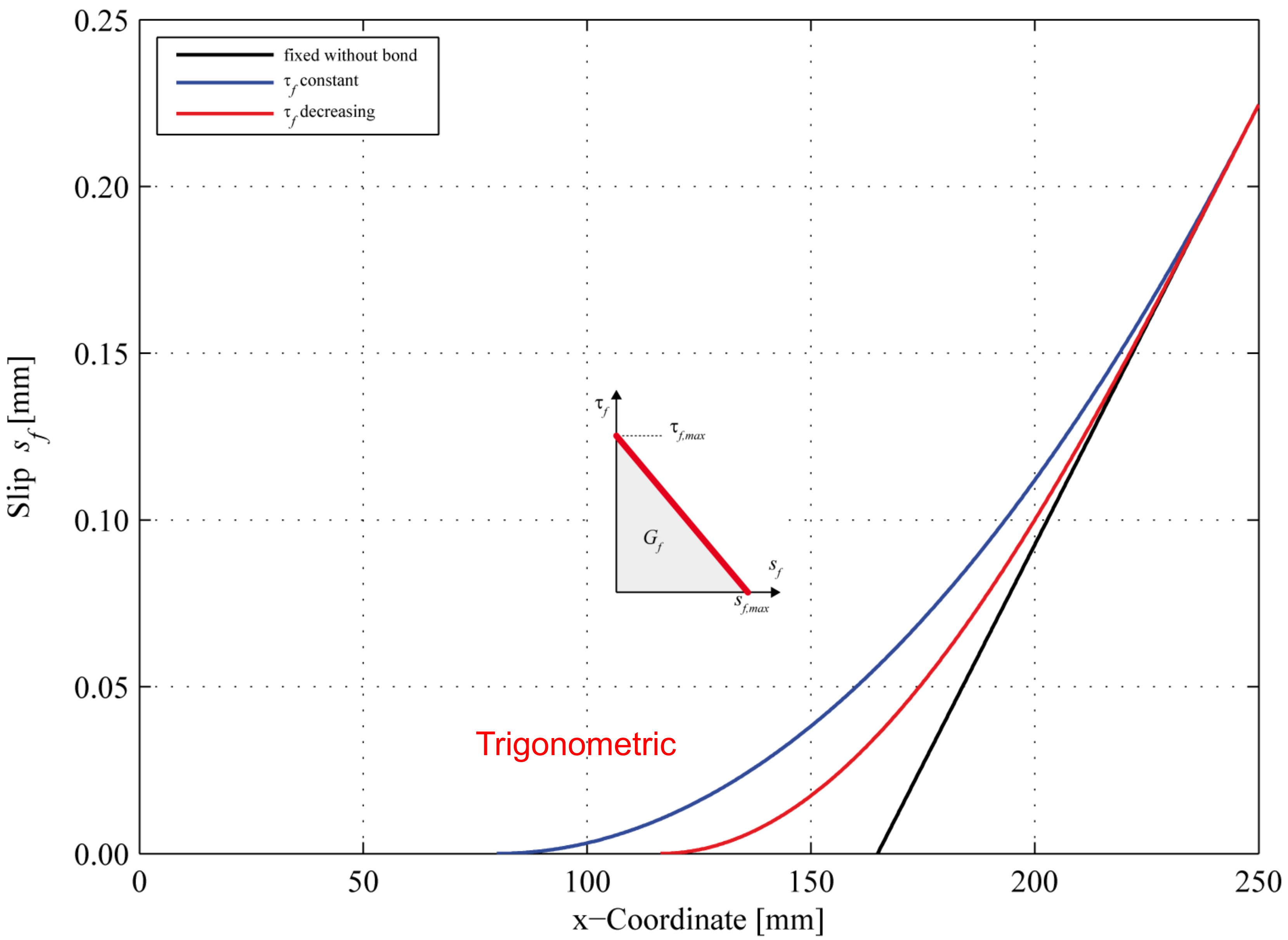
The specific fracture energy G_{Fc} corresponds to the work that must be expended to create a crack surface of 1mm^2.

G_{Fa} = specific fracture energy of EBR bonded on steel

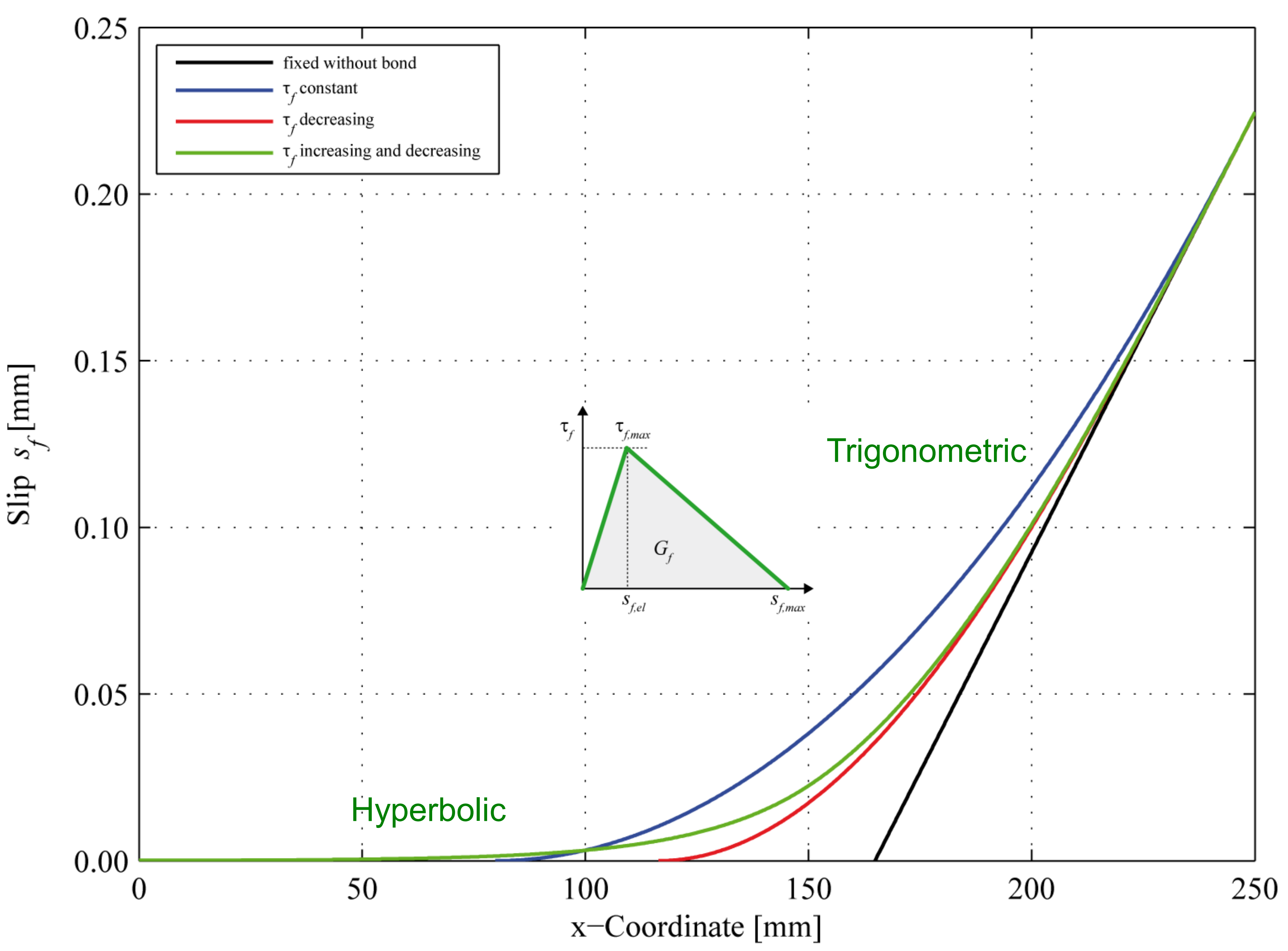

fixed without bond
Slip s_f [mm]
x−Coordinate [mm]
0.00
0.05
0.10
0.15
0.20
0.25
0
50
100
150
200
250



Empa
Materials Science and Technology

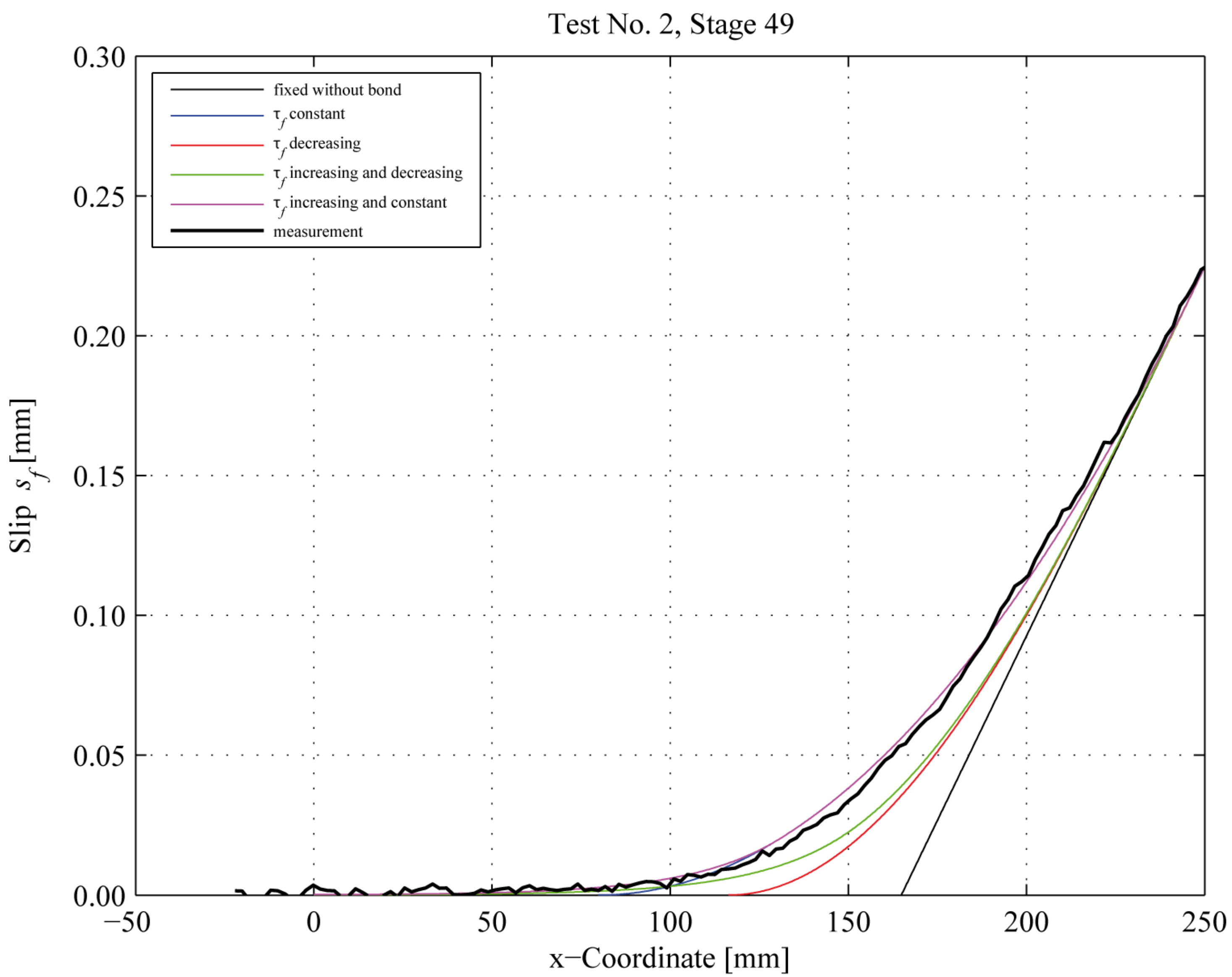


fixed without bond
τ_f constant
τ_f decreasing
τ_f
$\tau_{f,max}$
G_f
s_f
$s_{f,max}$
Trigonometric
Slip s_f [mm]
x−Coordinate [mm]
0.25
0.20
0.15
0.10
0.05
0.00
0
50
100
150
200
250

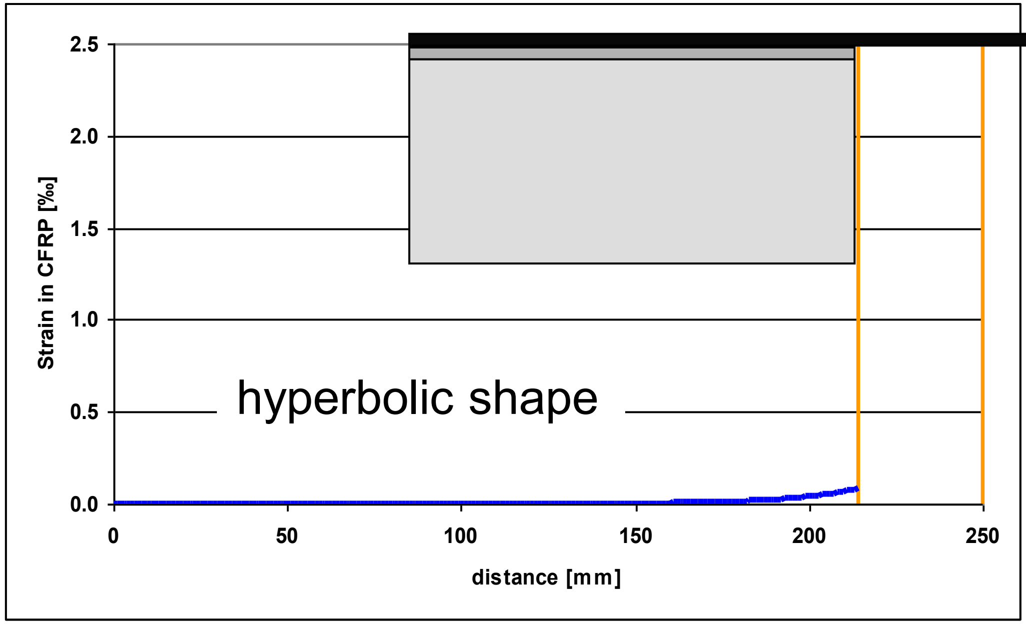


fixed without bond
τ_f constant
τ_f decreasing
τ_f increasing and decreasing
Trigonometric
Hyperbolic
τ_f
τ_f,max
G_f
s_f,el
s_f,max
0.25
0.20
0.15
0.10
0.05
0.00
Slip s_f [mm]
0
50
100
150
200
250
x−Coordinate [mm]

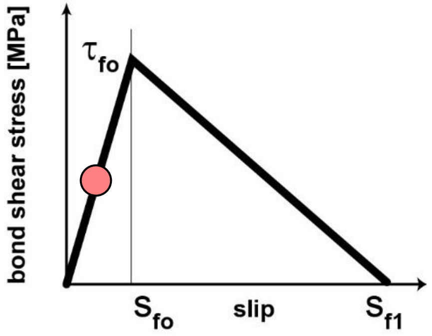
Comparison Modelling - Measurement

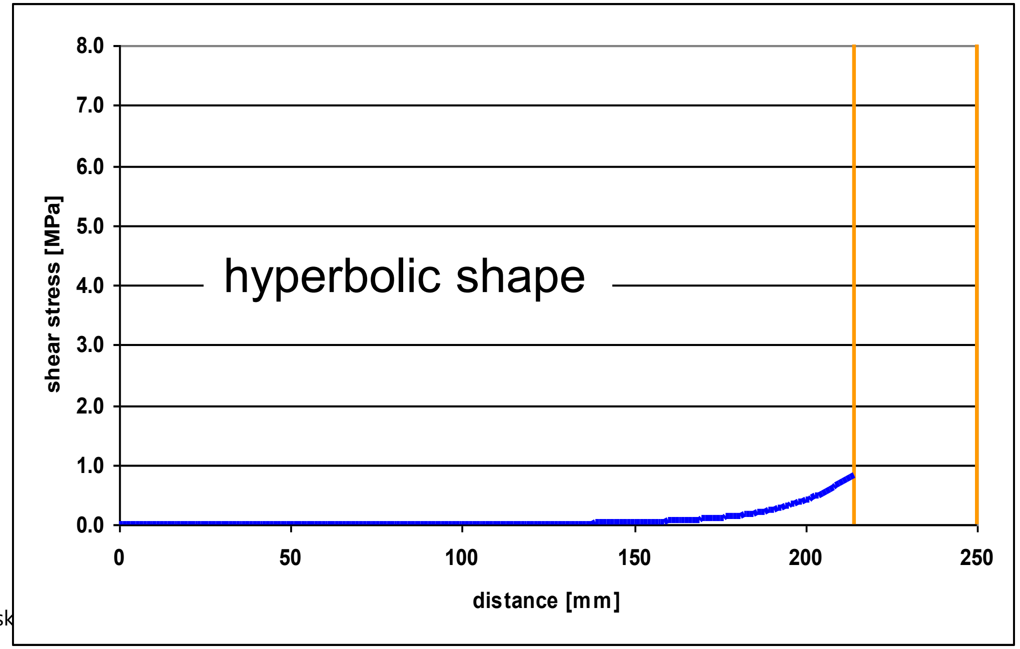


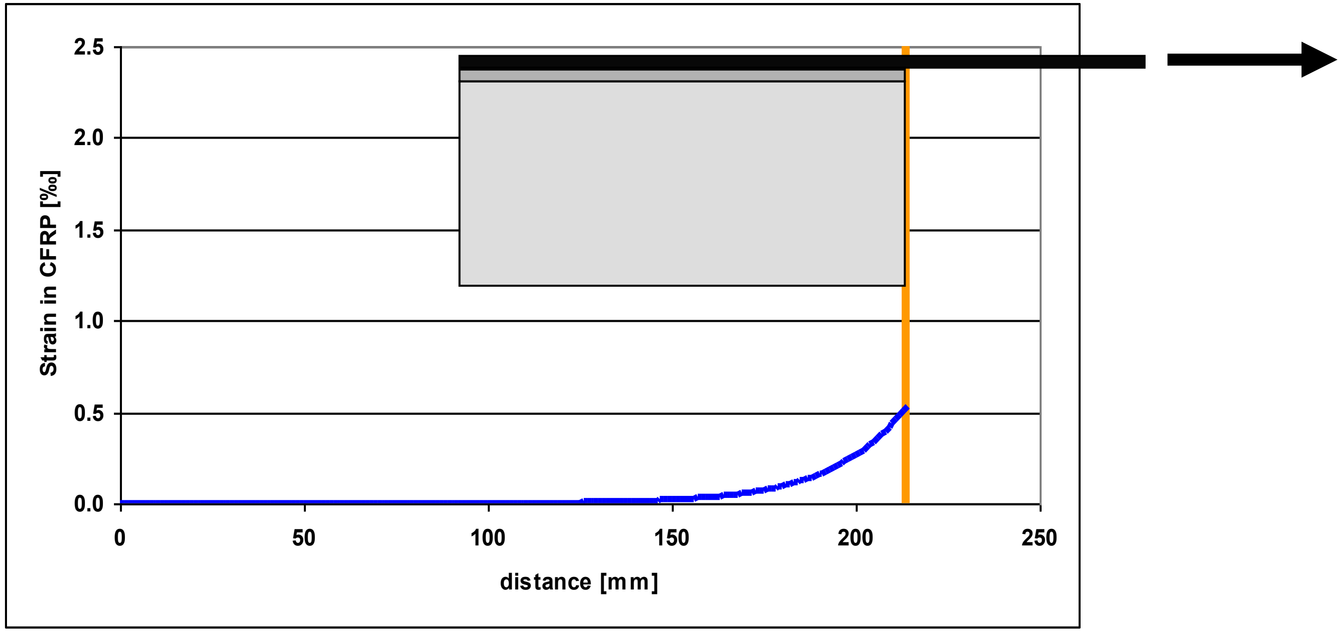
Longitudinal strain

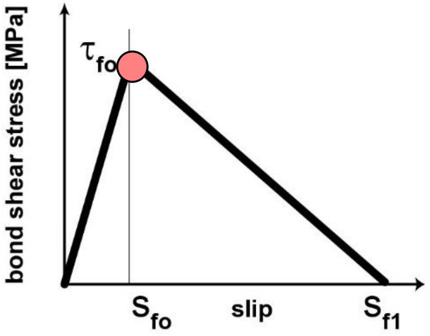


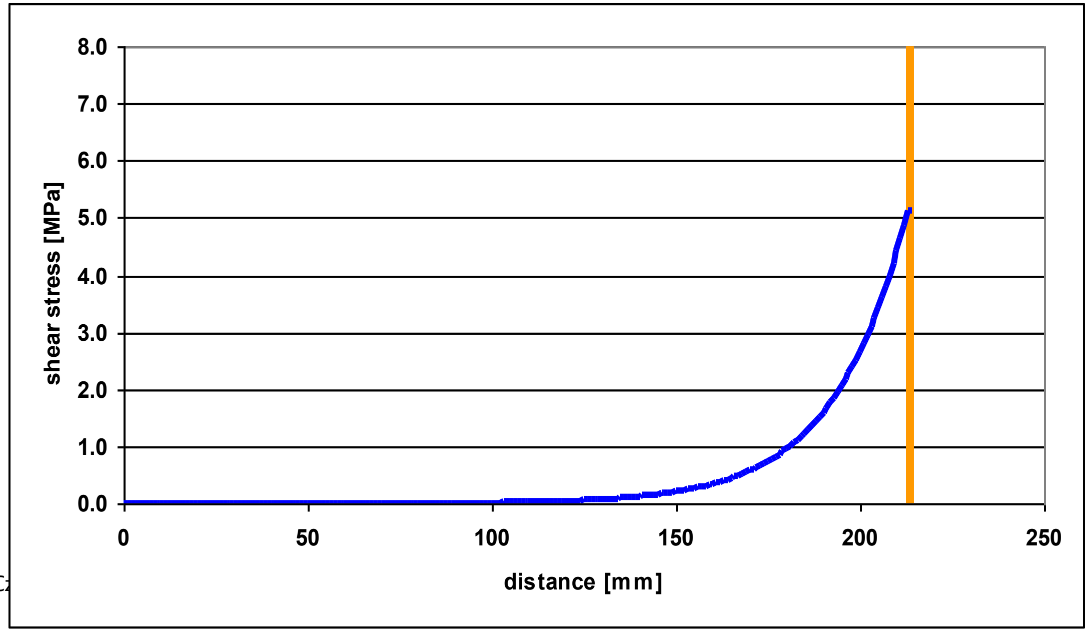
Shear stress between strip and concrete

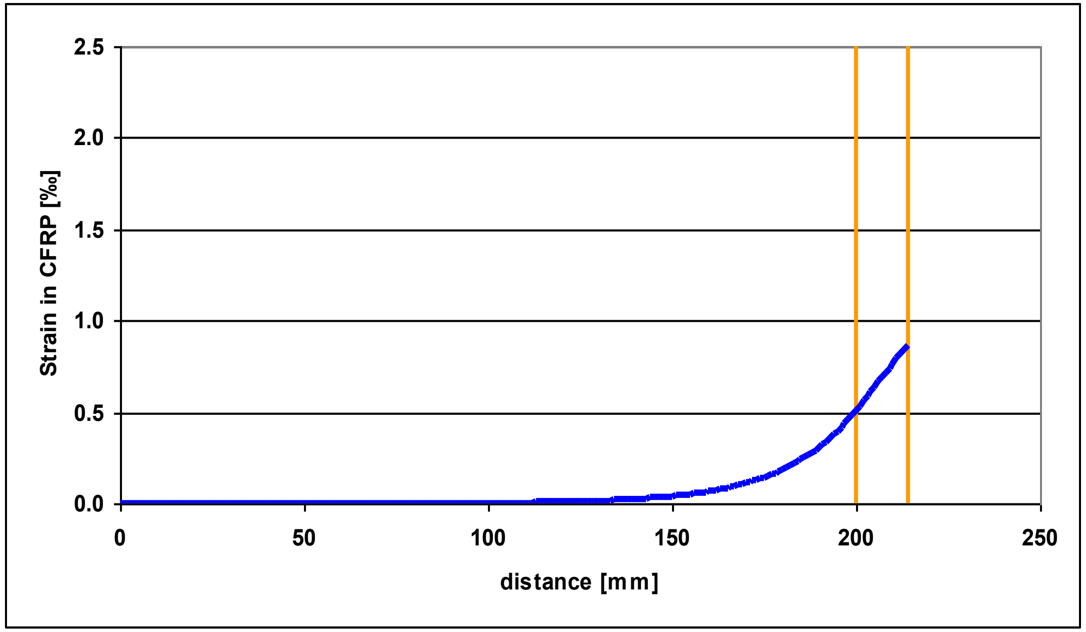




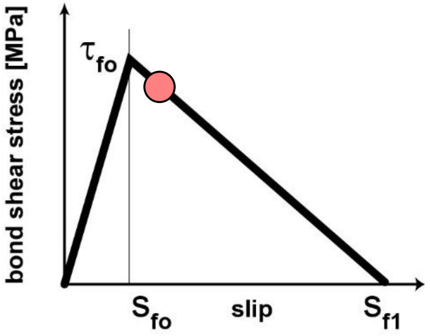

2.5
2.0
1.5
1.0
0.5
0.0
Strain in CFRP [‰]
0
50
100
150
200
250
distance [mm]


bond shear stress [MPa]
τ_{fo}
S_{fo}
slip
S_{f1}

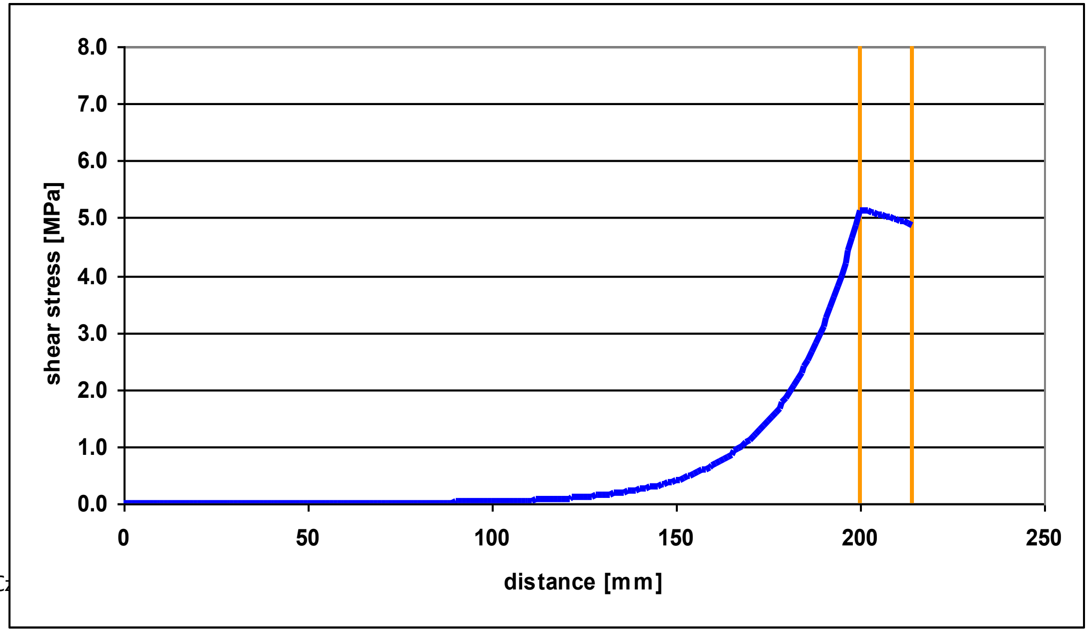

8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
shear stress [MPa]
0
50
100
150
200
250
distance [mm]



