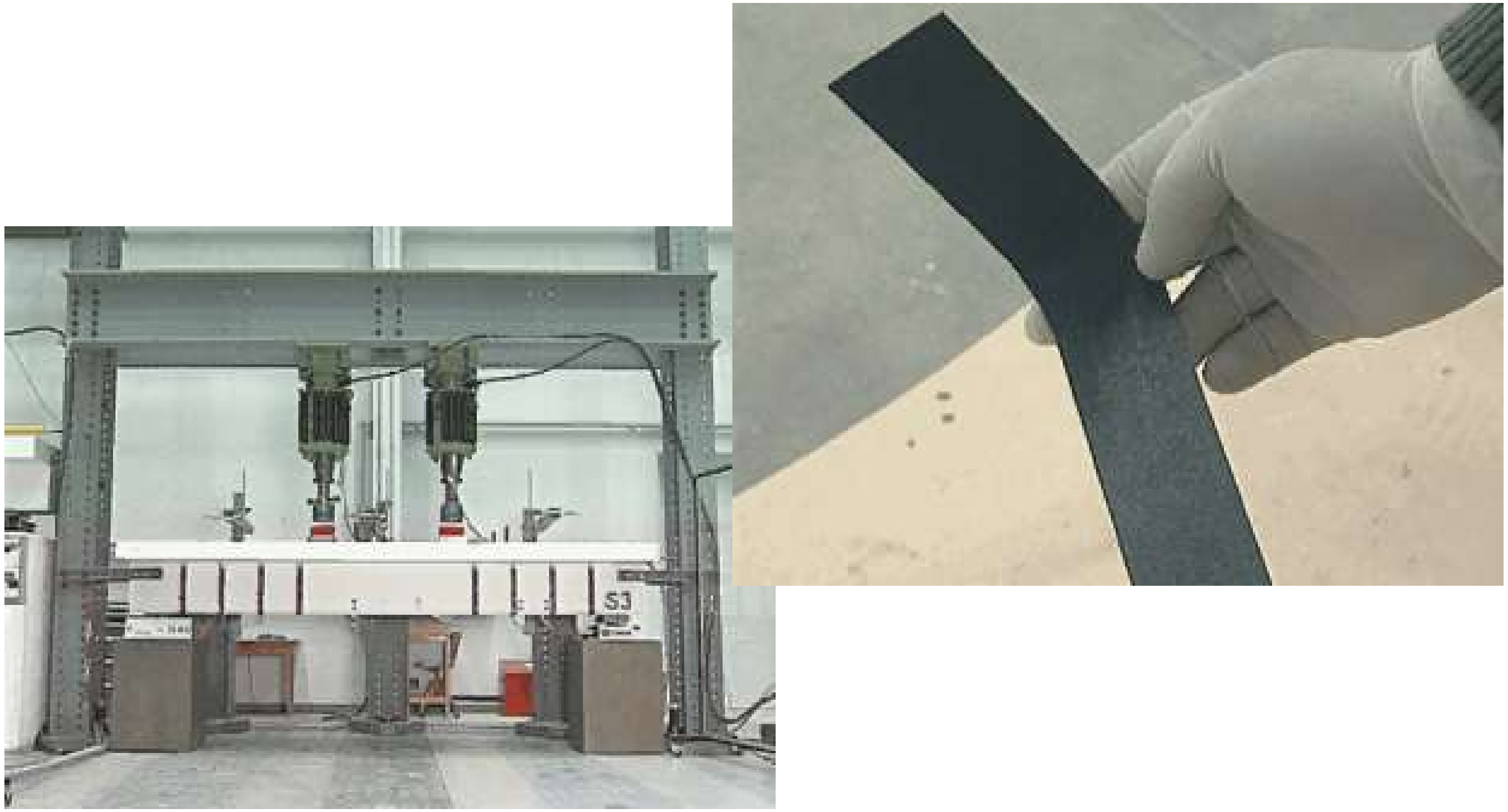
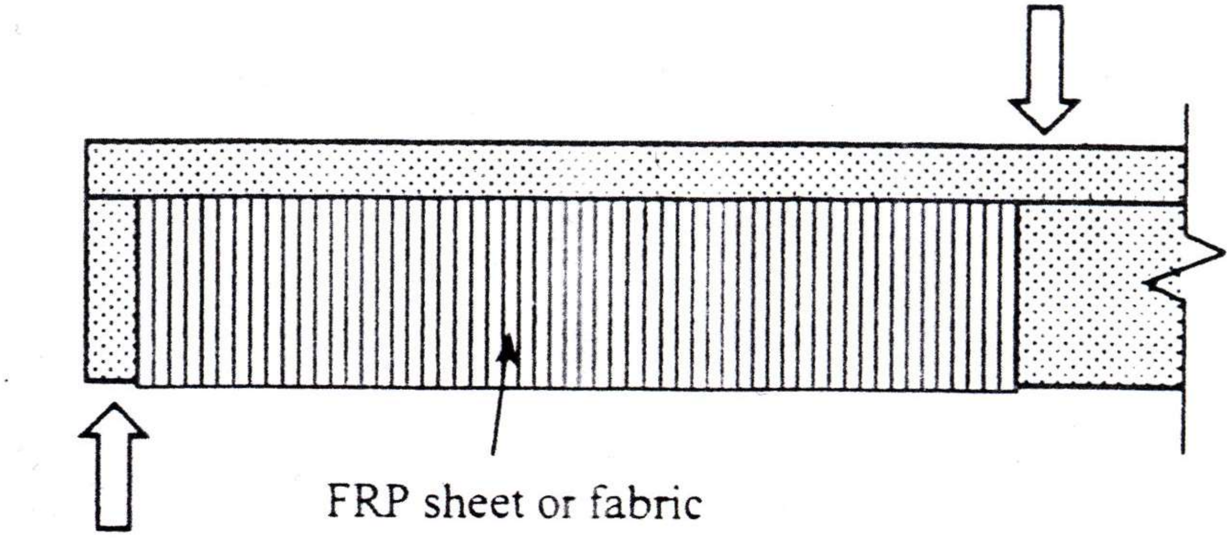
Table of content:

Strengthening in shear

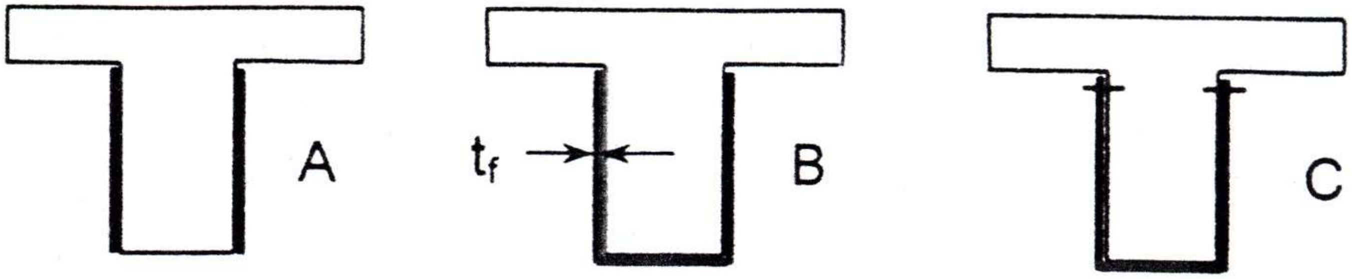
Book Composite for Construction, L. C. Bank, Chapter 10