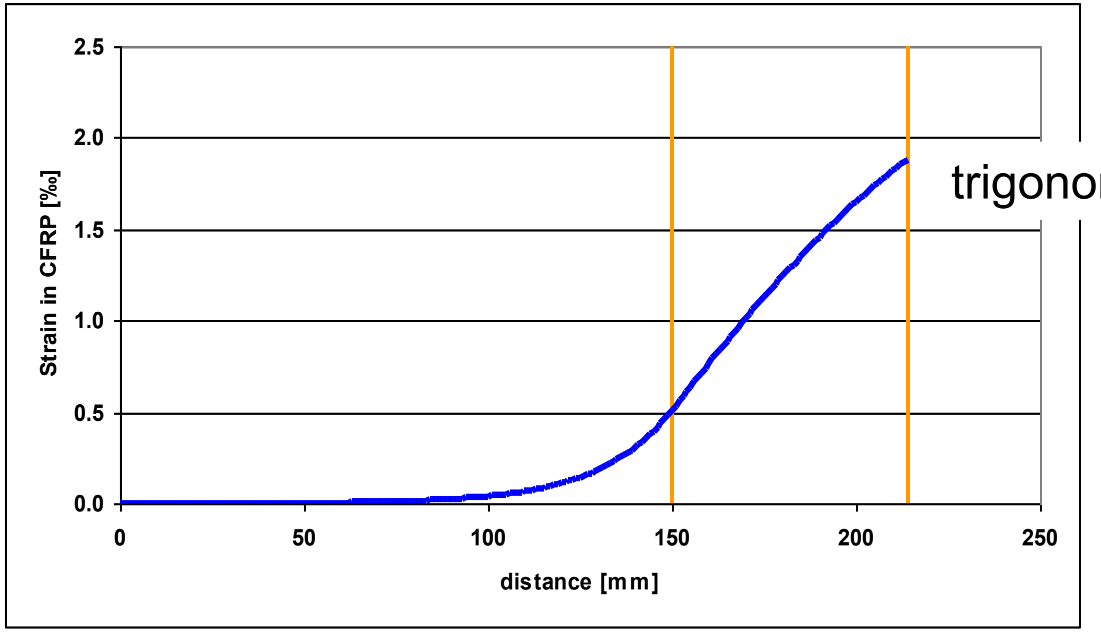
Strain in CFRP [‰]
2.5
2.0
1.5
1.0
0.5
0.0
0
50
100
150
200
250
distance [mm]



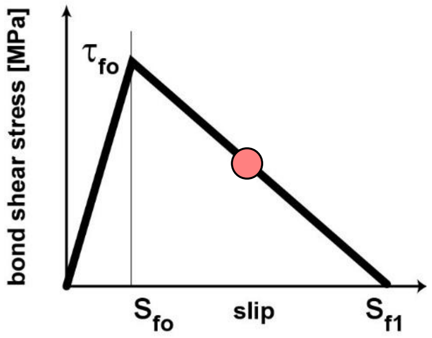
bond shear stress [MPa]
τ_{fo}
S_{fo}
slip
S_{f1}

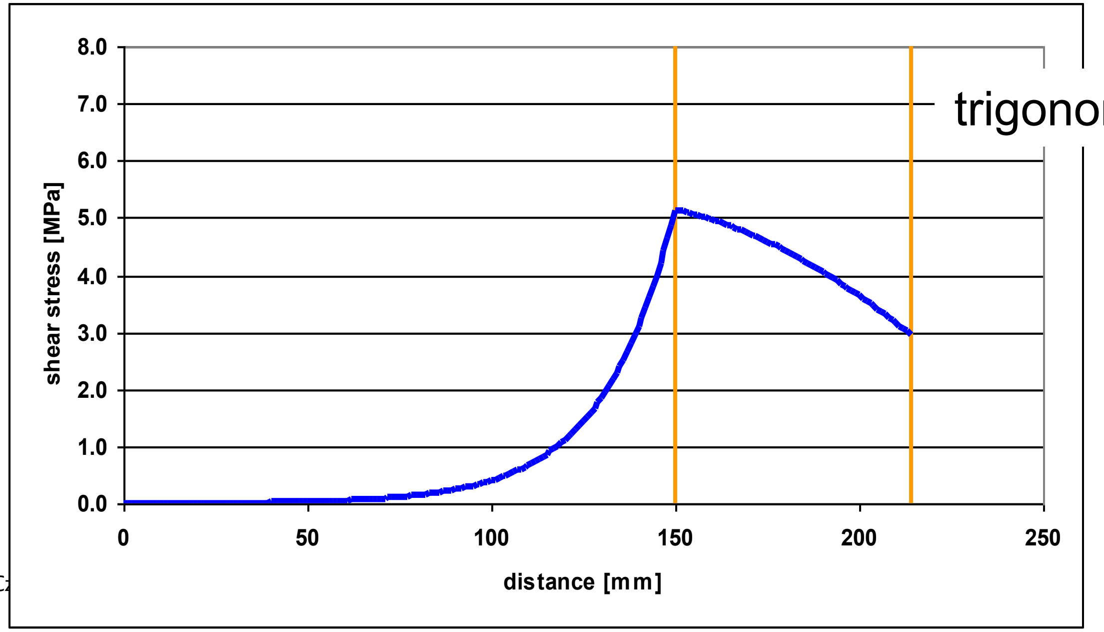


shear stress [MPa]
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
50
100
150
200
250
distance [mm]

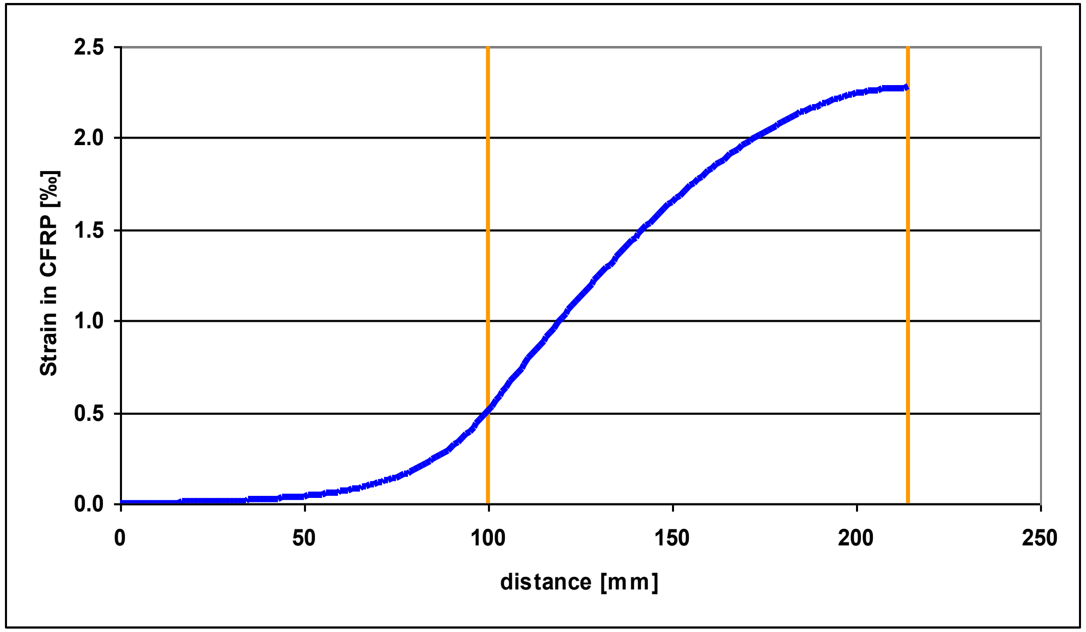


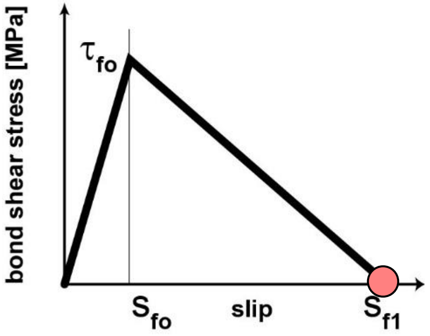
trigonometric shape

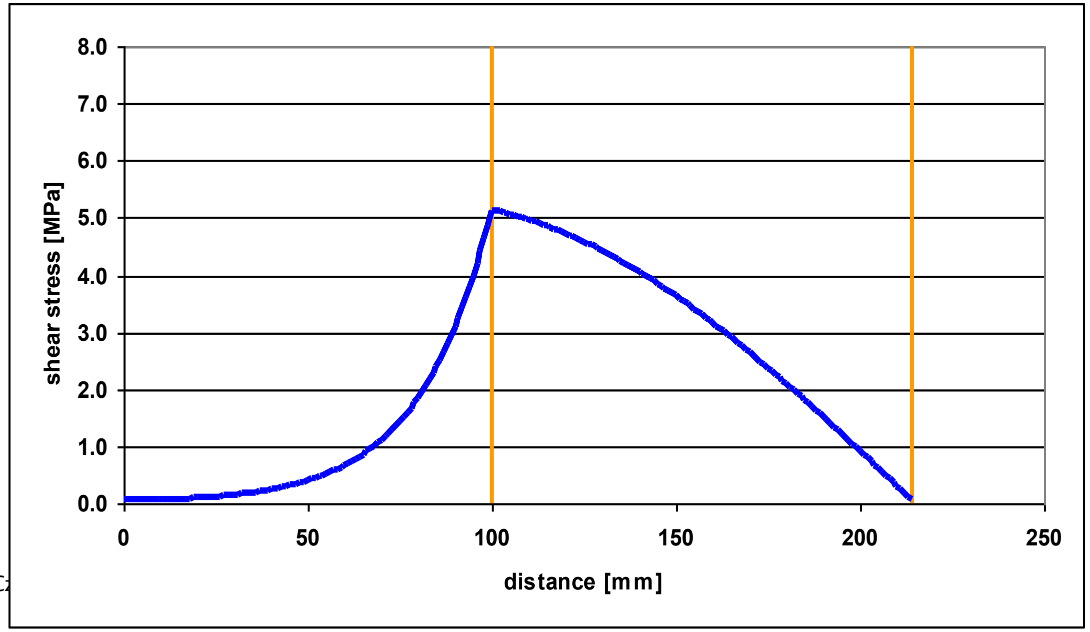




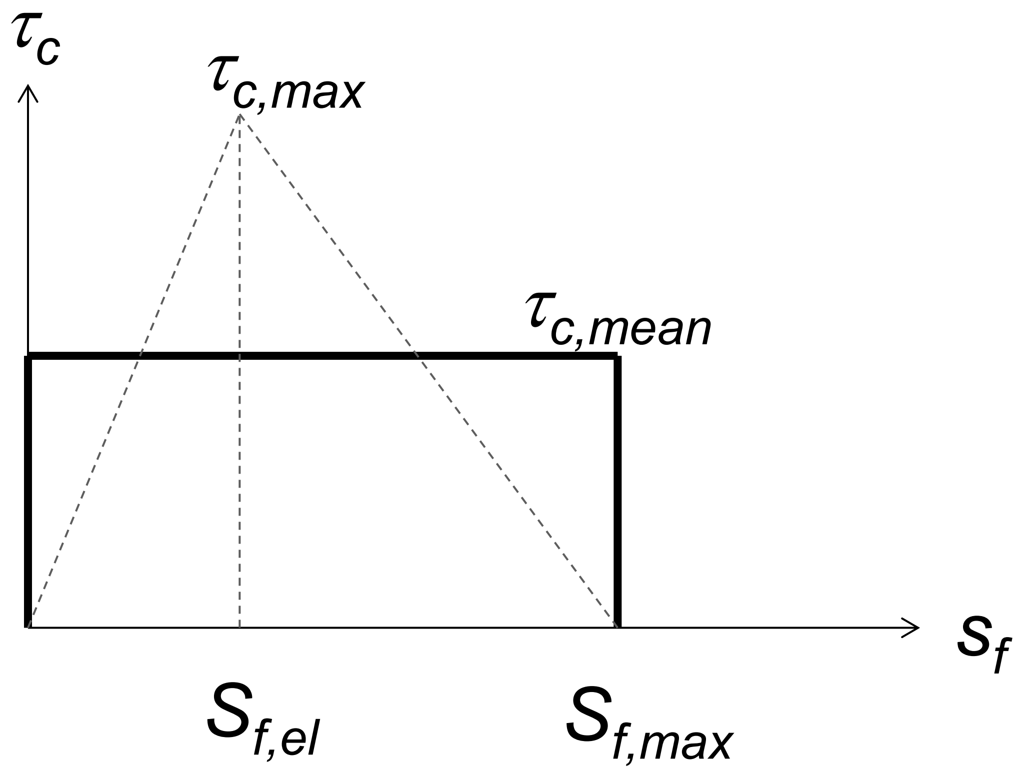
trigonometric shape


Strain in CFRP [‰]
2.5
2.0
1.5
1.0
0.5
0.0
0
50
100
150
200
250
distance [mm]


bond shear stress [MPa]
τ_{fo}
S_{fo}
slip
S_{f1}


shear stress [MPa]
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
50
100
150
200
250
distance [mm]

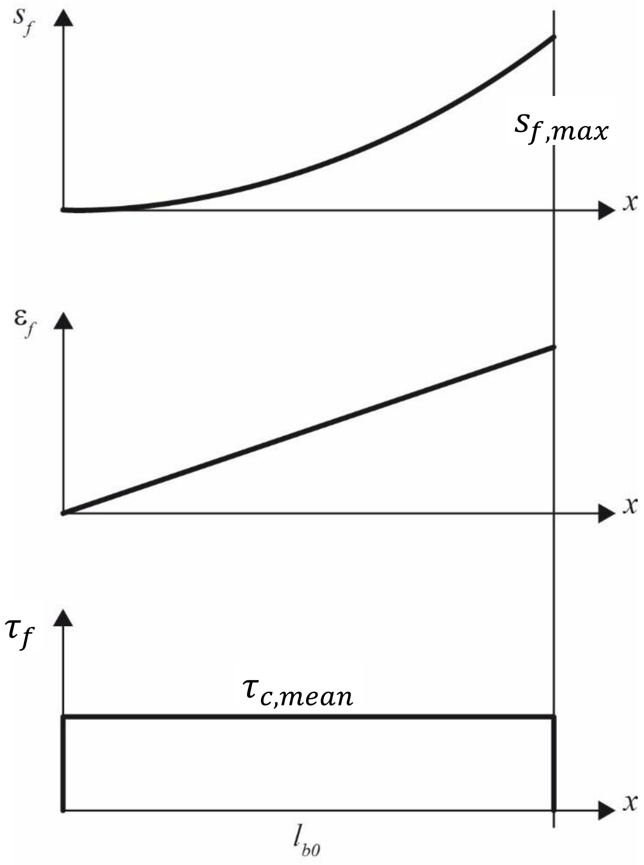
Simplified bond shear stress-slip relation



$$G_{Fc} = \frac{\tau_{c,max} s_{f,max}}{2}$$

$$G_{Fc} = \tau_{c,mean} s_{f,max}$$

$\tau_{c,max}$ = maximum bond shear stress on concrete
$\tau_{a,max}$ = maximum bond shear stress on steel



$$s_f(x) = \frac{\tau_f}{E_f t_f}\frac{x^2}{2} \qquad (1)$$

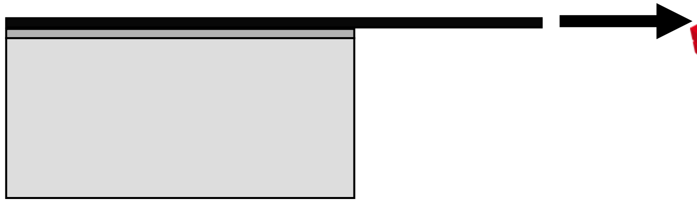
$$\varepsilon_f(x) = \frac{\tau_f}{E_f t_f} x \qquad (2)$$

$$\tau_c(x) = \tau_{f,mean} \qquad (3)$$

with $l_{b0} = \frac{F_{b0,R}}{\tau_{f,mean}\, b_f}$ and $s_f(x = l_{b0}) = s_{f,max}$ we get from Eq. (1): $s_{f,max} = \frac{F_{b0,R}^2}{2E_f t_f b_f^2 \tau_{f,mean}}$

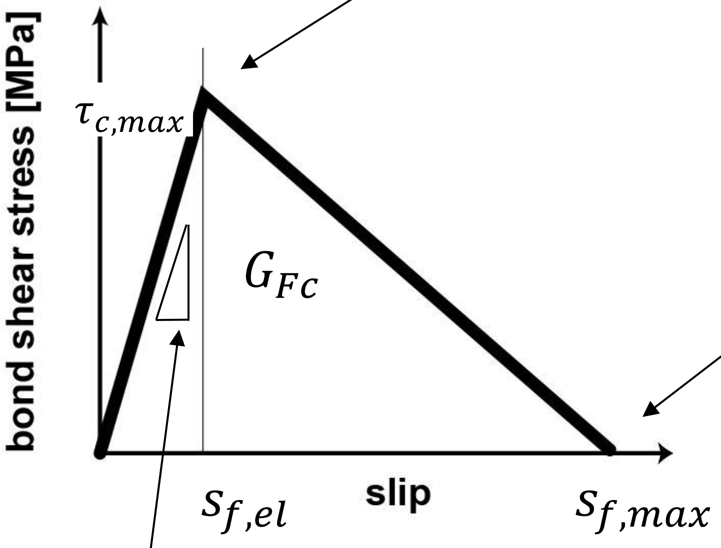
and with $G_{Fc} = \tau_{f,mean} s_{f,max}$ (slide 39) we get $\boxed{F_{b0,R} = b_f\sqrt{2G_{Fc}E_f t_f}}$

Bond behavior depends on



Bond damage (Entfestigungsbereich, Verbundschädigung)

- concrete quality



maximum slip

- constant (~0.2 mm)

Additionally, the stiffness ($E_f t_f$) of the strip influences also the bond behaviour.

Elastic deformation

- shear modulus of adhesive and concrete
- thickness of adhesive plus a layer of concrete

Comparison with reinforcement

Empa
Materials Science and Technology

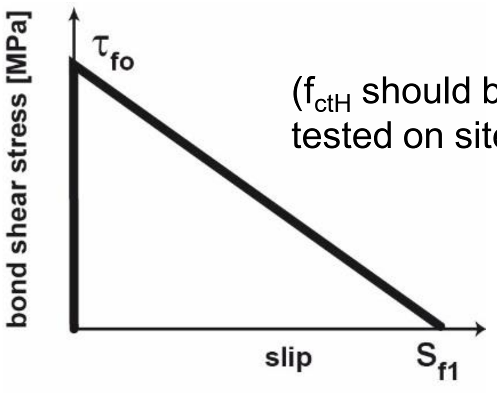
- Internal steel reinforcement is surrounded
  - Deflection perpendicular to longitudinal direction is not possible
  - Normal (confinement) stresses due to interlocking and friction

  → the longer the anchor length, the higher the anchor force up to yielding of the steel reinforcement

- Externally applied strip is free on one side
  - Deflection perpendicular to longitudinal direction is possible

  → maximum anchorage force (anchorage resistance) with corresponding length (active bond length)

CFRP strip according to SIA 166 (2004)



(fctH should better be tested on site)

$$\tau_{f0} = \frac{4}{3} f_{ctH} \qquad G_{Fb} = \frac{1}{8} f_{ctH}$$

f_{ctH} : **mean** value of bond strength (Haftfestigkeit) determined according to DIN 1048 part 2

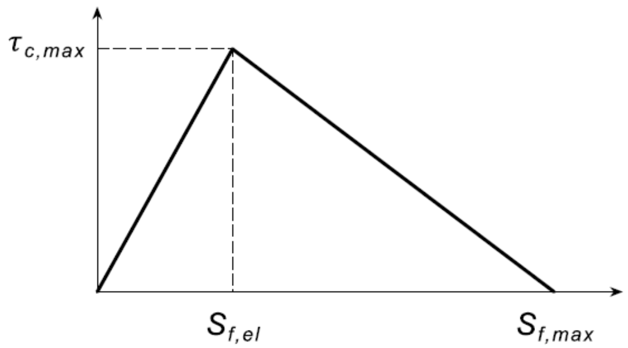
for C30/37: f_{ctm} = 2.9MPa

with $f_{ctm} = f_{ctH}$ (only as an example, the correlation is not correct!)

τ_{f0} = 3.9 MPa

G_{Fb} = 0.36 N/mm

CFRP strip according to SIA 166 (2024)



$$\tau_{c,max,k} = \frac{4}{3} f_{hk} \qquad G_{Fck} = \frac{1}{8} f_{hk}$$

f_{hk} : **characteristic** value of bond strength (Haftfestigkeit) determined according to SN EN 1542

for C30/37: f_{ctm} = 2.9MPa

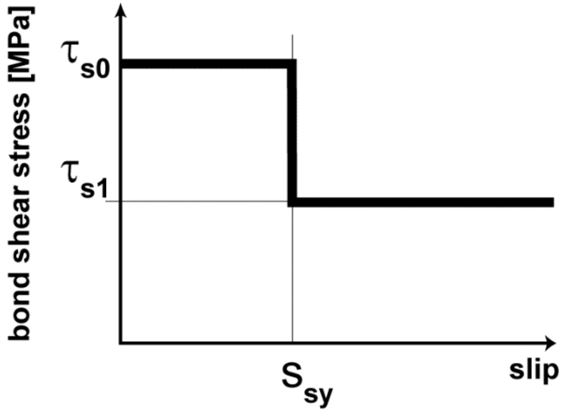
with $f_{hk} = 0.7\ f_{ctm} = 2.0$ (only as an example and to show the idea, the relation is not correct!)

$\tau_{c,max,k}$ = 2.7 MPa

G_{Fck} = 0.25 N/mm

Internal steel reinforcement according to Sigrist and Marti: (Skript Stahlbeton Marti 2009)

ology

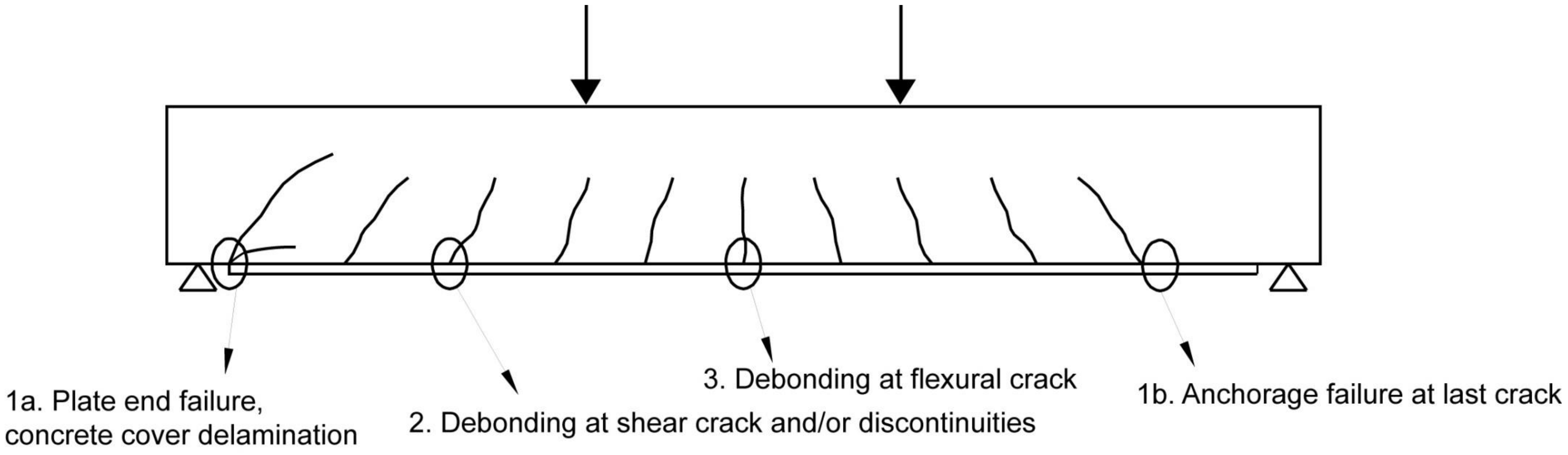


$$\tau_{s0} = 2 f_{ctm} \qquad \tau_{s1} = f_{ctm}$$

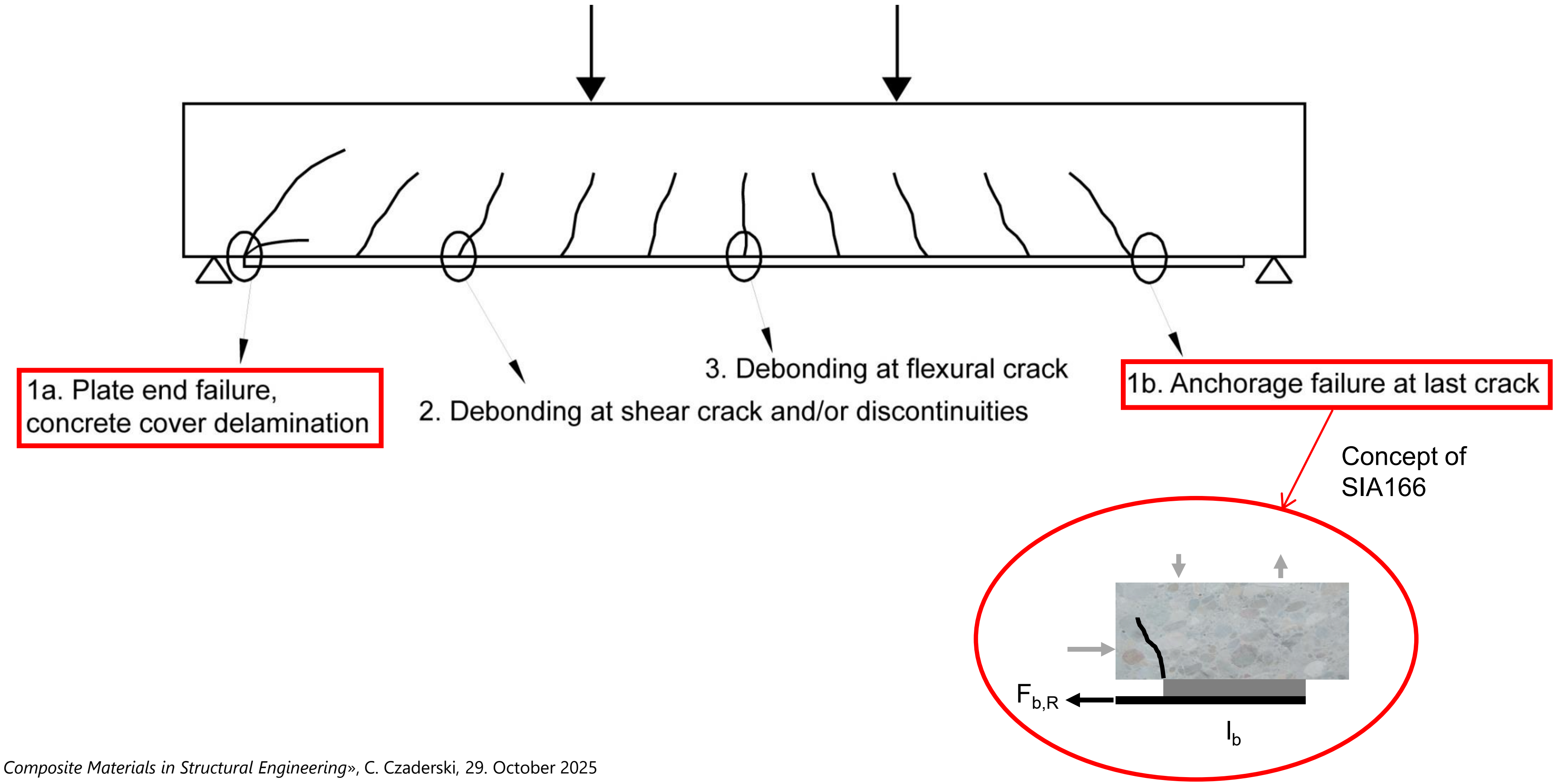
In SIA262 (2013): $f_b = 1.4 f_{ctm}$

τ_{s0} = 5.8 MPa **τ_{s1} = 2.9 MPa**

Debonding failure modes according to SIA 166



Debonding failure modes according to SIA 166



Equations for maximum anchorage resistance

Empa
Materials Science and Technology

SIA 166 (2024) $F_{b0,Rd} = b_f \sqrt{2 G_{Fcd} E_f t_f}$ with $G_{Fcd} = \frac{1}{8} \eta_u \eta_l \frac{f_{hk}}{\gamma_h}$ G_{Fcd} in N/mm f_{hk} in N/mm²

reduction factors $\eta_u \eta_l$ according to SIA166 (2024)

***fib* Bulletin 14** $N_{fa,max} = \alpha \cdot c_1 \cdot k_c \cdot k_b \cdot b_f \cdot \sqrt{E_f \cdot t_f \cdot f_{ctm}}$

TR55 $T_{k,max} = 0.5 \cdot k_b \cdot b_f \cdot \sqrt{E_{fd} \cdot t_f \cdot f_{ctk}}$

$$k_b = 1.06 \sqrt{\frac{2 - \frac{b_f}{b}}{1 + \frac{b_f}{400}}} \geq 1$$

Italian Code $F_{fd} = b_f \sqrt{2 \cdot E_f \cdot t_f \cdot \Gamma_{Fk}}$; $\Gamma_{Fk} = 0.03 \cdot k_b \cdot \sqrt{f_{ck} \cdot f_{ctm}}$

Take care to symbols. They depend on the reference.

Active bond length

(the length which is actively involved in the force transfer from the strip to the concrete)

If we assume a constant bond shear stress:

$$l_{b0} = \frac{F_{b0,R}}{\tau_{c,mean}\, b_f}$$

with

$$F_{b0,R} = b_f \sqrt{2 G_{Fc} E_f t_f}$$

$$\longrightarrow \quad l_{b0} = \frac{\sqrt{2 G_{Fc} E_f t_f}}{\tau_{c,mean}} \quad \longrightarrow \quad \text{proportional to} \quad \sqrt{\frac{G_{Fc} E_f t_f}{\tau_c^2}}$$

Equations for active bond length

(minimum necessary length for maximum anchor resistance $F_{b0,R}$)

Materials Science and Technology

SIA 166 (2024)	$l_{b0d} = 2.5\sqrt{\frac{G_{Fcd}E_f t_f}{\tau^2_{c,max,d}}}$	with $G_{Fcd} = \frac{1}{8}\eta_u\eta_l\frac{f_{hk}}{\gamma_h}$ $\tau_{c,max,d} = \frac{4}{3}\eta_u\eta_l\frac{f_{hk}}{\gamma_h}$ The reduction values η_u and η_l are tabulated in SIA 166(2024), they consider environment and loading conditions.
***fib* Bulletin 14**	$l_{b,max} = \sqrt{\frac{E_f \cdot t_f}{c_2 \cdot f_{ctm}}}$	
TR55	$l_{t,max} = 0.7 \cdot \sqrt{\frac{E_{fd} \cdot t_f}{f_{ctk}}}$	
Italian Code	$l_e = \sqrt{\frac{E_f \cdot t_f}{2 \cdot f_{ctm}}}$	

Example

Concrete C30/37

Sika CarboDur S512 — tensile strength > 2800 MPa

tensile strain > 17‰

$$G_{Fcd} = \frac{1}{8}\eta_u \eta_l \frac{f_{hk}}{\gamma_h}$$

$$\tau_{c,max,d} = \frac{4}{3}\eta_u \eta_l \frac{f_{hk}}{\gamma_h}$$

for concrete
C 20/25 to C 50/60

$$G_{Fcd} = \frac{1}{8} 1.0\ 1.0 \frac{2.9}{1.5} = 0.24\ \frac{N}{mm}$$

$$\tau_{c,max,d} = \frac{4}{3} 1.0\ 1.0 \frac{2.9}{1.5} = 2.6\ MPa$$

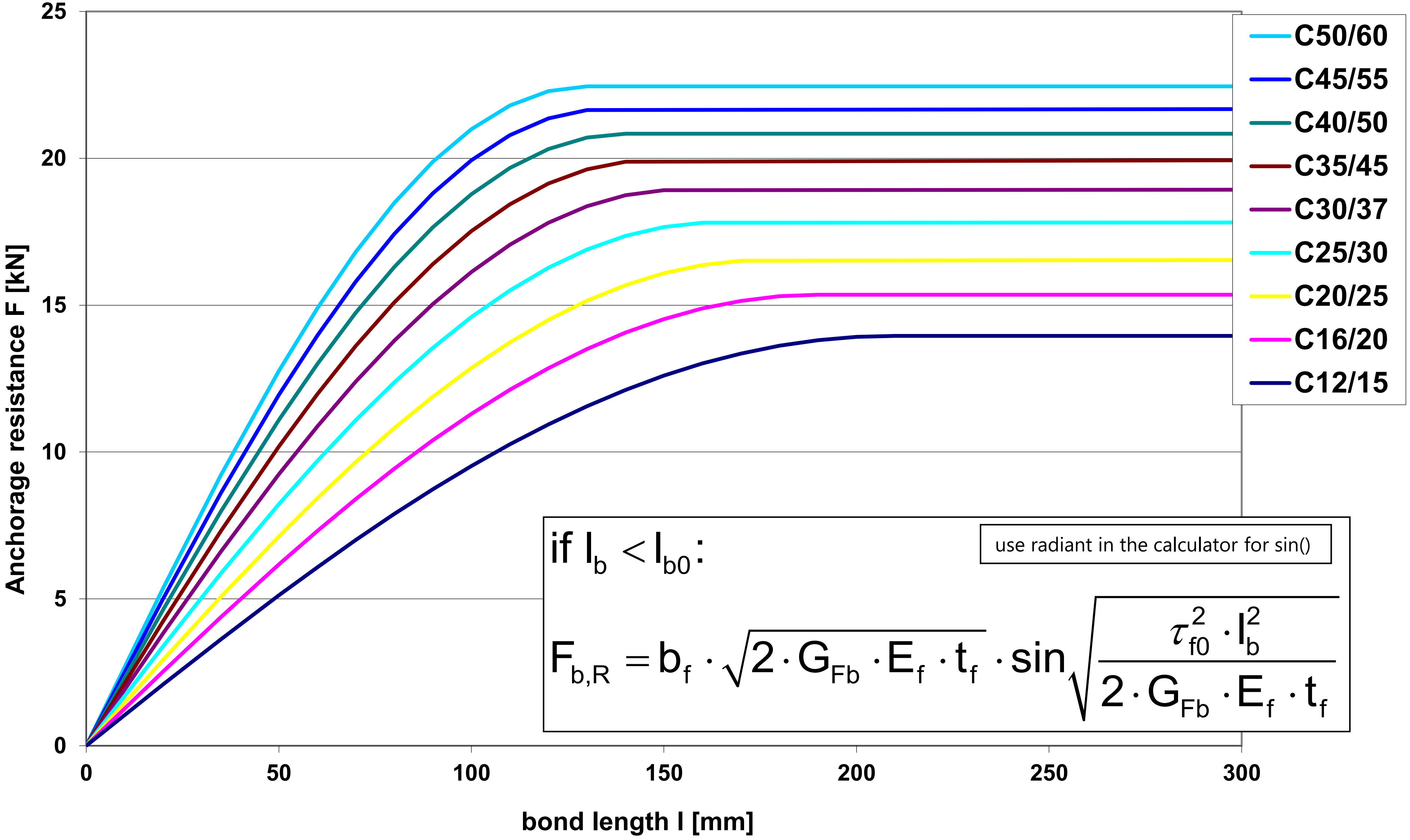
$$F_{b0,Rd} = b_f \sqrt{2 G_{Fcd} E_f t_f} = 50\sqrt{2 \cdot 0.24 \cdot 165000 \cdot 1.2} = 15.5\ kN$$

$$\sigma_{b0,Rd} = \frac{F_{b0,Rd}}{b_f t_f} = 258\ MPa$$

$$\varepsilon_{b0,Rd} = \frac{\sigma_{b0,Rd}}{E_f} = 1.56‰$$

old SIA166 (2004)

Empa
Materials Science and Technology

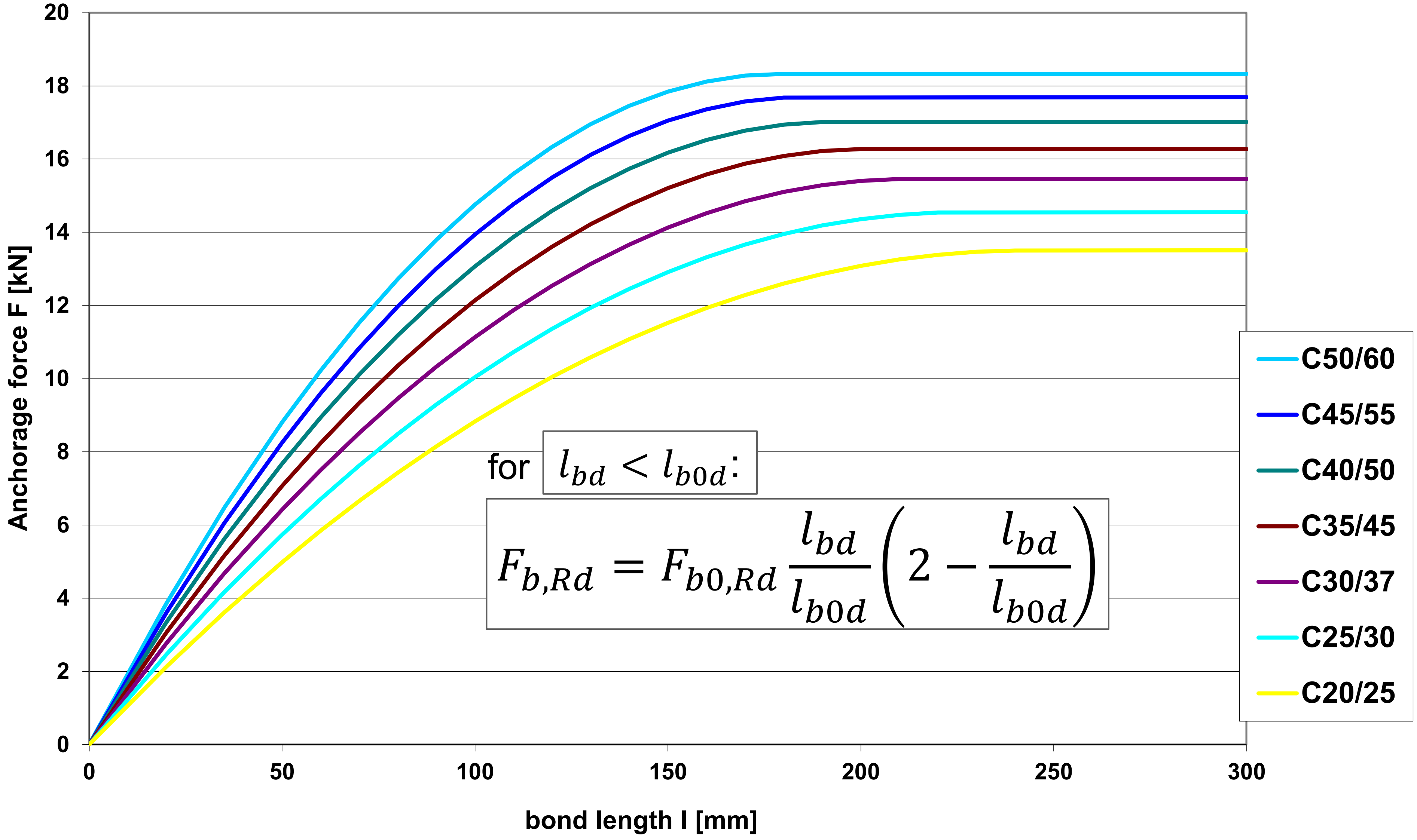


use radiant in the calculator for sin()

if $l_b < l_{b0}$:

$$F_{b,R} = b_f \cdot \sqrt{2 \cdot G_{Fb} \cdot E_f \cdot t_f} \cdot \sin\sqrt{\frac{\tau_{f0}^2 \cdot l_b^2}{2 \cdot G_{Fb} \cdot E_f \cdot t_f}}$$

SIA 166 (2024):

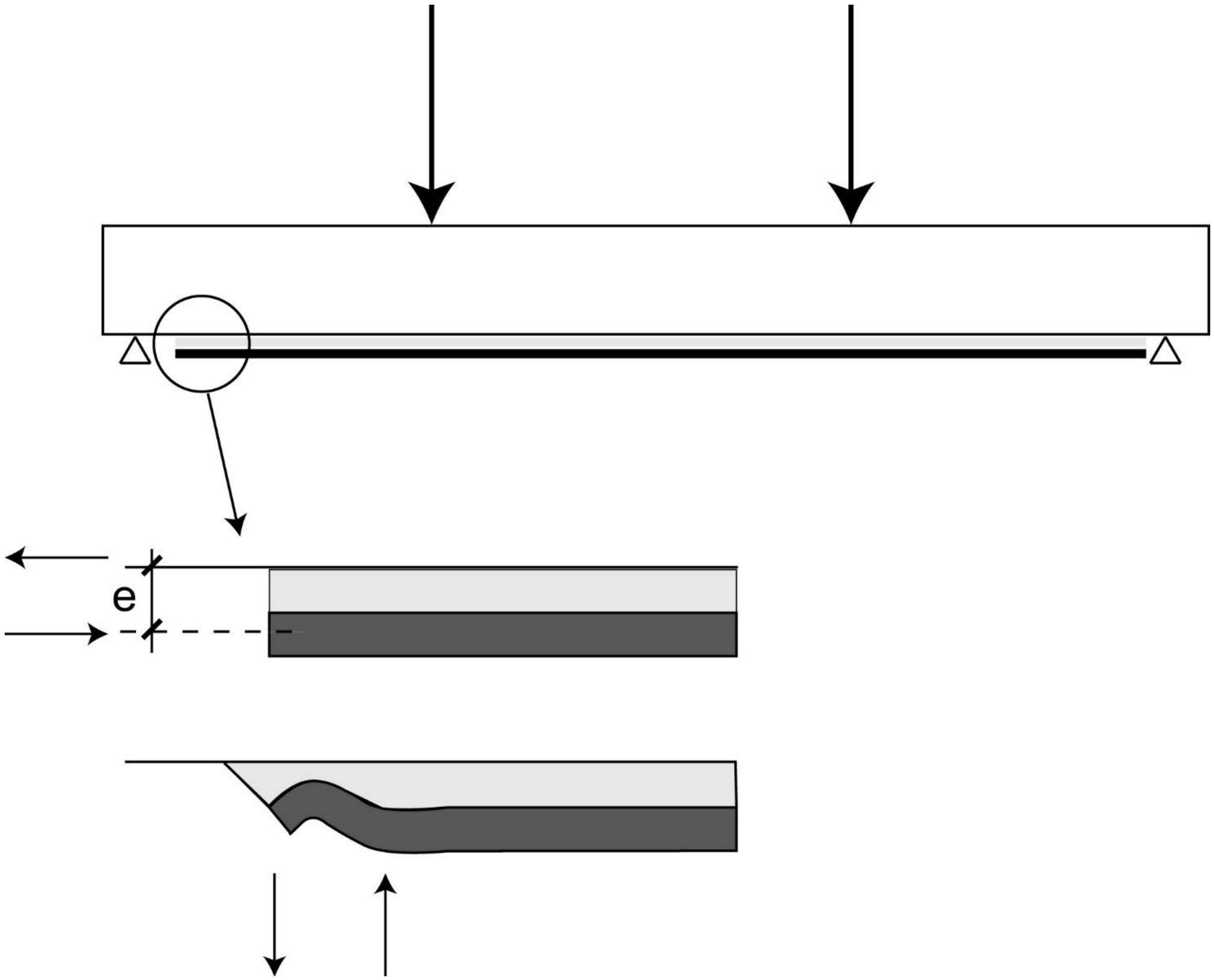

Anchorage force F [kN]
20
18
16
14
12
10
8
6
4
2
0
0
50
100
150
200
250
300
bond length l [mm]
C50/60
C45/55
C40/50
C35/45
C30/37
C25/30
C20/25
for $l_{bd} < l_{b0d}$:
$F_{b,Rd} = F_{b0,Rd} \frac{l_{bd}}{l_{b0d}} \left(2 - \frac{l_{bd}}{l_{b0d}} \right)$

Design philosopy of preventing end anchorage failure

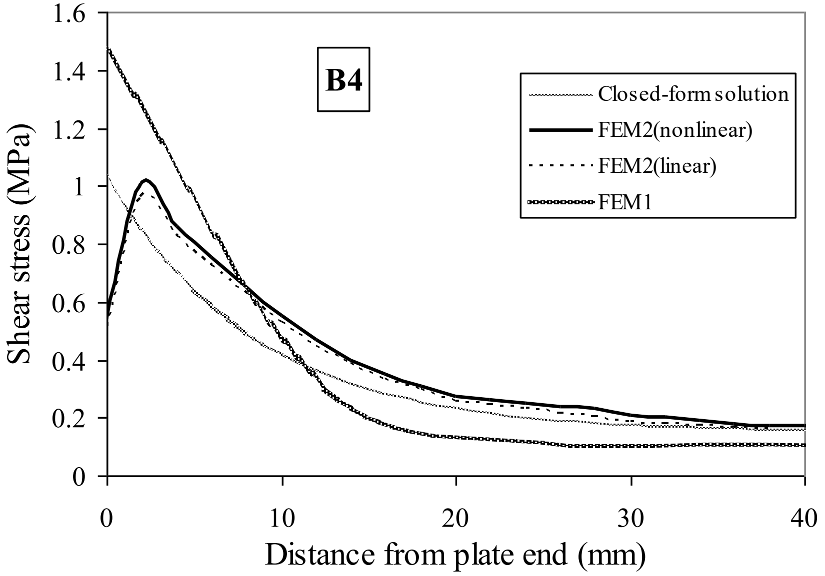
- SIA166: anchorage in the uncracked zone
- ACI:
  - If V_u > 0.67 V_c at strip end, then transverse reinforcement is necessary (they give a design equation for U-wrap reinforcement $A_{f,anchor}$)

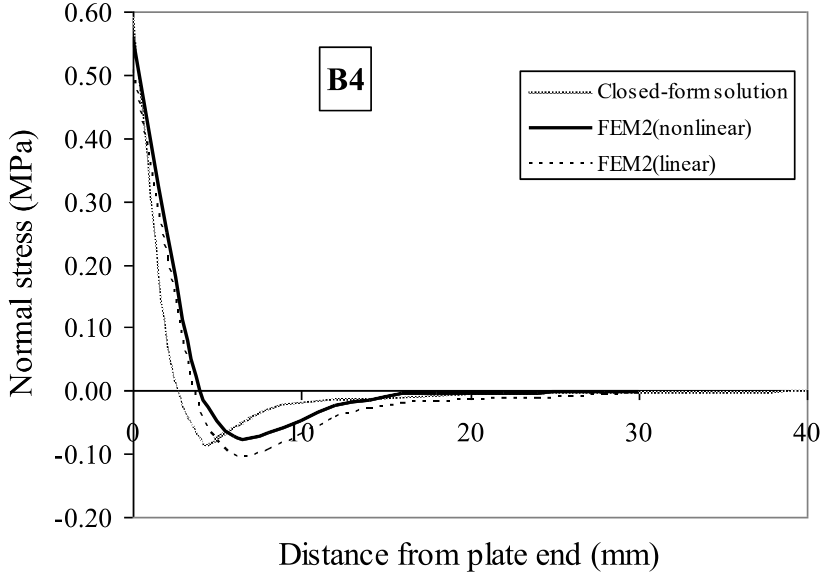
  or (instead of detailed analysis)
  - for simply supported beams: length l_{df} after last crack
- Elastic solutions for calculating shear and normal stresses at strip end

End anchorage, normal stresses at strip end



Shear and normal stresses at strip end





see:

Aram, M.R., C. Czaderski, and M. Motavalli, Debonding failure modes of flexural FRP-strengthened RC beams. Composites Part B: Engineering, 2008. 39(5): pp. 826-841.