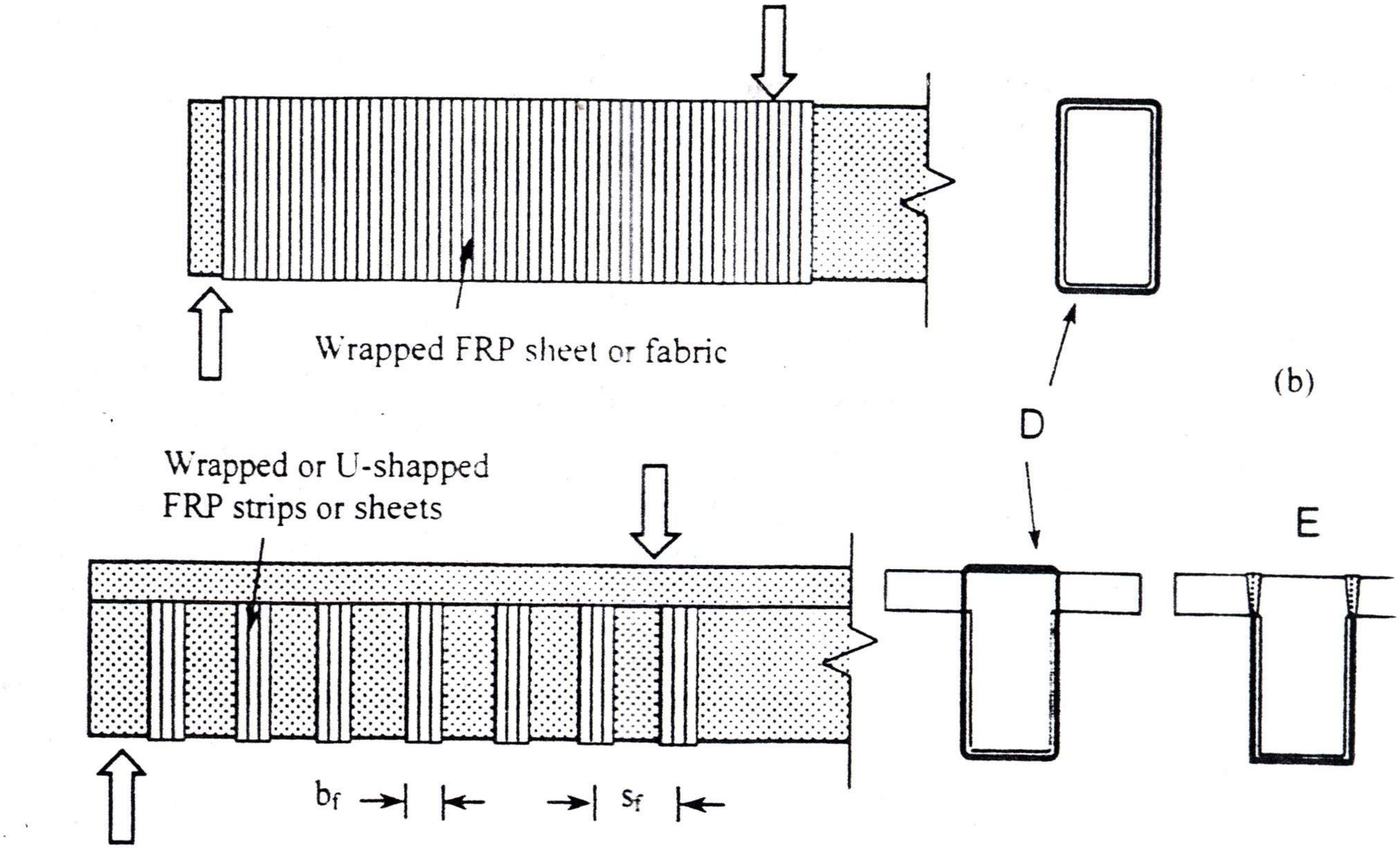


General



(a)