Safety concept of the new SIA 166 (2024)

Empa
Materials Science and Technology

Example: concrete tensile strength at the concrete surface

Mean value from tests f_{hm} = 4.0 GPa
Characteristic value (5% fractile) f_{hk} = 3.2 GPa

Design value:

$$f_{hd} = \frac{f_{hk}}{\gamma_h} = \frac{3.2}{1.5} = 2.1 MPa$$

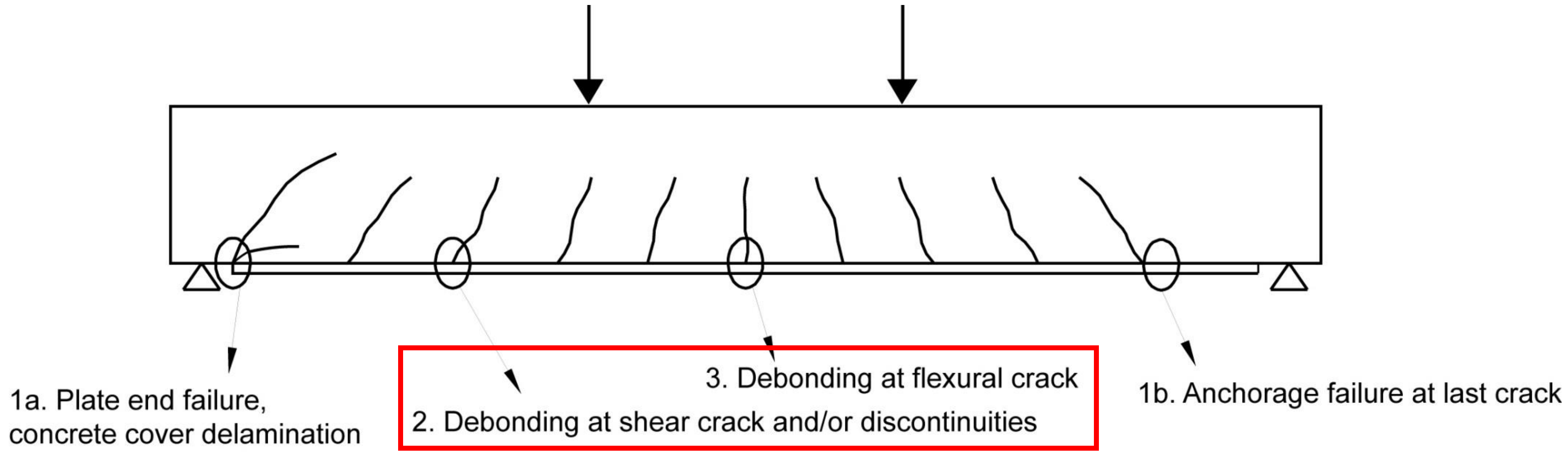
γ_h = 1.5, from Tabelle 5 in SIA166 (2024), see slide 94

Design value of fracture energy:

$$G_{Fcd} = \frac{f_{hd}}{8} = \frac{2.1}{8} = 0.26 N/mm$$

Debonding failure modes

Empa
Materials Science and Technology



Example

Concrete C30/37

Sika CarboDur S512 tensile strength > 2800 MPa

tensile strain > 17‰

$$F_{b0,Rd} = b_f \sqrt{2 G_{Fcd} E_f t_f} = 50\sqrt{2 \cdot 0.24 \cdot 165000 \cdot 1.2} = 15.5\ kN$$

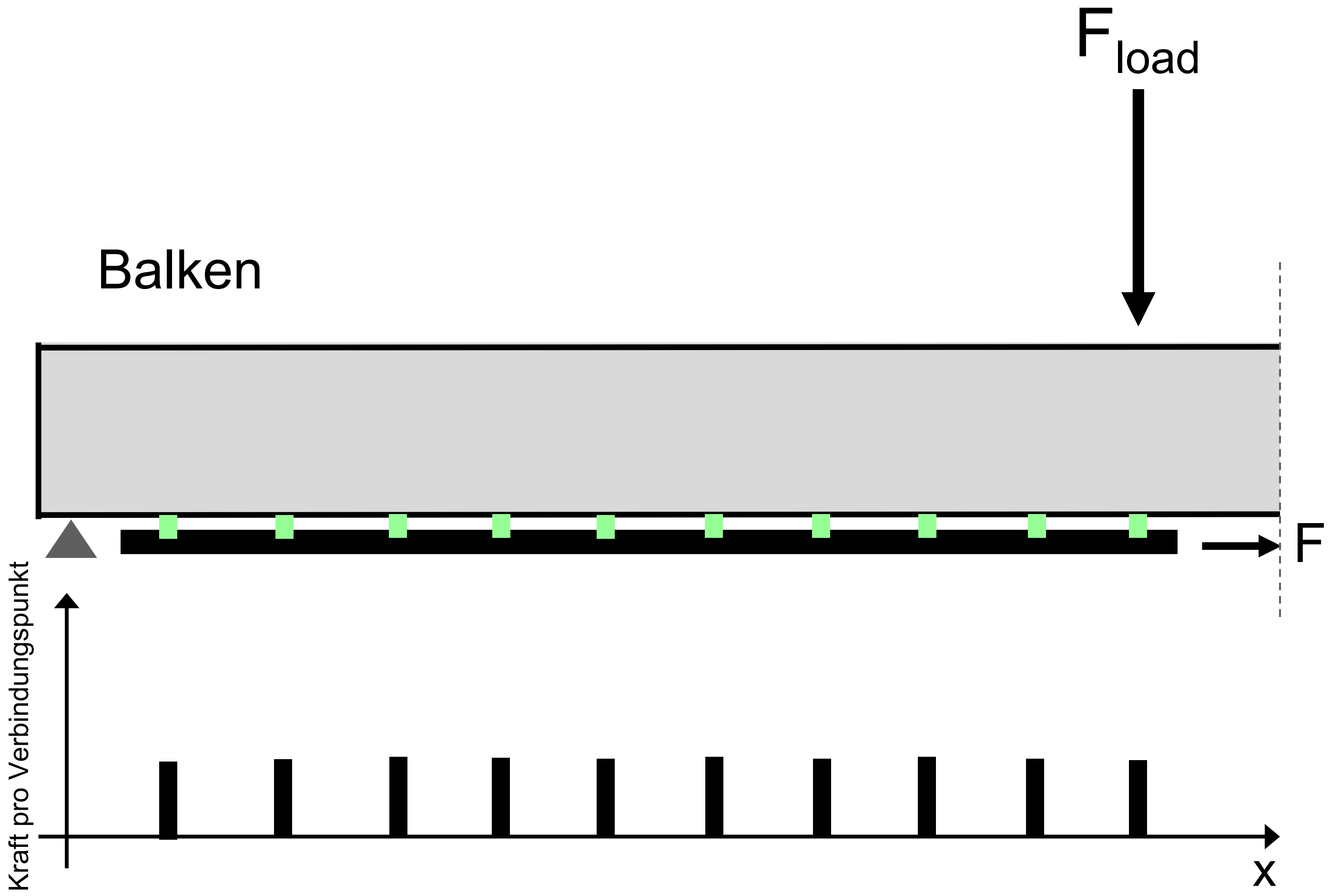
$$\sigma_{b0,Rd} = \frac{F_{b0,Rd}}{b_f t_f} = 258\ MPa$$

$$\varepsilon_{b0,Rd} = \frac{\sigma_{b0,Rd}}{E_f} = 1.56‰$$

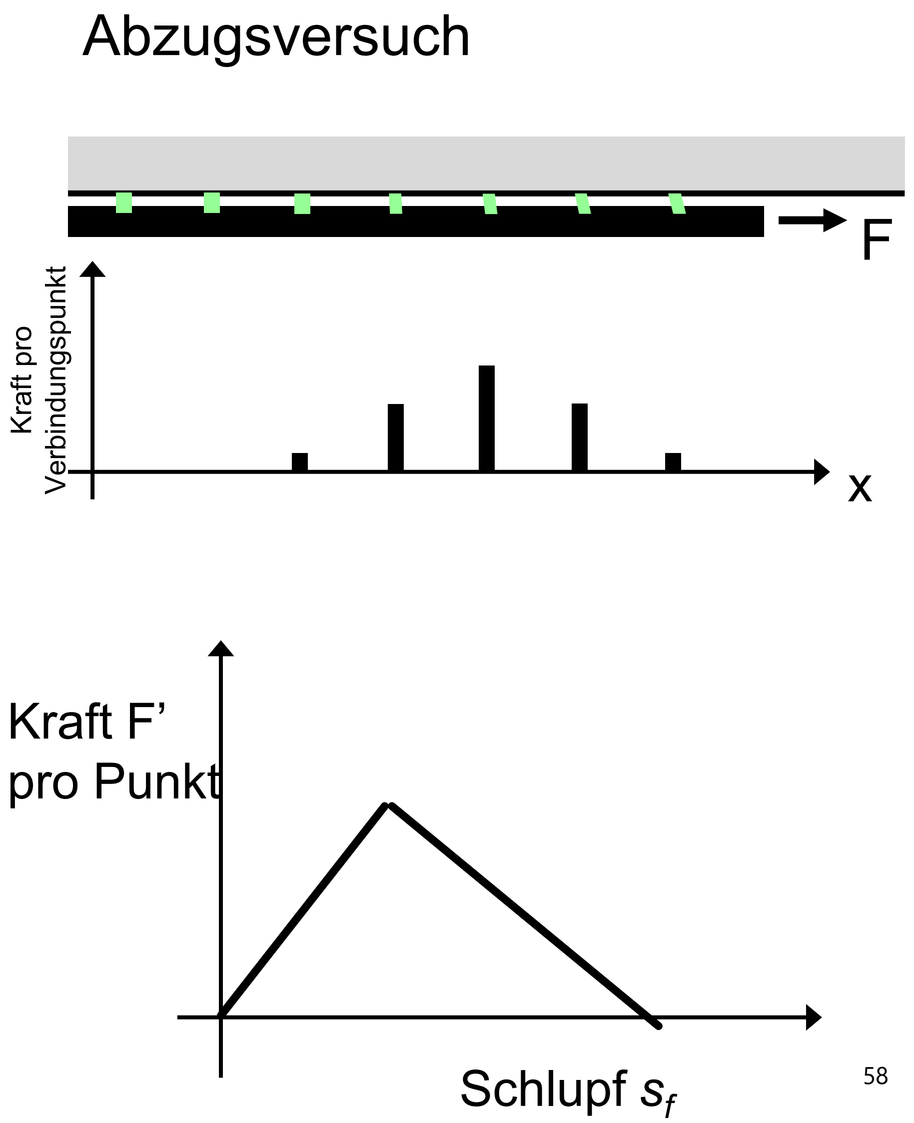
??

According to the old SIA166 (2004),

the maximum allowed strain was 8‰!

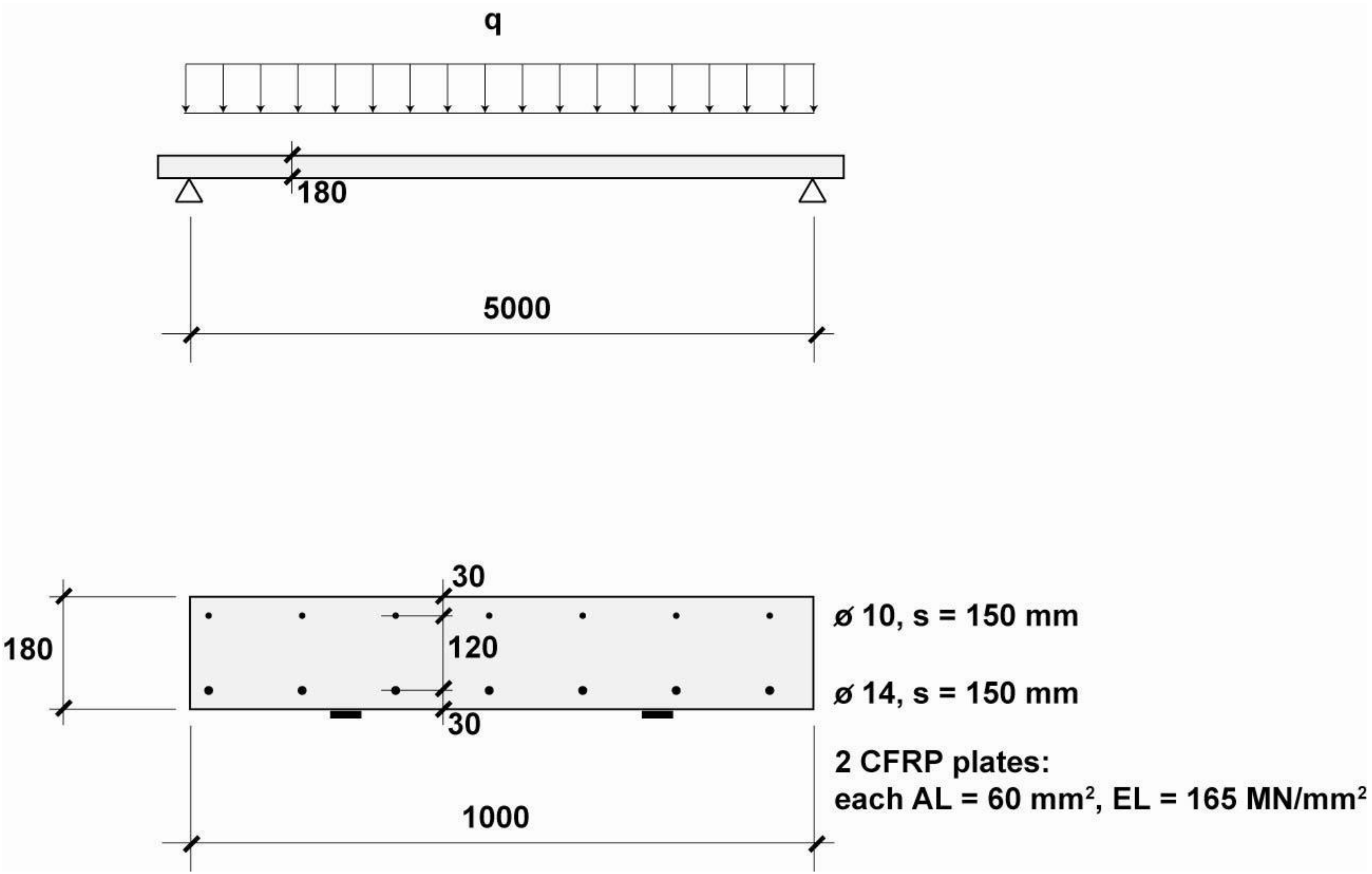


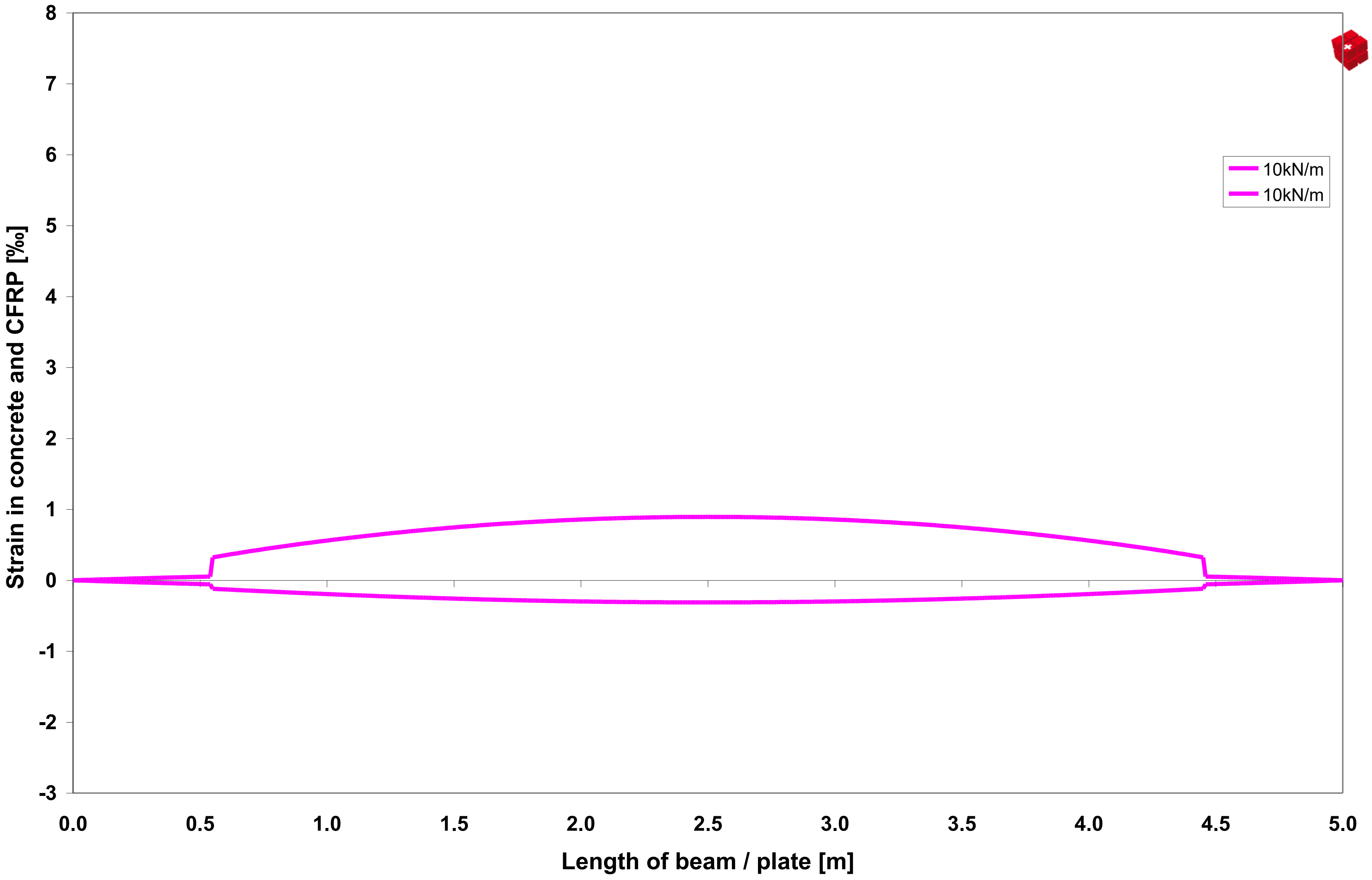
F_{load}
Balken
F
Kraft pro Verbindungspunkt
x



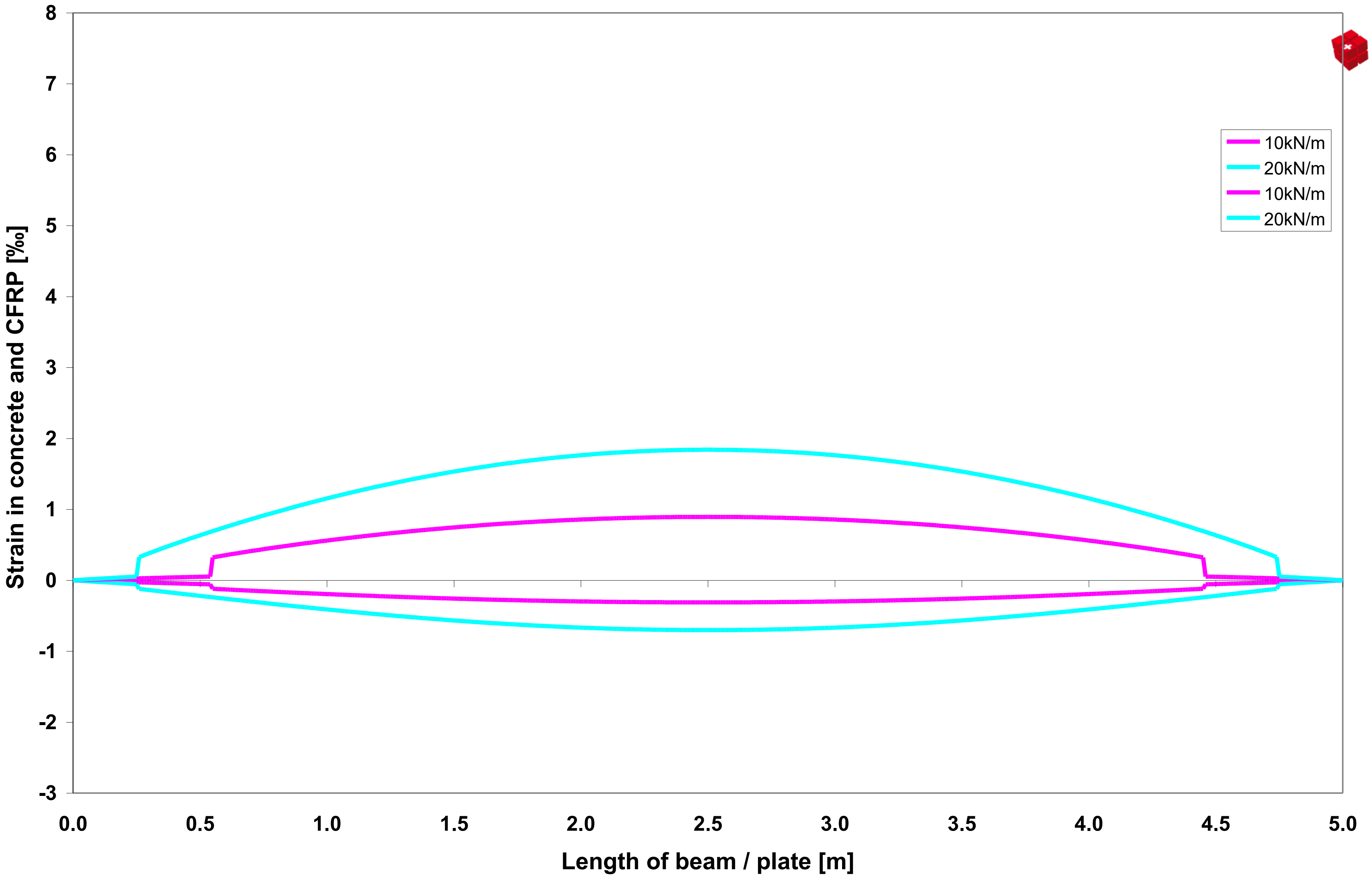
Abzugsversuch
F
Kraft pro Verbindungspunkt
x
Kraft F'
pro Punkt
Schlupf s_f

Simple supported plate

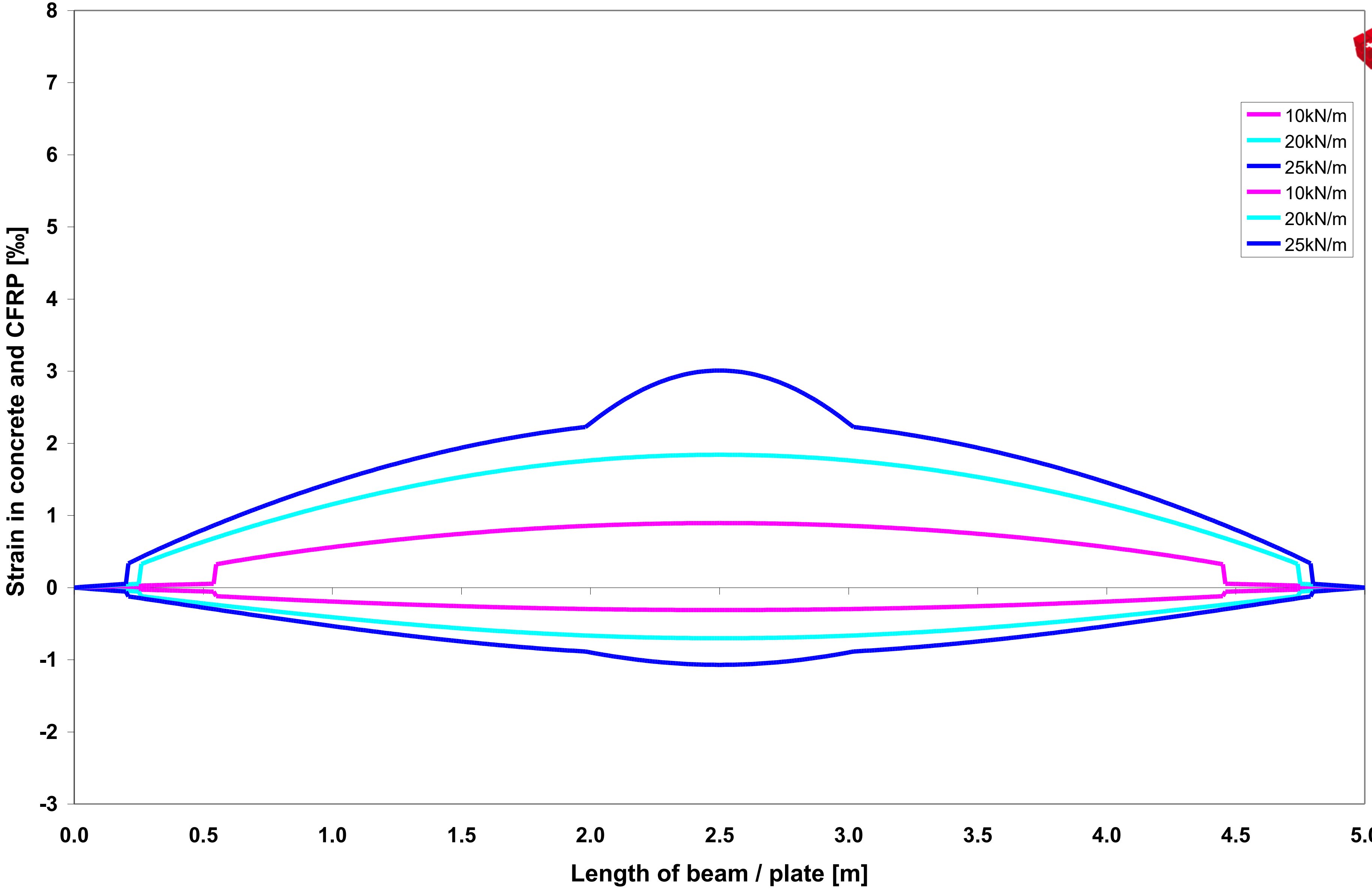




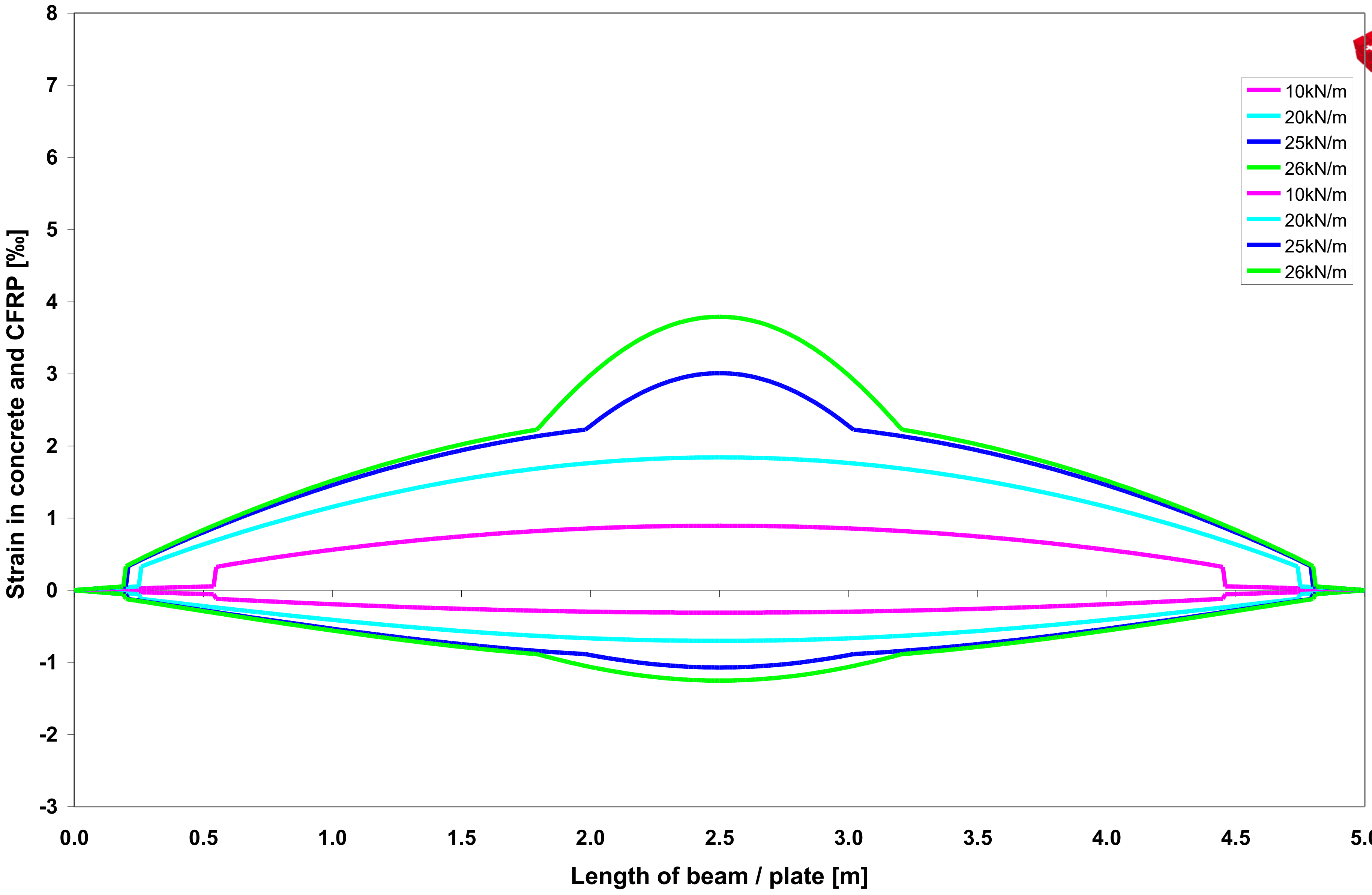
Empa
Materials Science and Technology

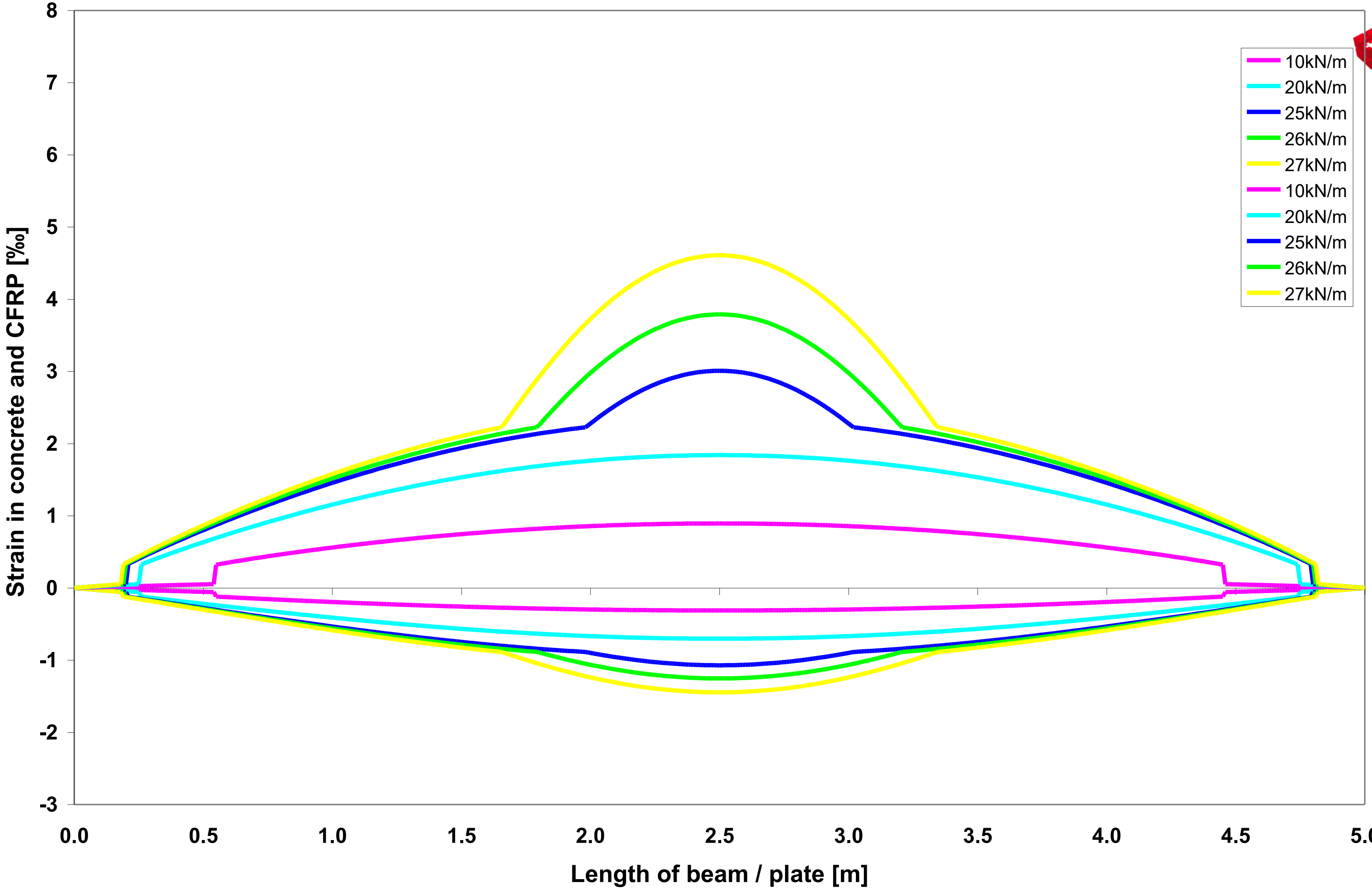


Strain in concrete and CFRP [‰]
8
7
6
5
4
3
2
1
0
-1
-2
-3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Length of beam / plate [m]
10kN/m
20kN/m
10kN/m
20kN/m

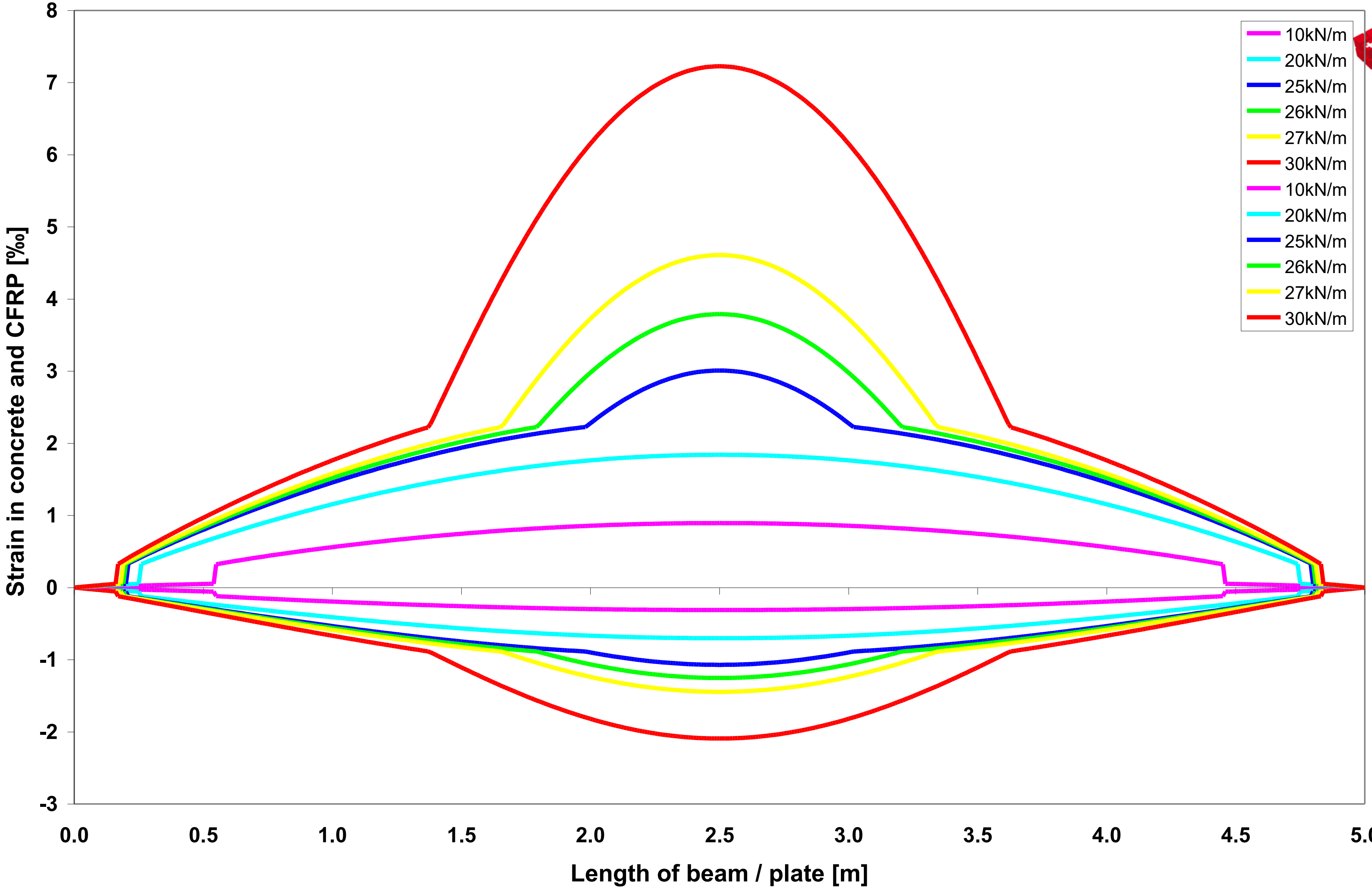


Strain in concrete and CFRP [‰]
8
7
6
5
4
3
2
1
0
-1
-2
-3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Length of beam / plate [m]
10kN/m
20kN/m
25kN/m
10kN/m
20kN/m
25kN/m

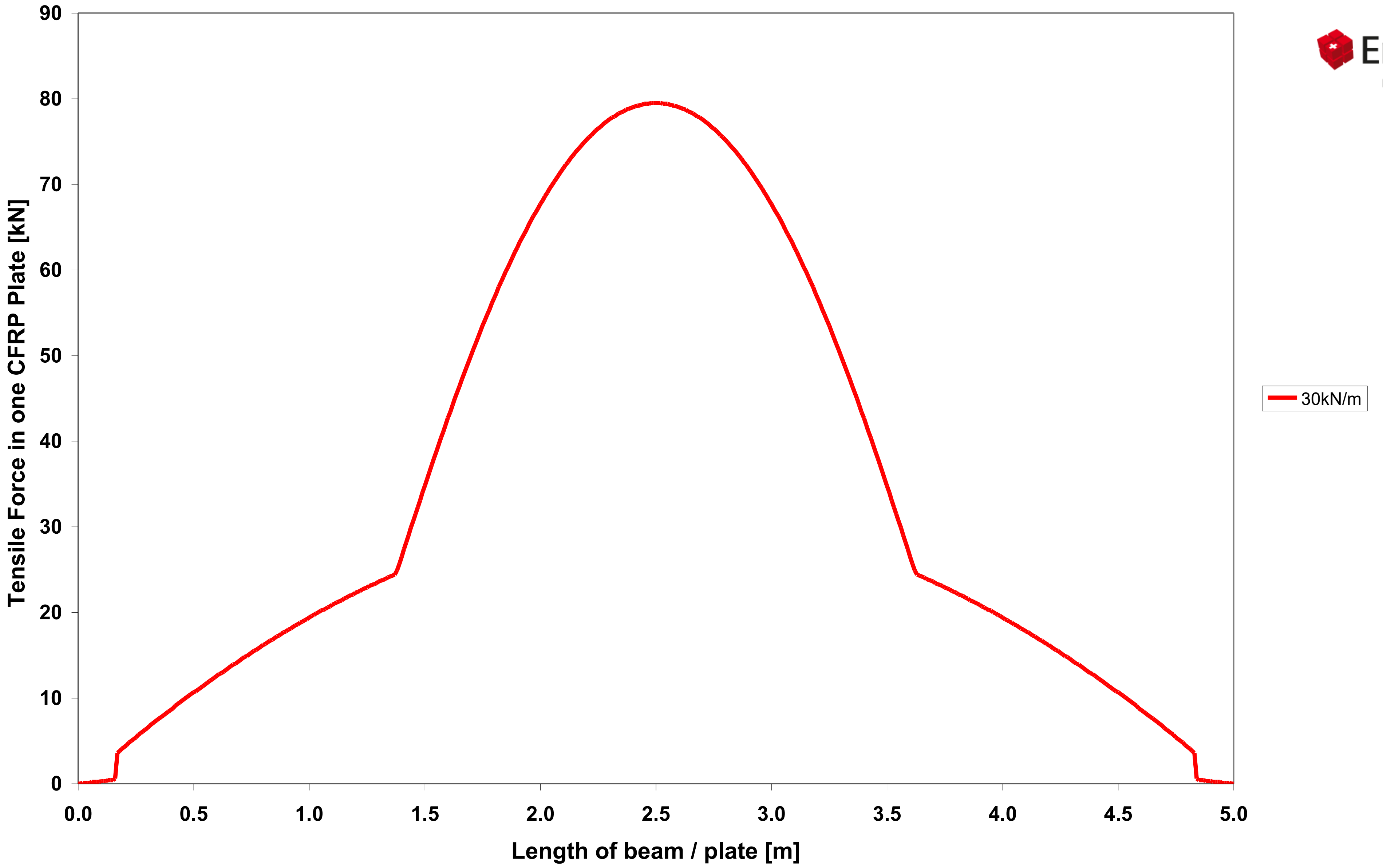

Strain in concrete and CFRP [‰]
8
7
6
5
4
3
2
1
0
-1
-2
-3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Length of beam / plate [m]
10kN/m
20kN/m
25kN/m
26kN/m
10kN/m
20kN/m
25kN/m
26kN/m



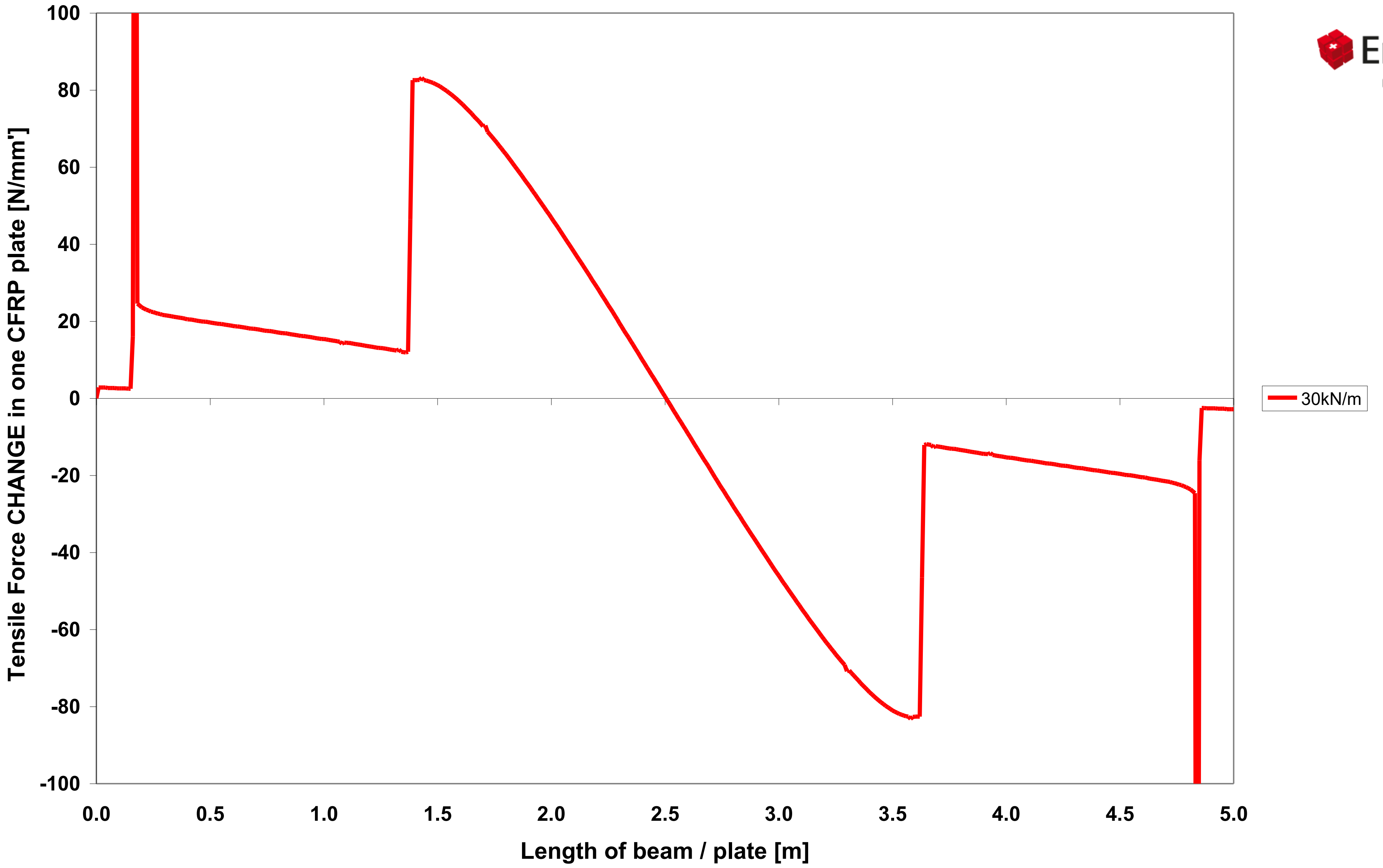
Empa
Materials Science and Technology



Empa
Materials Science and Technology



90
80
70
60
50
40
30
20
10
0
Tensile Force in one CFRP Plate [kN]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Length of beam / plate [m]
30kN/m



Empa
Materials Science and Technology

Maximum Tensile Force CHANGE according to the old SIA 166(2004)

$$\left(\frac{\Delta F_f}{\Delta x}\right)_R = \tau_{f,lim} \cdot b_l$$

$$\tau_{f,lim} = 2.5 \cdot \tau_c = 2.5 \cdot 0.3 \cdot \sqrt{f_{ck}}$$ (τ_c from SIA 262)

Example

Concrete C30/37

Sika CarboDur S512

$$\left(\frac{\Delta F_f}{\Delta x}\right)_R = \tau_{f,lim} \cdot b_f = 205 N / mm$$

$$\tau_{f,lim} = 2.5 \cdot \tau_c = 2.5 \cdot 0.3 \cdot \sqrt{30} = 4.1 MPa$$

please note: without safety factors!!

Shear stress limitations, other guidelines

fib Bulletin 14 (approach 3)

$$\tau_b \leq f_{cb}$$

$$f_{cb} = 1.8 \cdot f_{ctk} = 1.8 \cdot 0.7 \cdot 2.9 = 3.7MPa$$

fib Bulletin 14 (approach 2)

$\Delta\sigma_f$ **(between cracks …)**

TR55

$$\tau = V_{add}\alpha_f A_f \frac{h\text{-}x}{I_{cs} b_a}$$

$$\tau_{lim,c} = 0.8 f_{ctk}$$ **outside yield zone**

$$\tau_t = \underbrace{t_f\left[\frac{\sigma_{fmax} - \sigma_{fy}}{\Delta x}\right]}_{\tau_m} + \underbrace{7.8\left[1.1\text{-}\frac{M_y}{M_{ed}}\right] f_{ctk}}_{\tau_{sc}}$$

τ_m= mean shear stress

τ_{sc}=additional shear stress due to stress concentration at flexural cracks

$$\tau_{lim,y} = 4.5 f_{ctk}$$ **in the yield zone**

please note: without safety factors!!