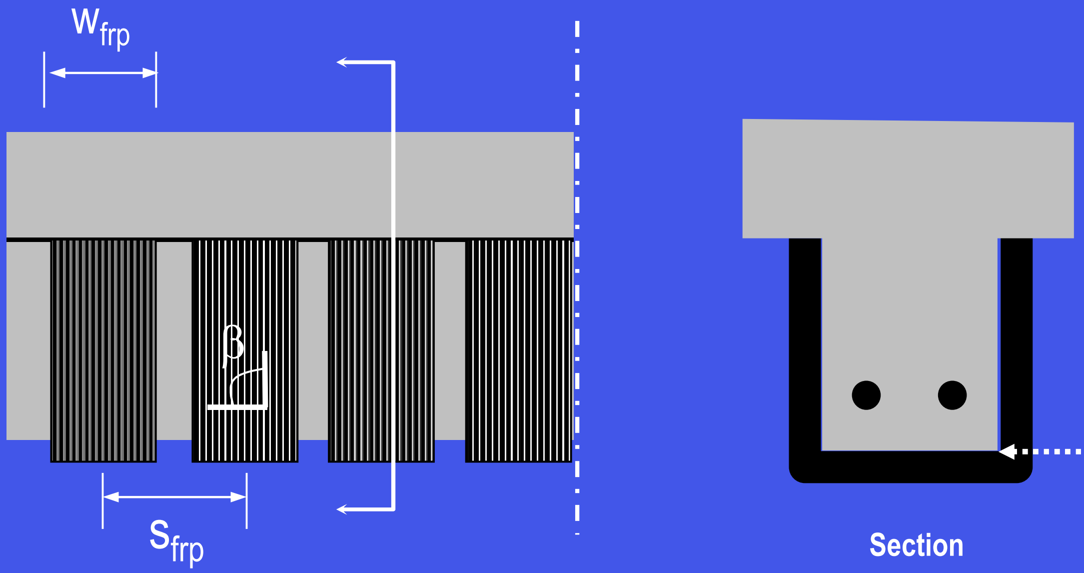



Wrapped FRP sheet or fabric
(b)
D
Wrapped or U-shapped FRP strips or sheets
E
b_f
s_f

Beam/One-Way Slab Strengthening

Shear Strengthening

Assumptions



To avoid stress concentrations, allow for a minimum radius of 15 mm

Beam/One-Way Slab Strengthening

External strengthening with FRPs:

Flexural failure → Generally fairly ductile

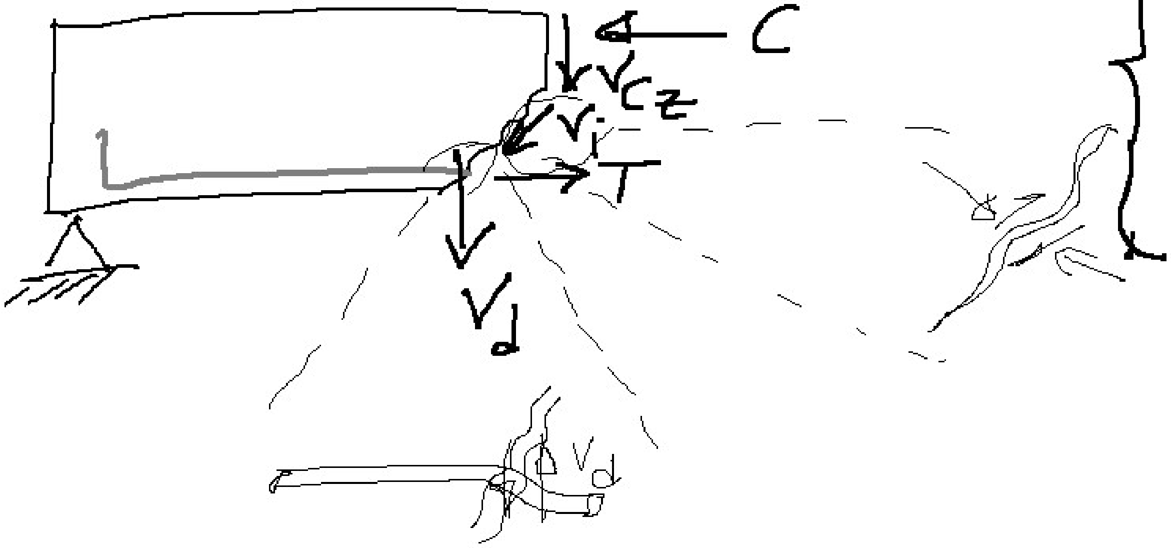
Shear failure → Sudden and brittle

↓

Undesirable failure mode

Control shear deformation to avoid sudden failure

Shear capacity of RC

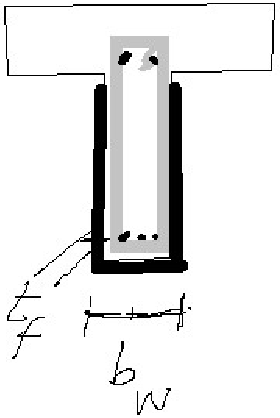


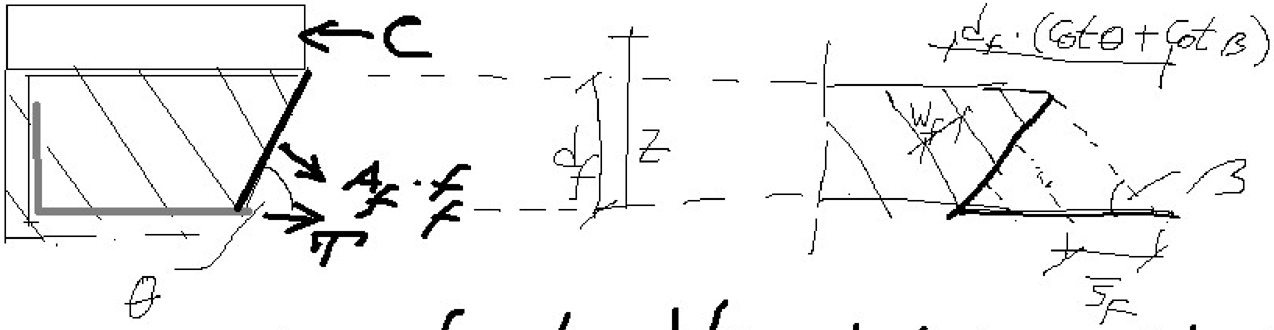
V_{cz} : Concrete strength
V_i : aggregate interlock
V_d : dowel effect

Stirrup increases all components: V_{cz}, V_i, V_d !

Design model for Ultimate Limit State (ULS)

Assuming that at the ultimate limit state in shear (concrete diagonal tension) the FRP develops an effective strain in the principal material direction, εf,e. The effective strain εf,e is, in general, less than the tensile failure strain, εfu.





$$V_R = V_c + V_w + V_f$$

$$V_f = 2 f_{fe} \cdot t_f \cdot \frac{w_f}{s_f} \cdot d_f \cdot (\cot\theta + \cot\beta) \cdot \sin\beta$$

$$\rho_f = \frac{2 \cdot t_f \cdot w_f}{b_w \cdot s_f}$$

$$\bar{s}_f = w_f / \sin\beta \Longrightarrow \rho_f = \frac{2 \cdot t_f \cdot \sin\beta}{b_w}$$

$$V_f = 0.9 \cdot \varepsilon_{fe} E_f \cdot \rho_f \cdot b_w \cdot d \cdot (\cot\theta + \cot\beta) \cdot \sin\beta$$

$$V_f = 0.9 \cdot \varepsilon_{fe} \cdot E_f \cdot \rho_f \cdot b_w \cdot d \cdot (\cot\theta + \cot\beta) \cdot \sin\beta$$

Where:

$\varepsilon_{fd,e}$: design value of effective FRP strain.

b_w : minimum width of cross section over the effective depth.

d : effective depth of cross section.

ρ_f : $(2\ t_f \sin \beta) / b_w$ for
continuously bonded shear reinforcement of thickness
tf (b_w = minimum width of concrete cross section over
the effective depth), or
$(2t_f/b_w)(b_f/s_f)$ for
FRP reinforcement in the form of strips or sheets of width
bf at the spacing s_f.

E_{fu} : elastic modulus of FRP in the principal fiber orientation.

θ : diagonal crack angle, assumed to be 45°.

β : fiber orientation

Circular cross-sections

$$V_{fd} = \frac{\varepsilon_{fe}}{\gamma_f} E_{fu} \rho_f \frac{1}{2} \frac{\pi D^2}{4} \cot\theta$$

Where:

ε_{fe} : 0.006 (experimentally verified by Priestley et al, 1995)

D : column diameter

$$V_{cd} = 0.2\Phi_c.b.d.\sqrt{f'_c}$$

$$\Phi_c = 0.6$$

$$V_{wd} = A_s . \varepsilon_s . E_s . \frac{z_s}{s_f} . Cotg\theta$$ If: $$\varepsilon_s \leq \frac{f_y}{E_s}$$

$$V_{wd} = A_s . f_y . \frac{z_s}{s_f} . Cotg\theta$$ If: $$\varepsilon_s > \frac{f_y}{E_s}$$

Where:
z_s : internal lever arm.
s_f : spacing of stirrups.
f_y : yield strain.
θ : crack angle, assumed to be 45^o.

- Fully wrapped (or properly anchored) CFRP-FRP fracture controls:

$$\varepsilon_{f,e} = 0.17\left(\frac{f_{cm}^{2/3}}{E_{fu}\rho_f}\right)^{0.3}\varepsilon_{fu}$$

- Side or U-shaped CFRP jackets:

$$\varepsilon_{f,e} = \min\left[0.65\left(\frac{f_{cm}^{2/3}}{E_{fu}\rho_f}\right)^{0.56}\times 10^{-3}\ \&\ 0.17\left(\frac{f_{cm}^{2/3}}{E_{fu}\rho_f}\right)^{0.30}\varepsilon_{fu}\right]$$

Peeling off *Fracture*

Note that in all equations fcm is in *MPa* and Efu is in *GPa*.

Design recommendations

It should point out that the spacing of strips (s_f), if they are used vertically, should not exceed:

$$s_f \le 0.9d - \frac{b_f}{2}$$ for rectangular cross sections

$$s_f \le d - h_f - \frac{b_f}{2}$$ for T-beams

In which h_f is slab thickness and b_f is FRP width.

See IIFC Webinar: Shear Strengthening of RC Beams,

Fib, ACI, Italian and Australian Guidlines,

EBR and NSMR

Given by: Professor J. Barros, University of Minho, Portugal

https://www.youtube.com/watch?v=AMWziRZuWHk&list=PLsdGDOBT-H8E9FWUgrURcto9jU9Pr0I45&index=2&t=0s