Maximum strain in the strip according to old SIA 166(2004)

- Local debonding due to compatibility problems between strip and concrete at flexural cracks

$$F_{f,R} = A_f \cdot E_f \cdot \varepsilon_{f,lim} \leq A_f \cdot E_f \cdot \varepsilon_{fu}$$

$$\varepsilon_{f,lim,d} = 8‰$$ design value!

$$\varepsilon_{fu} =$$ supplier of the material

Maximum strain in strip, other guidelines

Empa
Materials Science and Technology

***fib* Bulletin 14 (approach 1)**	$\varepsilon_{f,lim} = 6.5 - 8.5‰$	
ACI	$$\varepsilon_{fd} = 0.41\sqrt{\frac{f_c'}{n \cdot E_f \cdot t_f}}$$ (equation for DFM 2 and 3)	**in SI units**
Italian Code	$$\varepsilon_{fdd} = k_{cr}\sqrt{\frac{0.06 k_b \sqrt{f_{ck} f_{ctm}}}{E_f \cdot t_f}}$$ (equation for DFM 2 and 3)	**With k_{cr} = 3.0**
TR55	$$\varepsilon_{fmax} + \underbrace{0.114 \frac{\tau_{sc}}{\sqrt{E_{fd} t_f}}}_{} \leq \varepsilon_{fd}$$ τ_{sc}=localised strain increase at flexural cracks	

please note: without safety factors!!

Flexural strengthening

Empa
Materials Science and Technology

- SIA 166 (2004) 3 Verifications

1. End anchorage
2. Tensile force change
3. Limitation of strains ε=0.8%

- SIA 166 (2024) 2 Verifications

1. End anchorage Ziffer 4.3.2
2. Limitation of strains Ziffer 4.3.4
3. ~~Limitation of strains ε=0.8%~~

4.3.4.10 Die maximale Dehnung $\varepsilon_{fd,lim}$ von Klebebewehrungen, die auf Beton aufgeklebt sind, ist in der Wirkungszone begrenzt durch das Verbundversagen (oder durch das Zugversagen der Klebebewehrung) und kann für die Aufnahme von symmetrisch angeordneten Lasten für Betone bis C50/60 mit folgender Beziehung ermittelt werden:

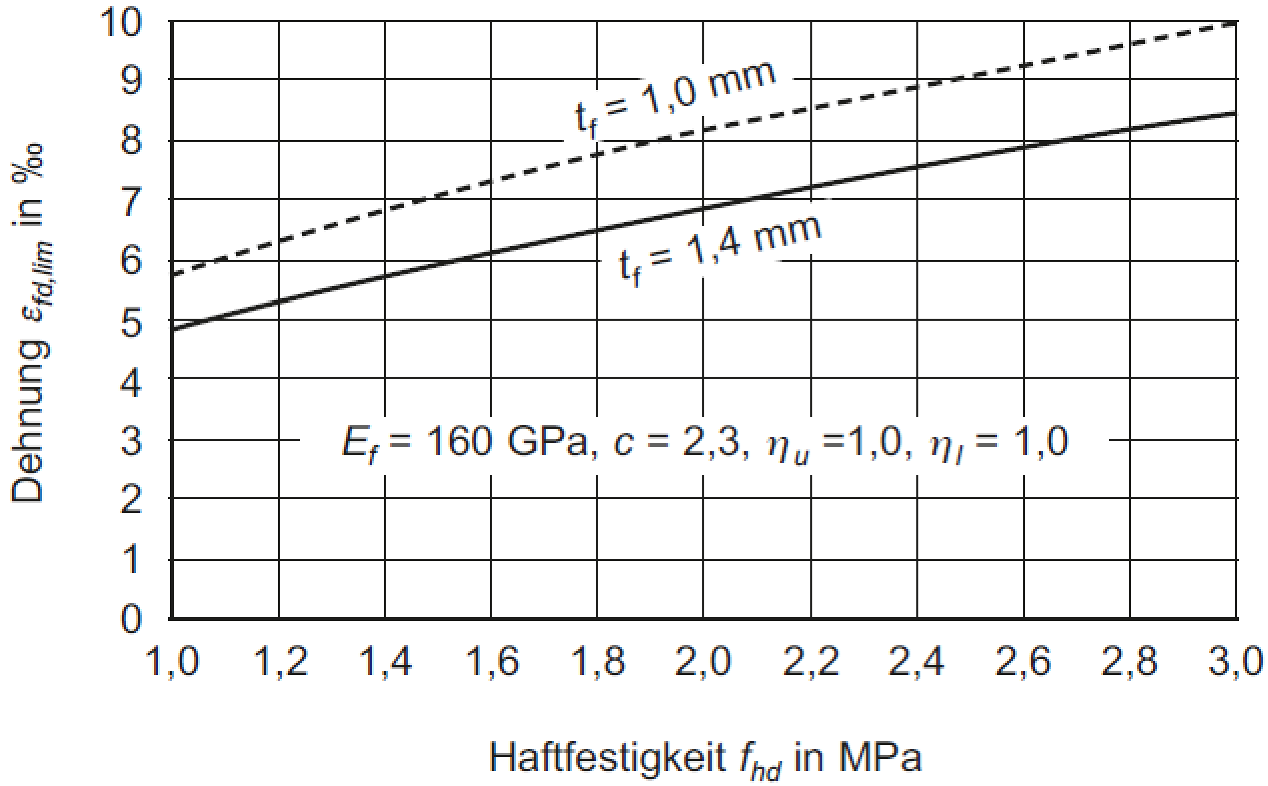
$$\varepsilon_{fd,lim} = c \cdot \eta_u \cdot \eta_l \cdot \sqrt{\frac{f_{hd}}{E_{fd} \cdot t_f}} + \varepsilon_{p\infty} \leq \varepsilon_{fud} \qquad f_{hd} \text{ in MPa, } E_{fd} \text{ in MPa, } t_f \text{ in mm} \qquad (33)$$

mit c = 2,3 für Lamellen aus Faserverbundwerkstoff und Stahl,
c = 1,6 für Gewebe und Gelege aus Faserverbundwerkstoff,
$\varepsilon_{p\infty} = F_{p\infty}/A_f$,
η_u und η_l gemäss Tabelle 2 und Tabelle 3.

Beispiele für maximale Dehnungen können Figur 5 entnommen werden.

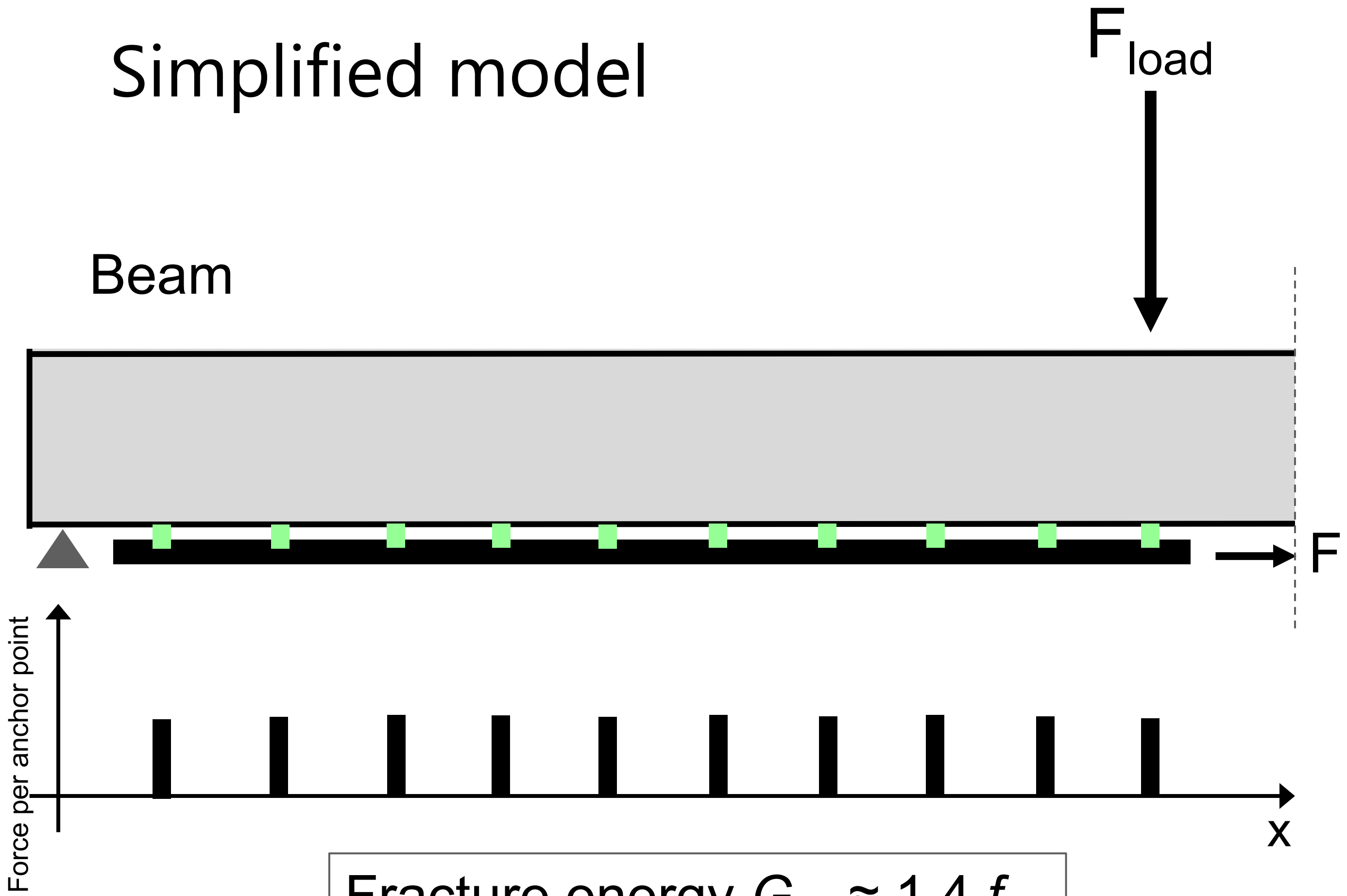
SIA 166 (2024)

Figur 5 Beispiele für Bemessungswerte der maximalen Dehnung $\varepsilon_{fd,lim}$ für Lamellen gemäss Gleichung (33) in Abhängigkeit des Bemessungswertes der Haftfestigkeit des oberflächennahen Betons f_{hd}

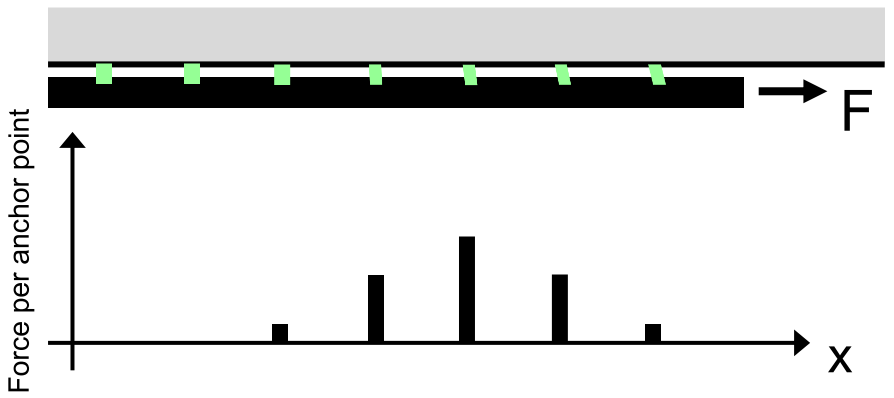


Simplified model

Beam



Lap-shear test



Fracture energy $G_{Fc} \approx 1.4\ f_h$

Fracture energy $G_{Fc} \approx f_h/8$

Specific fracture energy [N/mm] = [Nmm/mm²]: $\approx \tau_{f,mean} s_{f,max}$

$$F_{b0,Rd} = b_f \sqrt{2G_{Fcd}E_f t_f} \longrightarrow F_{b0,Rd} = \varepsilon_f E_f b_f t_f = b_f \sqrt{2G_{Fcd}E_f t_f} \longrightarrow \varepsilon_{fd,lim} = \sqrt{\frac{2G_{Fcd}}{E_f t_f}}$$

Cross-section analysis

- Cross-section analysis to determine the strains in the CFRP strips so that the debonding failure modes 1 and 2 according to SIA 166 can be verified.

by using e.g.

- Excel
- Matlab
- Software programs e.g. FAGUS
- …

Equations for cross-section analysis

Empa
Materials Science and Technology

Equilibrium:

$$C=T \rightarrow F_c+F_{s'}=F_s''+F_f''$$

Tensile force:

$$T=F_s''+F_f''=\frac{\varepsilon_s}{\kappa_s}E_sA_s+\frac{\varepsilon_f}{\kappa_f}E_fA_f$$

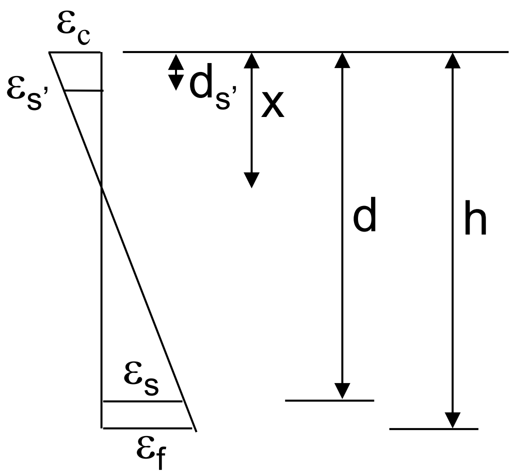
Compression force:

$$\begin{cases} F_{s'}=\varepsilon_{s'}E_sA_{s'} \\ F_c=\text{ see literature} \end{cases}$$

Compatibility:

$$\frac{\varepsilon_c}{x}=\frac{\varepsilon_f}{h-x}$$

$$\frac{\varepsilon_c}{x}=\frac{\varepsilon_s}{d-x}$$

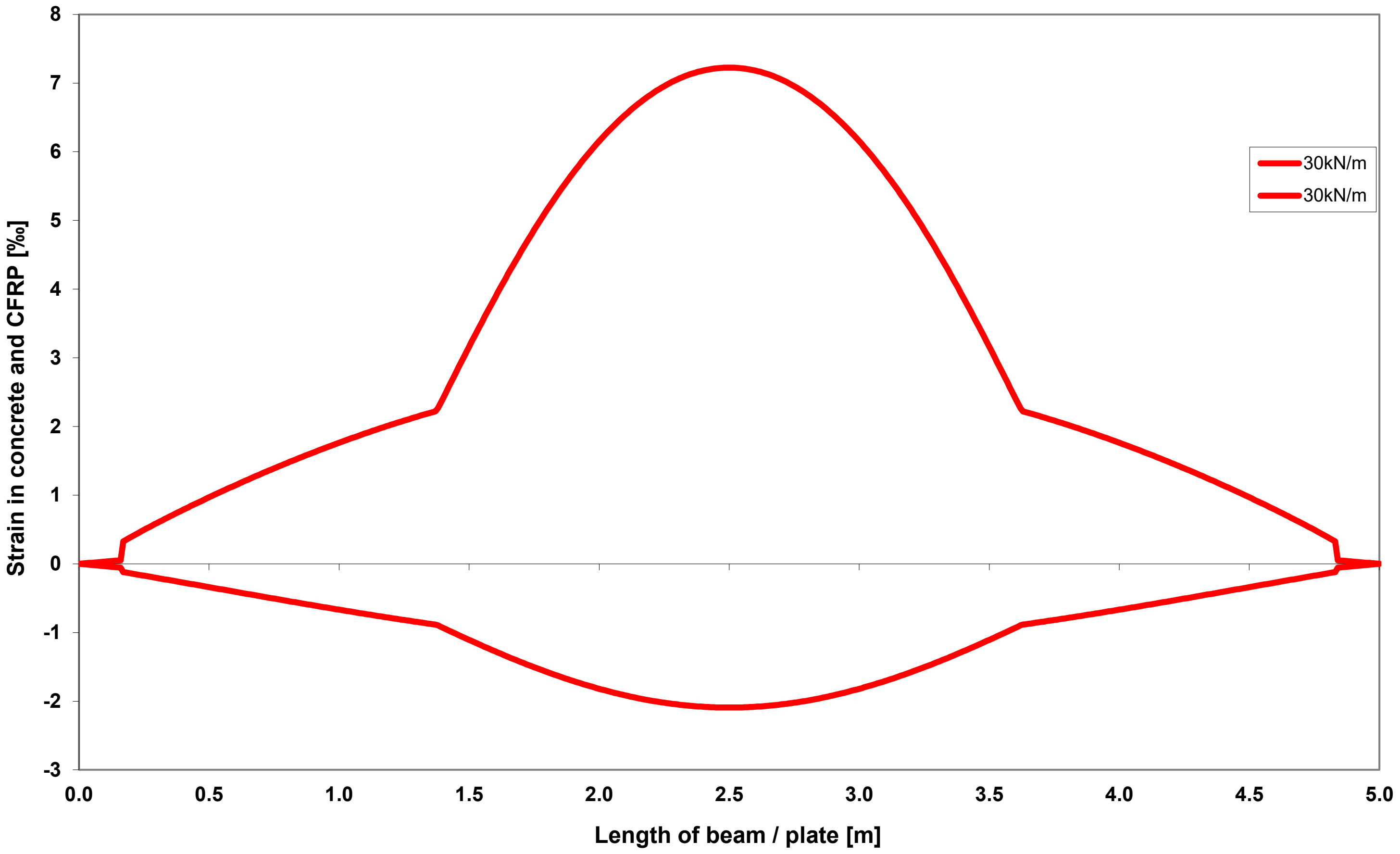


$$\frac{\varepsilon_c}{x}=\frac{\varepsilon_{s'}}{d_{s'}-x}$$

→ one equation with unknowns ε_c and x

Summary of calculation procedure: first step

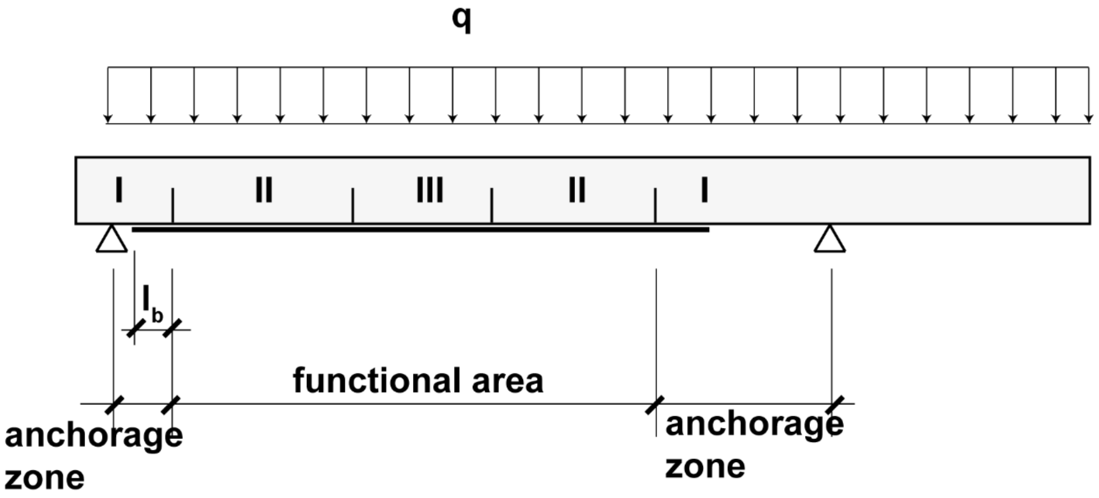
Cross-section analysis to determine the strain and forces along the structure for an assumed maximum load



Check debonding failure mode 1: end strip failure

Determine the location of last crack and define anchorage zone

Compare the existing force in the strip at the last crack with anchorage resistance which can be anchored in the anchorage zone

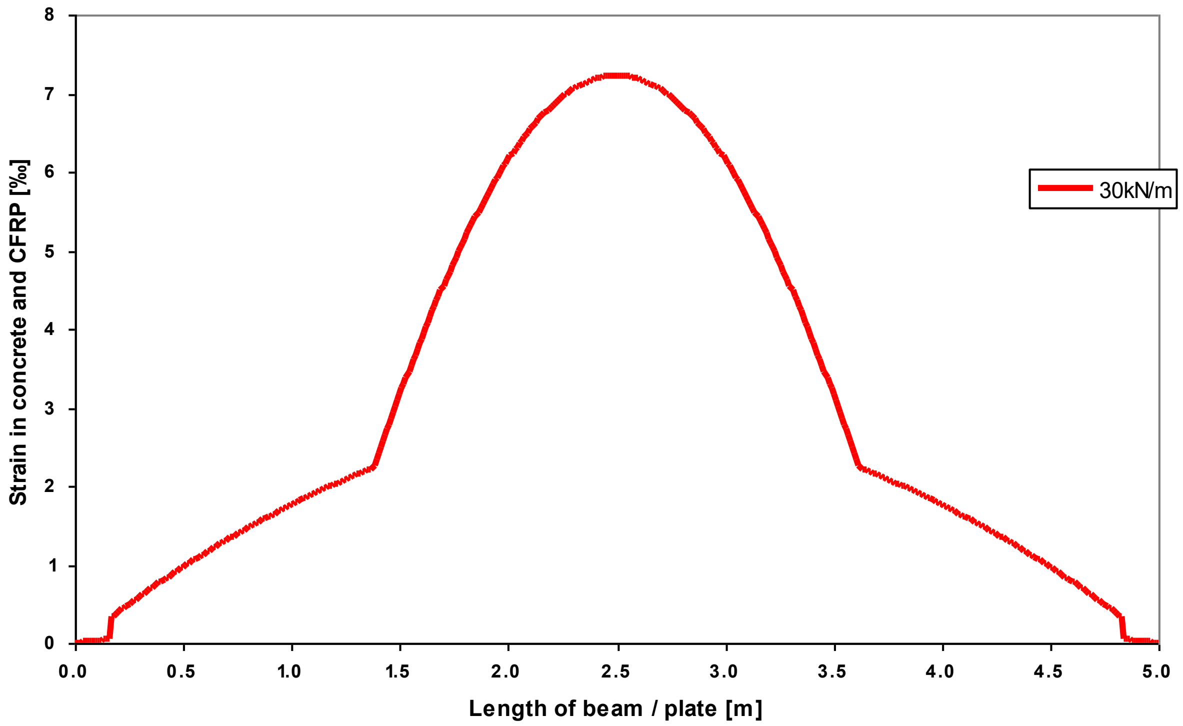


$$\boxed{F_{fcr} \leq F_{b,R}}$$

- I: uncracked cross-section
- II: cracked cross-section, internal steel in elastic state
- III: cracked cross-section, internal steel in yielding state

Check debonding failure mode 2:

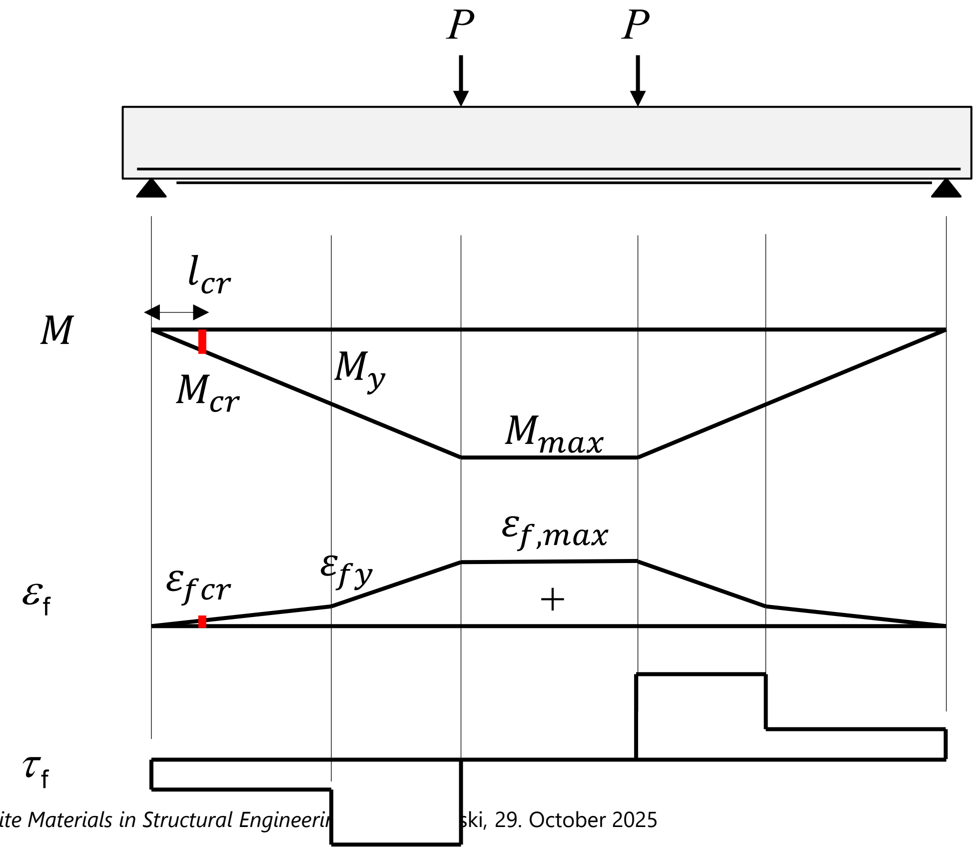
Compare maximum existing strain with maximum admissible strain



$$\boxed{\varepsilon_f \leq \varepsilon_{fd,lim}}$$

Calculation procedure for a four point beam

1. Determination of the characteristic (or design) bond properties G_{Fck}, $\tau_{c,max,k}$, $F_{b0,Rk}$, $l_{b0,Rk}$, $\varepsilon_{fk,lim}$
2. By using cross-section analysis (CSA) calculation of cracking moment M_{cr} and maximum moment M_{max} at maximum strain
   → Strip strain at last crack and at the maximum strain
3. Calculation of the failure load P_{ult}
4. Drawing of the bending moment, strip strain, shear stress along the beam axis
5. Calculation of the location of the last crack l_{cr} and then the length l_b , the two SIA verifications (if not fulfilled, the verifications shall be repeated with a lower failure load P_{ult}



Summary of the Equations from SIA

Empa
Materials Science and Technology

$$\boxed{F_{b0,Rd} = b_f \sqrt{2 G_{Fcd} E_f t_f}}$$

$$\boxed{l_{b0d} = 2.5 \sqrt{\frac{G_{Fcd} E_f t_f}{\tau_{c,max,d}^2}}}$$

$$\boxed{G_{Fcd} = \frac{1}{8} \eta_u \eta_l \frac{f_{hk}}{\gamma_h}}$$

G_{Fcd} in N/mm $\quad$ f_{hk} in N/mm²

$$\boxed{\tau_{c,max,d} = \frac{4}{3} \eta_u \eta_l \frac{f_{hk}}{\gamma_h}}$$

for $\boxed{l_{bd} < l_{b0d}:}$

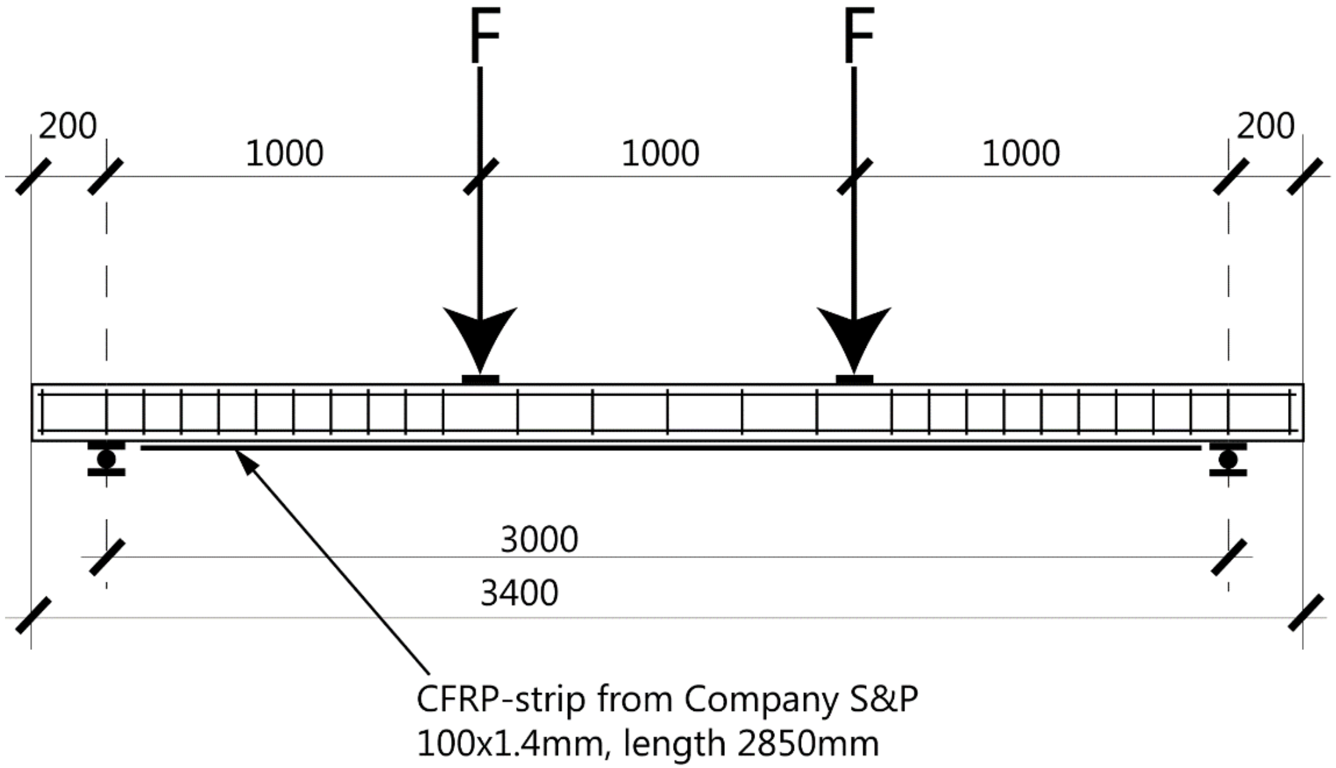
$$\boxed{F_{b,Rd} = F_{b0,Rd} \frac{l_{bd}}{l_{b0d}} \left(2 - \frac{l_{bd}}{l_{b0d}}\right)}$$

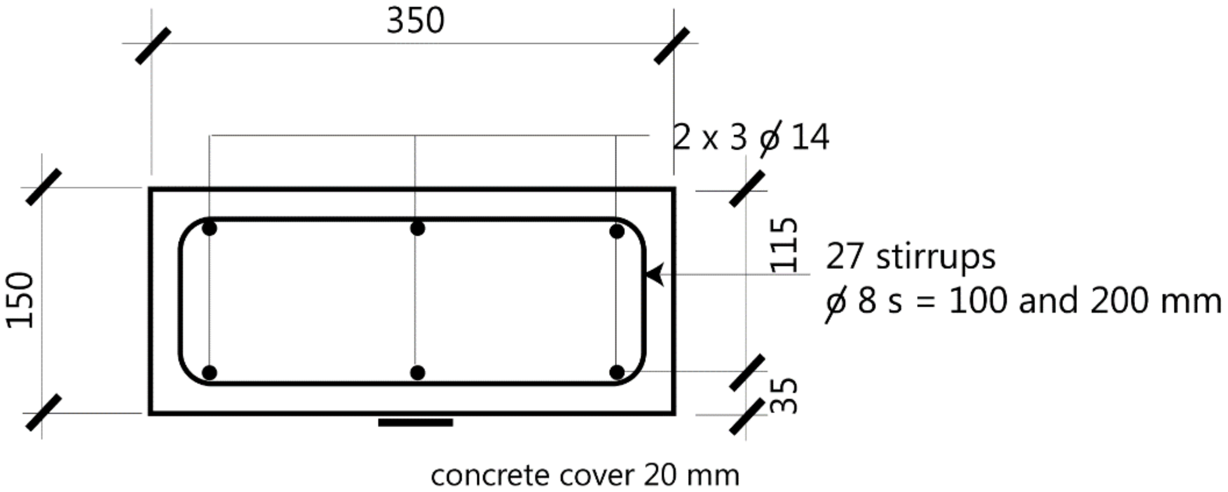
$$\boxed{\varepsilon_{fd,lim} = c\, \eta_u \eta_l \sqrt{\frac{f_{hd}}{E_f t_f}} \leq \varepsilon_{fud}}$$

c= 2.3 for strips, c=1.6 for fabrics

The reduction values η_u and η_l are tabulated in SIA 166(2024), they consider environment and loading conditions.

Example


F
F
200
1000
1000
1000
200
3000
3400
CFRP-strip from Company S&P
100x1.4mm, length 2850mm


350
2 x 3 ∅ 14
150
115
35
27 stirrups
∅ 8 s = 100 and 200 mm
concrete cover 20 mm


Empa

Materialeigenschaften

Empa
Materials Science and Technology

Concrete

- Cylinder compressive strength $f_{ck} = f_{cm} = 42.3$ MPa
- Tensile strength $f_{ctm} = 3.6$ MPa

≈ C40/50 (assumption for QSA)
$f_{hk} \approx 2.5$MPa (assumption)

CFRP

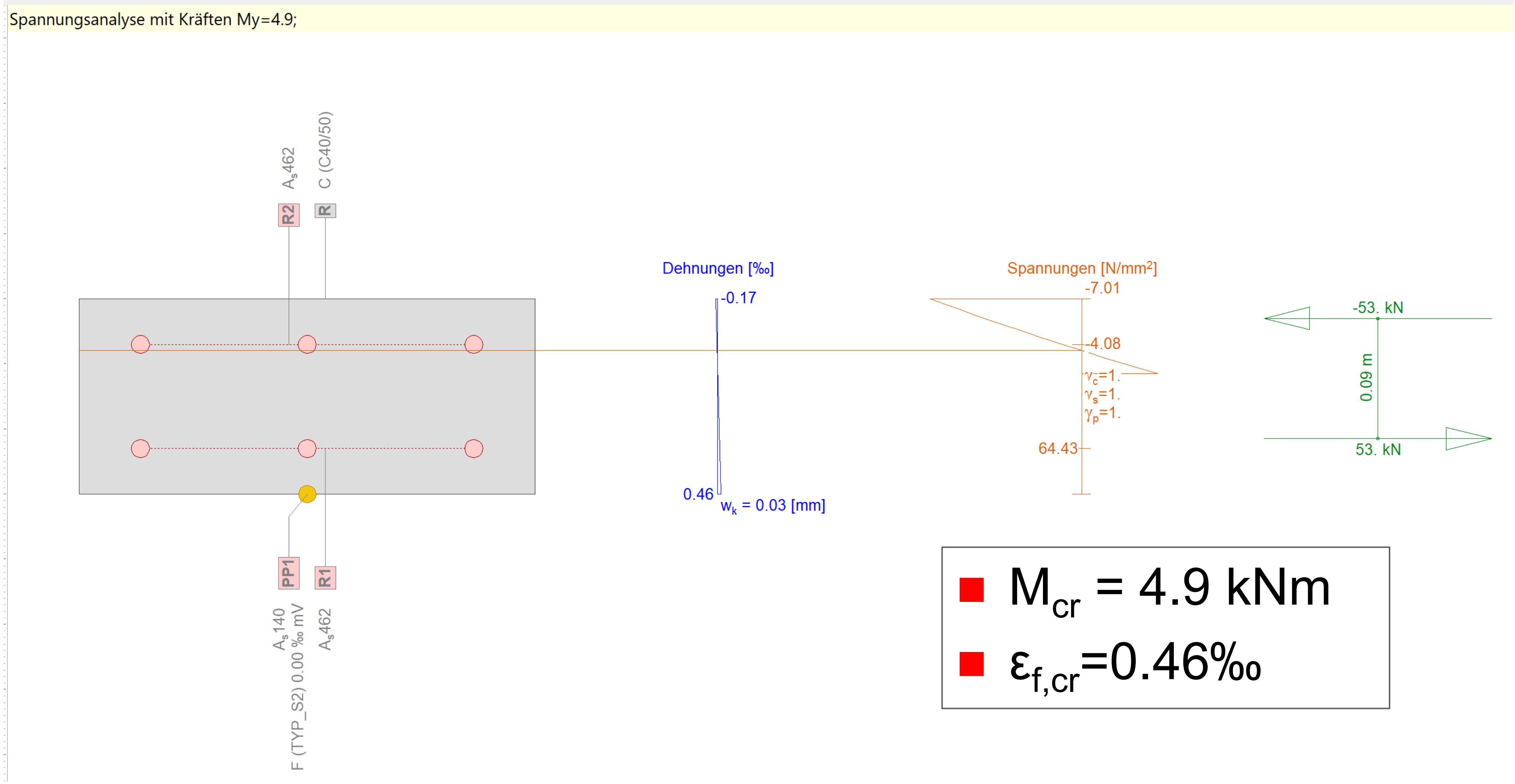
- Elasticity modulus $E_f = 170$ GPa
- Tensile strength $f_f = 2971$ MPa

Reinforcement

- $f_s = 545$ MPa
- $E_s = 205$ GPa

Calculation with FAGUS from the Company Cubus

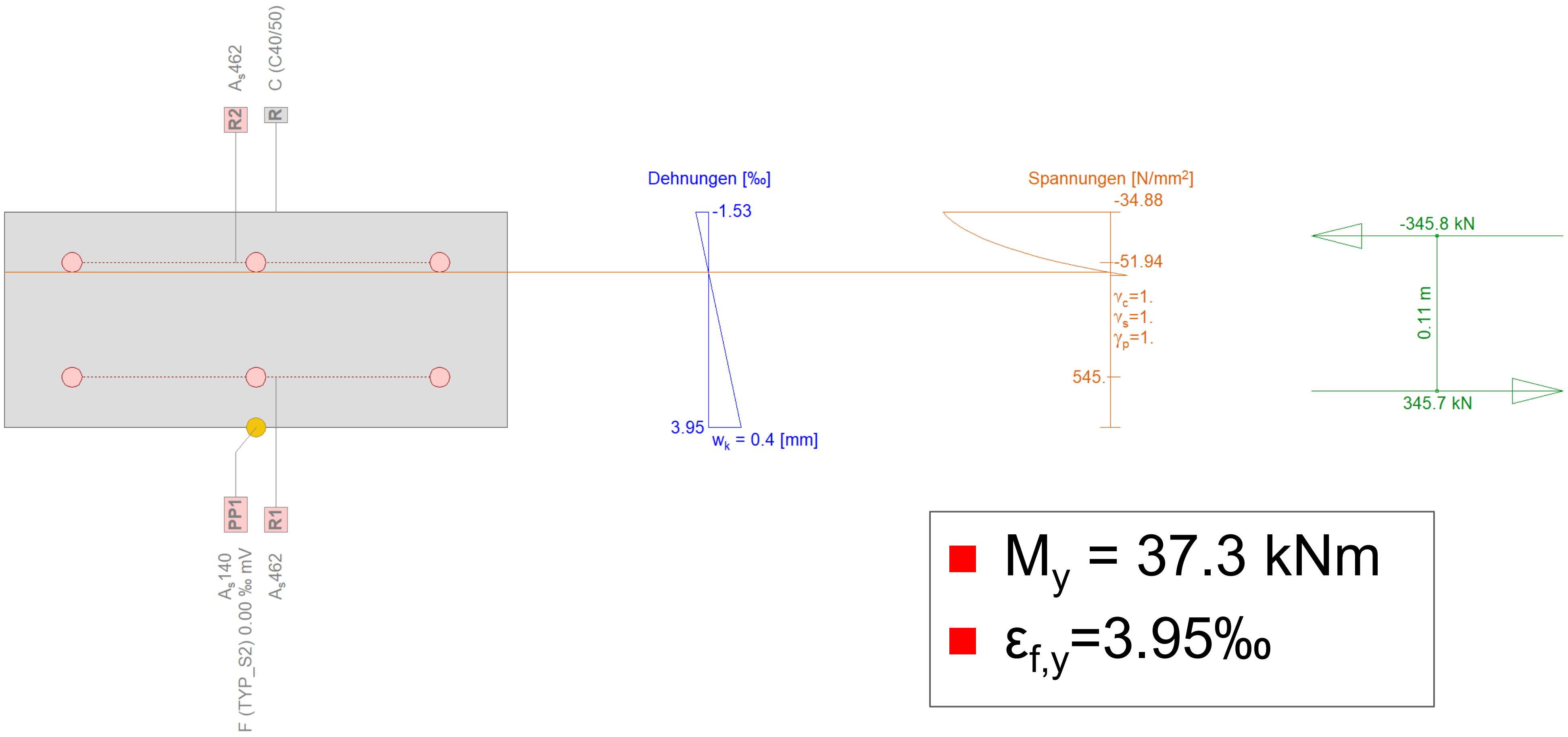
Cracking moment



- M_{cr} = 4.9 kNm
- $\varepsilon_{f,cr}$=0.46‰

Yielding moment

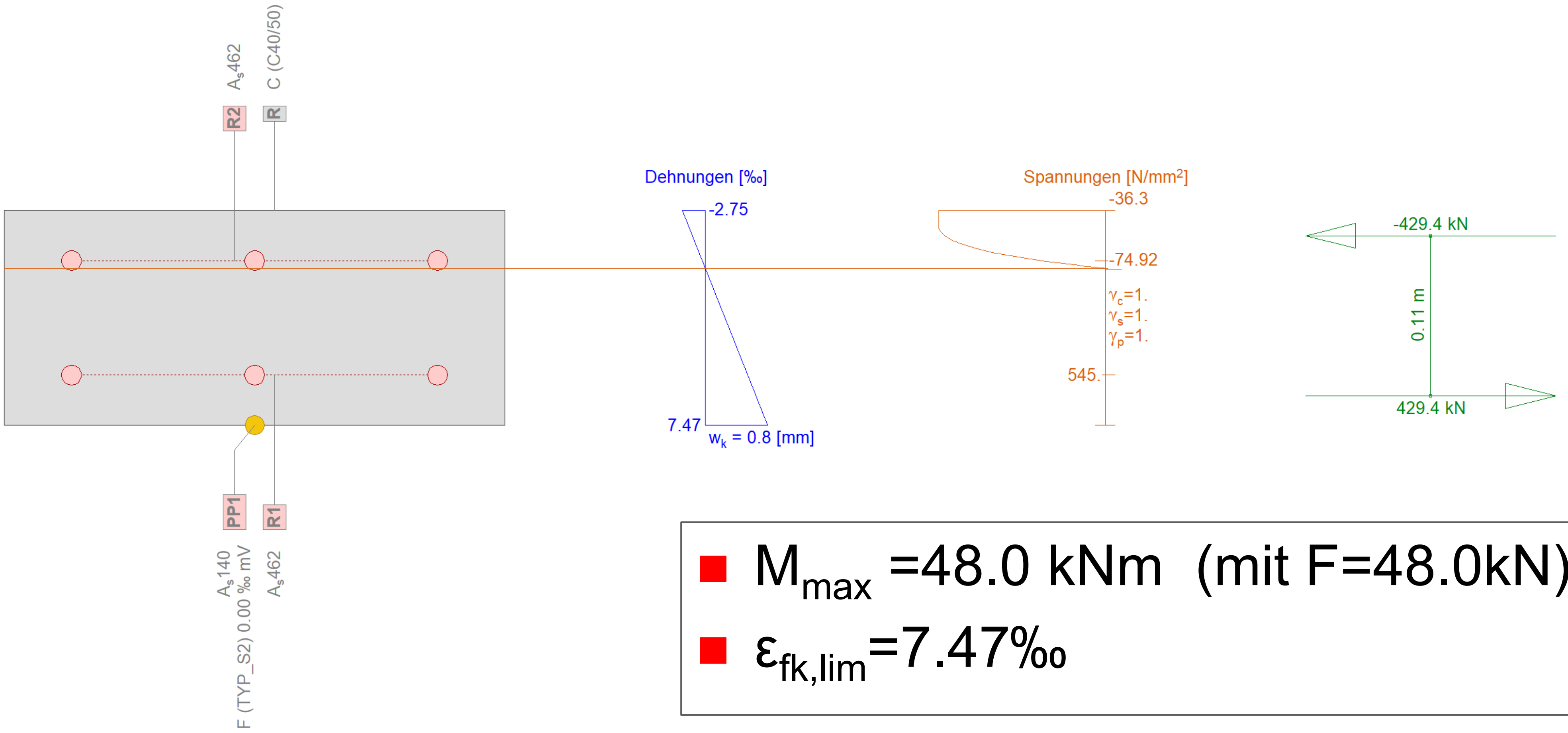
Spannungsanalyse mit Kräften My=37.3;



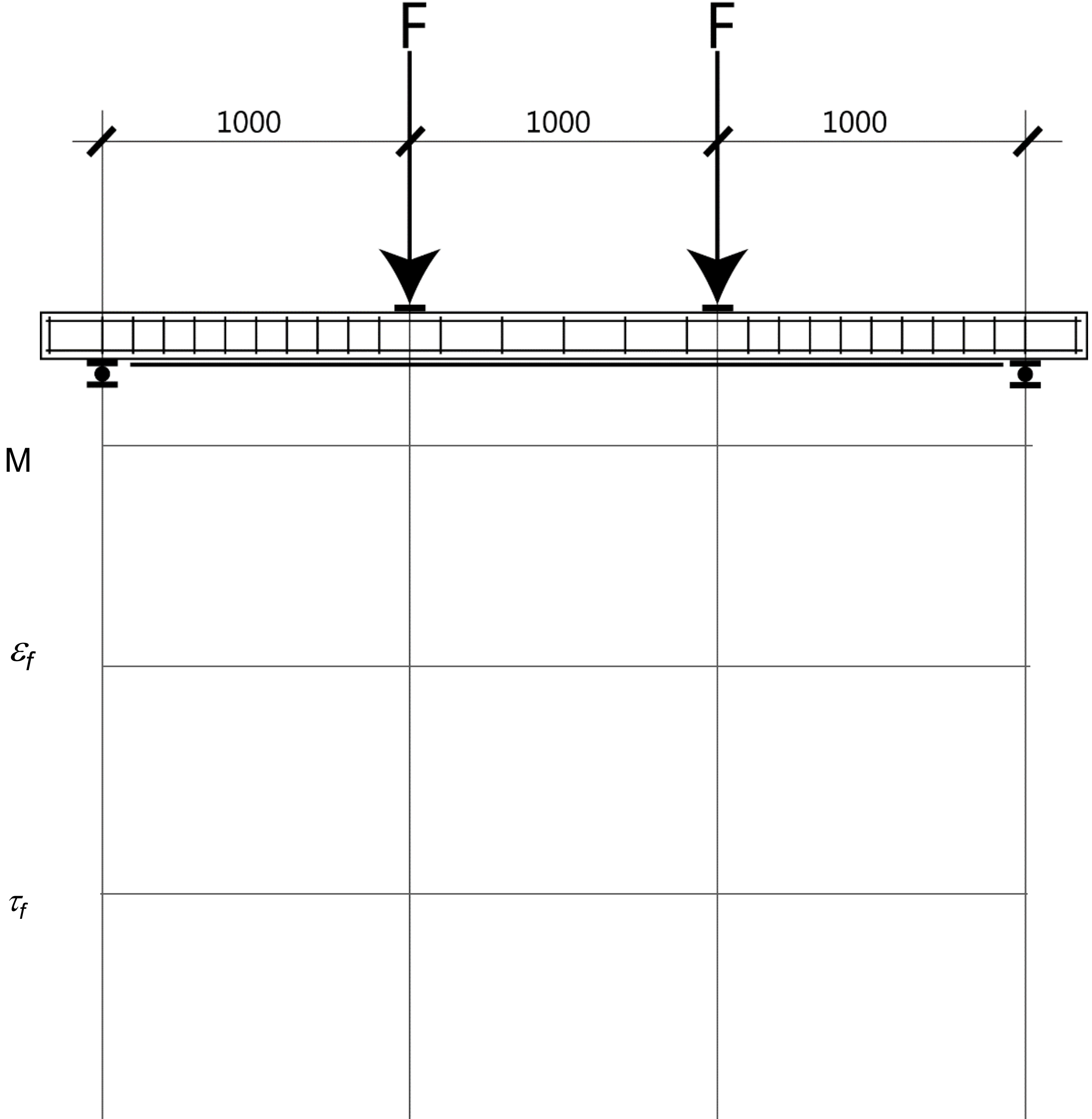
- $M_y = 37.3$ kNm
- $\varepsilon_{f,y}$=3.95‰

Bruchmoment

Spannungsanalyse mit Kräften My=48.0;



- M_{max} =48.0 kNm (mit F=48.0kN)
- $\varepsilon_{fk,lim}$=7.47‰



aus QA:

M_{cr} 4.9 kNm

$\varepsilon_{f,cr}$=0.46‰

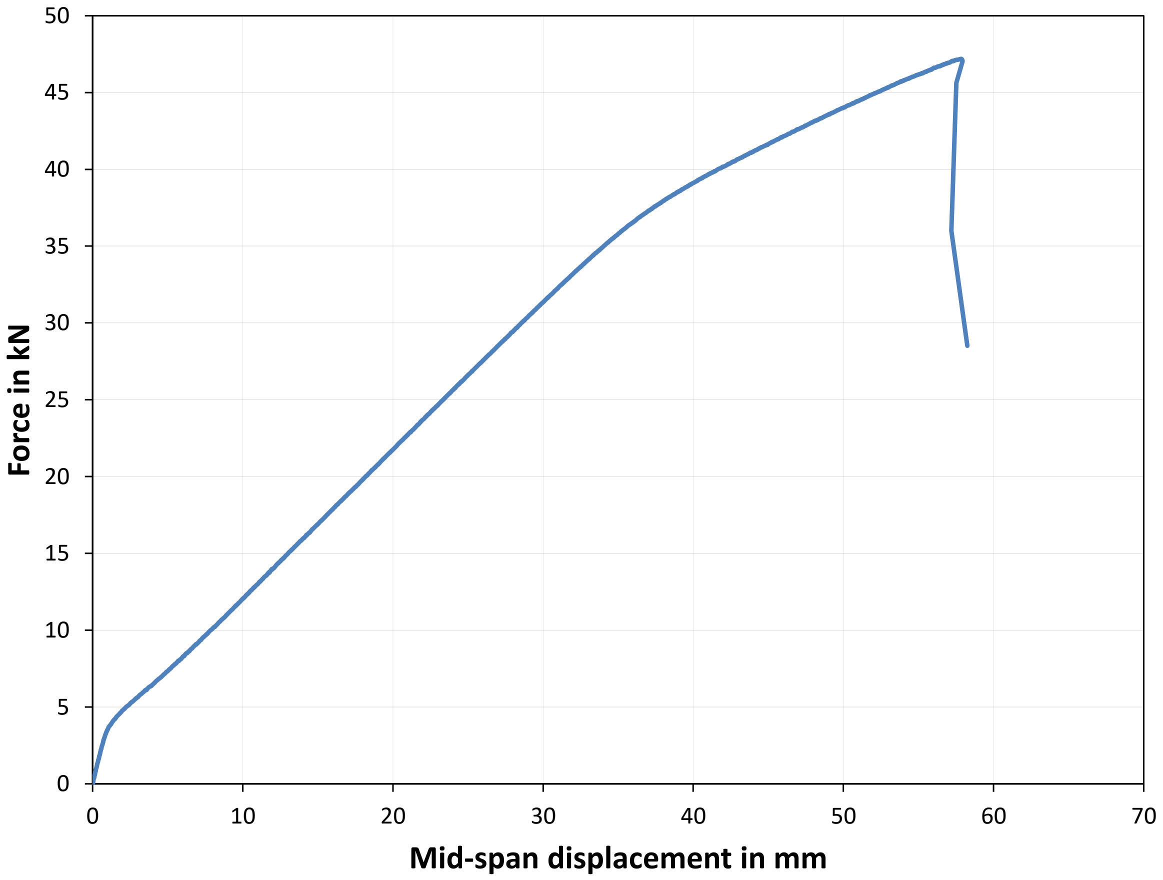
M_y = 37.3 kNm

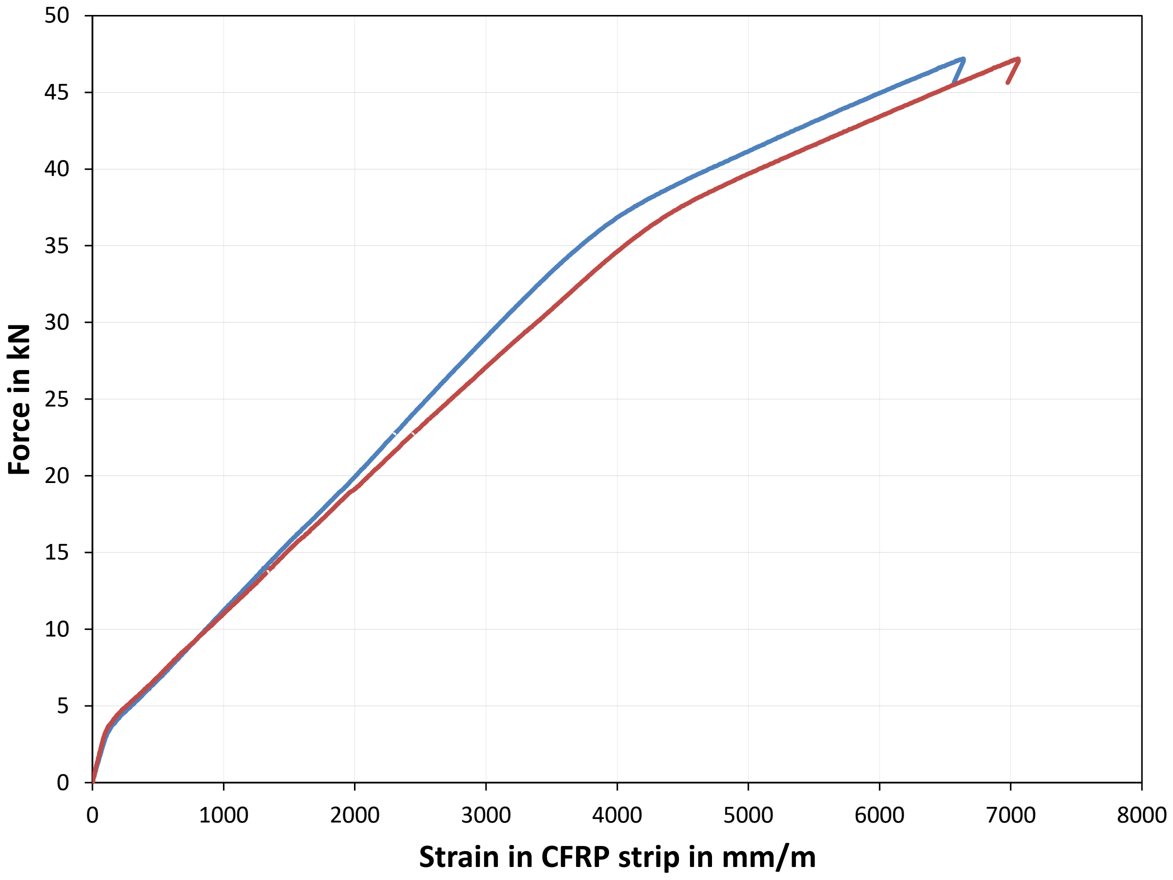
$\varepsilon_{f,y}$=3.95‰

M_{max} =48.0 kNm (mit F=48.0kN)

$\varepsilon_{f,max}$=7.47‰

Resultate





→ **Video: Failure test**

Several additional topics according to Swiss code 166 (2024)

- Assessment of existing structures
- Ultimate limit state (ULS)
- Serviceability (SLS)
  - Stresses
  - Deflections
- Check of deformation capacity

Assessment of existing structure

Empa Materials Science and Technology

- Consider existing stresses and deflections in the structure before strengthening
- Good assessment: geometry, strength
- Internal reinforcement (we need a minimum value..., otherwise we risk a brittle behavior, risk of premature concrete crushing if steel is neglected in calculations)
- Concrete property (tensile strength good enough?)
- Static system
- ...
- Use conversation codes (Erhaltungsnormen) to check if a strengthening is necessary or not!
  - SIA269 Basics
  - SIA269/1 Actions
  - SIA269/2 Reinforced Concrete
- If a strengthening is necessary, use SIA260, 261, 262 and 166

Ultimate limit state

$$E_d \leq R_d$$

Design value of effects of action ≤ Design value of ultimate resistance

Definition of hazard scenarios (Gefährdungsbilder)

do not forget the hazard scenario “failure of externally bonded reinforcement” (accidential scenario)

Determination of the design values according to SIA 166 (2024)

Empa
Materials Science and Technology

4.2 **Bemessungswerte**

4.2.1 Für den Bemessungswert X_d der Baustoffeigenschaft gilt:

$$X_d = \frac{\eta \cdot X_k}{\gamma_f} \quad \text{bzw.} \quad X_d = \frac{\eta \cdot X_k}{\gamma_h} \qquad (5)$$

wobei γ_f, γ_h und η nach Tabelle 5 ermittelt werden.

X_k = characteristic value of material property

Depending on failure in strengthening material or bond failure:

η = reduction factor

γ_f or γ_h = resistance factor

→ see Table 5 in SIA166 (2024)

Table 5 in SIA 166(2024)

Empa
Materials Science and Technology

4.2.2 Für die spezifischen Versagensarten von Klebebewehrungen sind für Festigkeitswerte die Beiwerte gemäss Tabelle 5 einzusetzen.

Tabelle 5 Beiwerte für die Bestimmung der Bemessungswerte der Baustoffeigenschaften

Verstärkungsversagen		
Stahllamelle	γ_f = 1,05	$\eta = \eta_e \cdot \eta_l$ mit η_e gemäss Tabelle 1 und η_l gemäss Tabelle 3
Faserverbundwerkstoff-Lamelle	γ_f = 1,10	
Gewebe/Gelege aus Faserverbundwerkstoff	γ_f = 1,30	
Verbundversagen		
Verbundversagen im Untergrund Beton	γ_h = 1,50, weitere η-Werte gemäss SIA 262	$\eta = \eta_u \cdot \eta_l$ mit η_u gemäss Tabelle 2 und η_l gemäss Tabelle 3
Verbundversagen im Untergrund Stahl bzw. im Klebstoff	γ_h = 1,50, weitere η-Werte gemäss SIA 263	
Verbundversagen im Untergrund Holz	γ_M und weitere η-Werte gemäss SIA 265 und SIA 265/1	
Verbundversagen im Untergrund Mauerwerk	γ_M entsprechend SIA 266	
Versagen von Verankerungshilfsmitteln		
Bei der Verwendung von Verankerungshilfsmitteln entsprechend 4.1.6 sind die Beiwerte unter Berücksichtigung des Einsatzzwecks mit einer wissenschaftlichen Methode zu bestimmen.		

SLS and evenness

- SIA 166: Serviceability (SLS)
  - Stresses in the internal reinforcement should not exceed the stresses as defined in SIA262 (depending on the requirements: normal, increased, high)
  - Deflections have to be checked

- Evenness of concrete surface
  - for 2 m measurement length (Messlatte): max. 5 mm tolerance
  - for 0.3 m measurement length (Messlatte): max. 1 mm tolerance

Deformation capacity according EN 166 (2024)

Empa
Materials Science and Technology

4.3.4.7 Bei vorwiegend auf Biegung beanspruchten Bauteilen ist zur Gewährleistung einer minimalen Verformbarkeit, d. h. zur Verhinderung der Bruchart «Betonstauchen vor Stahlfliessen», die Höhe der Druckzone beim Bruch wie folgt zu beschränken:

$$\frac{x}{d} \leq 0{,}5 \cdot \frac{435}{f_{sd}} \quad (27)$$

x: height of bending compression zone
d: static height
f_{sd}: design value of yield strength of internal reinforcement steel

Literature: Codes/Guidelines

- SIA166 (2024) Klebebewehrungen für die Verstärkung bestehender Tragwerke (Adhesively bonded reinforcement for strengthening existing structures). Schweizerischer Ingenieur- und Architektenverein SIA
- SIA166 (2004) Klebebewehrungen (Externally bonded reinforcement). Schweizerischer Ingenieur- und Architektenverein SIA.
- SIA (2004) D 0209, Dokumentation, Klebebewehrung, Einführung in die Norm SIA 166.
- *fib* (2001) Externally bonded FRP reinforcement for RC structures - Bulletin 14. International Federation for Structural Concrete (fib), Switzerland.
- *fib* (2019) Externally applied FRP reinforcement for concrete structures - Bulletin 90. International Federation for Structural Concrete (fib), Switzerland.
- CNR (2004) Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Existing Structures, CNR-DT 200/2004. CNR - Advisory Committee on Technical Recommendations for Construction, Rome, Italy.
- ACI (2008) ACI440.2R-08, Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures. American Concrete Institute.
- TR55 (2012) Design guidance for strengthening concrete structures using fibre composite materials, Third Edition. Technical Report No. 55 of the Concrete Society, UK.

Literature

PhD Theses (can freely be downloaded from www.research-collection.ethz.ch)

- Ulaga T (2003) Dissertation ETH Nr. 15062, Betonbauteile mit Stab- und Lamellenbewehrung: Verbund- und Zuggliedmodellierung, http://dx.doi.org/10.3929/ethz-a-004525392
- Czaderski C (2012) Dissertation ETH No. 20504, Strengthening of reinforced concrete members by prestressed, externally bonded reinforcement with gradient anchorage, http://dx.doi.org/10.3929/ethz-a-007569614

Book chapters

- Motavalli, M., C. Czaderski, A. Schumacher, and D. Gsell, Fibre reinforced polymer composite materials for building and construction, in *Textiles, polymers and composites for buildings*, G. Pohl, Editor. 2010, Woodhead Publishing Limited: Cambridge UK. p. 69-128.
- Czaderski C., Flexural and Shear Strengthening of Reinforced Concrete Structures, in *The International Handbook of FRP Composites in Civil Engineering*, CRC Press, 2013.
  p. 235-252

Visit at Empa, 26. November and 10. December 2025, Transport from ETH-Hönggerberg to Empa:

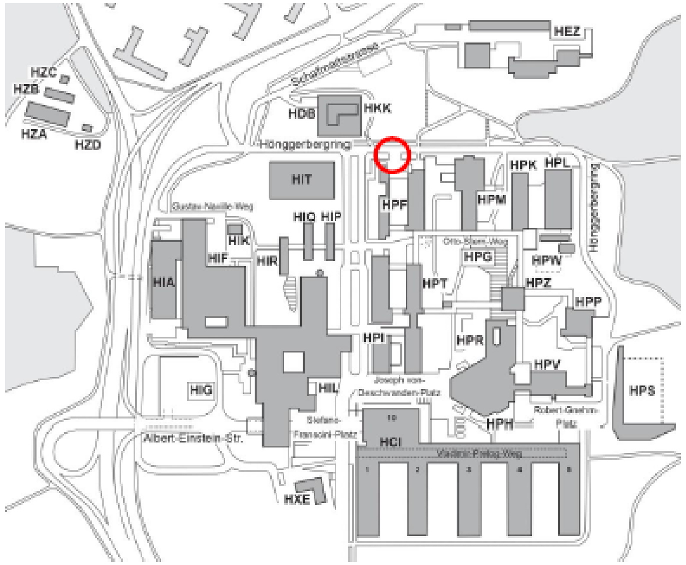
Transport with Empa-Bus:

Meeting point at the ETH Hönggerberg at the *Anlieferung HPF 51* (opposite of restaurant Bellavista), **see red circle**

Time: **15:30**

Contact persons:

Christoph Czaderski Phone: 058 765 42 16
Kilian Strasser Phone: 058 765 65 94



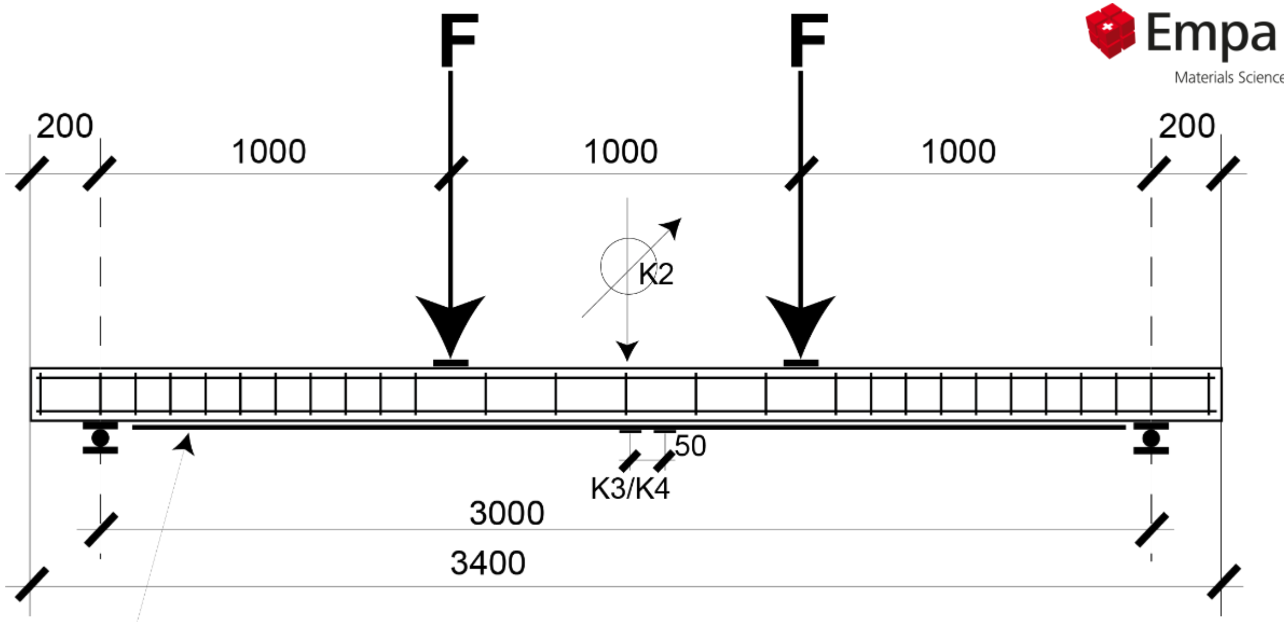
Individual transport to Empa:

Meeting point at Empa "Empfang" (NEST building) at Überlandstrasse 129 in Dübendorf

Time: **approx. 15:45**

Please carry working dresses! After the exercise, individual return journey from station Dübendorf. Please let me know, if you cannot attend! (christoph.czaderski@empa.ch)

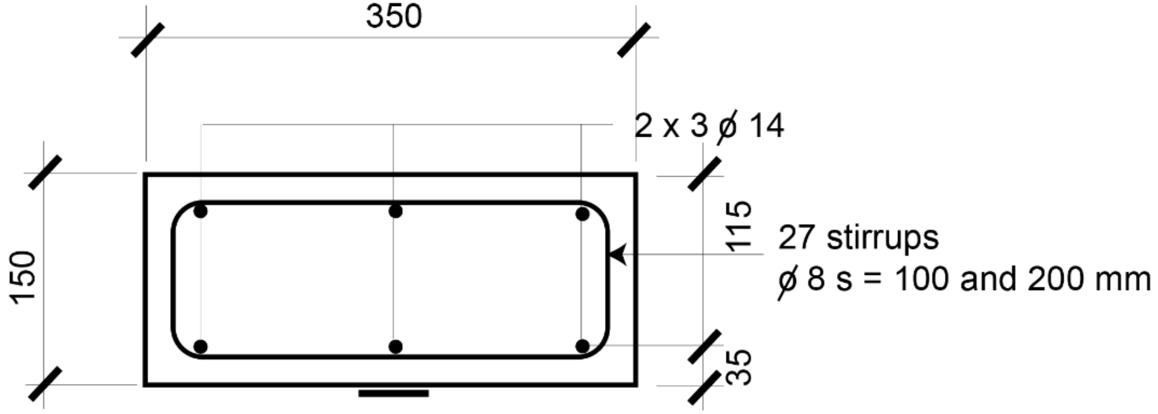
First part of the laboratory competition: prediction of the failure load of this beam



Concrete
C35/45

Steel
$f_s = 487$ N/mm^2
$f_t = 566$ N/mm^2

CFRP
$E_f = 150'000$ N/mm^2



$f_{c,cube}$ 68 days = 47.2 MPa
($f_c = 0.8 * f_{c,cube} = 37.8$ MPa)
Possible assumption for calculations:
concrete C35/45 ($f_{ctm} = 3.2$ MPa, $f_{ck} = 35$ MPa)

→ Video: application of the CFRP strip

Procedure for the determination of the failure loads
(see the example in slides 83-88 and its «Musterlösung», and see slide 81):

1. Assumption of f_{hk}
2. Determination of the characteristic bond properties
3. CSA
4. Drawing of the bending moment, strip strain, shear stress
5. Determination of l_{cr}, l_b and then the verification of end anchorage and maximum strain

Second part of the laboratory competition: prediction of the failure load of four concrete cylinders

Empa
Materials Science and Technology

Empa
Abteilung Beton / Asphalt

Prüfprotokoll (gilt nur zusammen mit dem Berichtsdeckblatt)

Auftragsnummer: 5214036589

Seriebezeichnung: W/Z 0.55_28d

Prüfung:	Druckversuch	Herstelldatum:	26.06.2025
Norm:	EN 12390-3:2009	Probenalter:	28
Prüfkörper:	Würfel 150x150x150	Prüfdatum:	24.07.2025
Betonklasse	-----	Prüfer:	kap

Bezeichnung	**Länge [mm]**	**Breite [mm]**	**Höhe [mm]**	**Masse [g]**	**Rohdichte [kg/m³]**	**Höchstkraft [kN]**	**Festigkeit [MPa]**
13	149.7	149.5	146.5	7730	2358	1046.5	46.8
14	149.5	150.2	146.3	7855	2391	1081.8	48.2
Mittelwert					2374		47.5
Std. Abw.					24		1.0

Empa
Abteilung Beton / Asphalt

Prüfprotokoll (gilt nur zusammen mit dem Berichtsdeckblatt)

Auftragsnummer: 5214036589

Seriebezeichnung: W/Z 0.55 28d

Prüfung:	Druckversuch	Herstelldatum:	26.06.2025
Norm:	EN 12390-3:2009	Probenalter:	28
Prüfkörper:	Zylinder 150x300	Prüfdatum:	24.07.2025
Betonklasse	-----	Prüfer:	kap

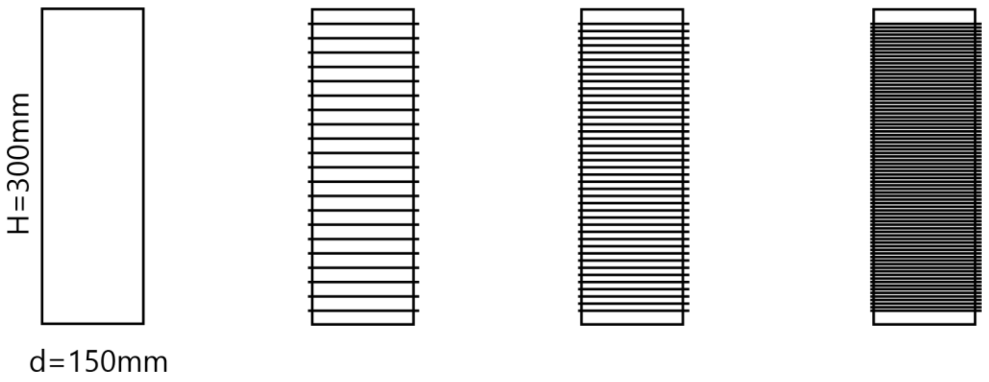
Konformitätskriterien nach EN 206:2013 für Erstherstellung: nicht möglich

Name	**Durchmesser [mm]**	**Höhe [mm]**	**Masse [g]**	**Rohdichte [kg/m³]**	**Bruchlast [kN]**	**Festigkeit [MPa]**
1	149.7 Bruchtyp:	296.0	12625	2423	708.0	40.2
2	149.5 Bruchtyp:	296.0	12525	2411	688.1	39.2
Mittelwert				2417		39.7
Std. Abw.				9		0.73

Compression tests on four concrete cylinders

Second part of the laboratory competition: prediction of the failure load of four concrete cylinders

Concrete cylinder 150x300mm



unstrengthened | strengthened with one layer of CFRP | strengthened with two layers of CFRP | strengthened with three layers of CFRP

CFRP: SikaWrap-231 C applied with Sikadur-330

Time schedule:

Casting on 26.06.2025
24.07.2025 after 28 days: Testing of concrete cube compressive strength
26.11.2025: Application of CFRP sheet on the cylinders
10.12.2025: Failure tests

ETH Course „Fibre Composite Materials in Structural Engineering"
Christoph Czaderski / 23.10.2025

Materials Science and Technology

Lab competition

Empa
Materials Science and Technology

- Video of the experiment on the beam will be presented on 26.11.2025
- Lap experiments on the cyclinders will be performed on 10.12.2025

- Who makes the best prediction? The best predictions are awarded with a price.
- Predictions (in kN):
  Failure load of the Beam (by 25.11.2025)
  Failure loads of Cylinders 1 to 4 (by 09.12.2025)

- →Submission of the calculations and the failure loads by email to: christoph.czaderski@empa.ch