Attachments

Beam/One-Way Slab Strengthening

Canadian Guideline

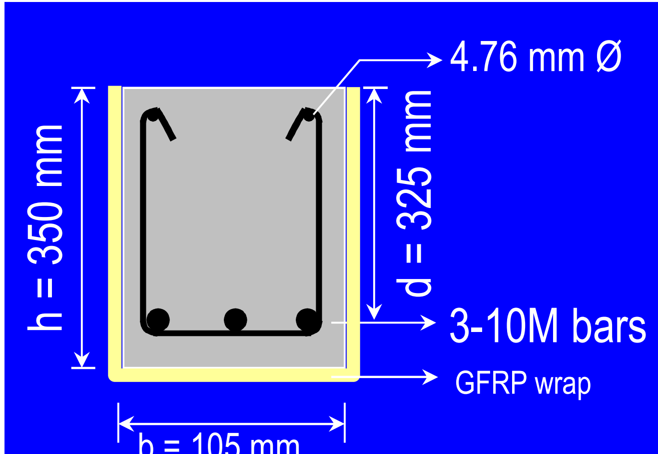
Shear Strengthening

Example

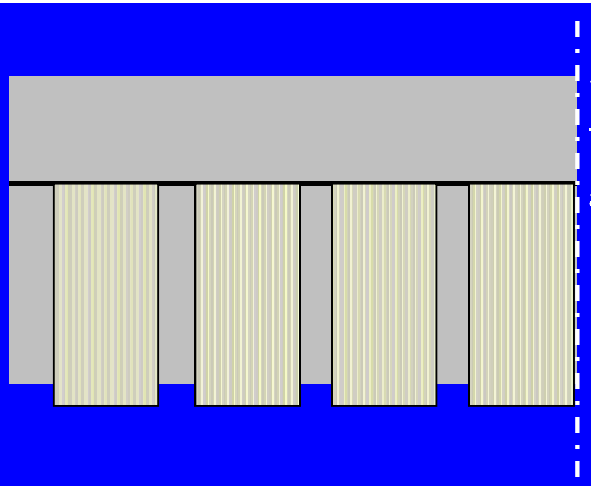
Problem statement

Calculate the shear capacity (V_r) for an FRP-strengthened concrete section

Section information



Section



Elevation

$\lambda = 1.0$

$f'_c = 45$ MPa

$\varepsilon_{frpu} = 2.0$ %

$t_{frp} = 1.3$ mm

$w_{frp} = 100$ mm

$s_{frp} = 200$ mm

$E_{frp} = 22.7$ GPa

$s_s = 225$ mm c/c

$f_y = 400$ MPa (rebar)

$f_y = 400$ MPa (stirrup)

Beam/One-Way Slab Strengthening

Solution

Step 1: Calculate concrete and steel contributions

Concrete:

$$V_c = 0.2\ \phi_c \sqrt{f'_c}\ b_w d$$

$$V_c = 0.2\ (0.6)\ \sqrt{45}\ (105)\ (325)$$

$$V_c = 27470\ \text{N} = 27.47\ \text{kN}$$

Steel:

$$V_s = \frac{\phi_s f_y A_v d}{s} = \frac{0.85\ (400)\ (36)\ (325)}{225}$$

$$V_s = 17680\ \text{N} = 17.68\ \text{kN}$$

Beam/One-Way Slab Strengthening

Shear Strengthening

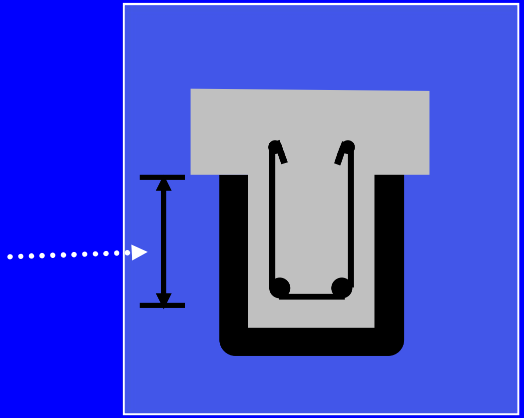
Design Principles

Shear resistance of a beam:

$$V_{frp} = \frac{\phi_{frp} A_{frp} E_{frp} \varepsilon_{frpe} d_{frp} (\sin\beta + \cos\beta)}{s_{frp}}$$

⇨ $A_{frp} = 2\, t_{frp}\, w_{frp}$

⇨ d_{frp}: distance from free end of FRP to bottom of internal steel stirrups



Beam/One-Way Slab Strengthening

Shear Strengthening

Design Principles

- Effective strain in FRP, ε_{frpe}:

$$\boxed{\varepsilon_{frpe} = R\,\varepsilon_{frpu} \leq 0.004}$$

Prevents shear cracks from widening beyond acceptable limits

Ensures aggregate interlock!

- Reduction factor, R:

$$\boxed{R = \alpha\lambda_1 \left(\frac{f'^{\,2/3}_c}{\rho_{frp} E_{frp}}\right)^{\lambda_2}}$$

α: 0.8

Carbon: $\lambda_1 = 1.35$, $\lambda_2 = 0.30$

Glass: $\lambda_1 = 1.23$, $\lambda_2 = 0.47$

Beam/One-Way Slab Strengthening

- FRP shear reinforcement ratio, ρ_{frp}:

$$\boxed{\rho_{frp} = \left(\frac{2\,t_{frp}}{b_w}\right)\left(\frac{w_{frp}}{s_{frp}}\right)}$$

Beam/One-Way Slab Strengthening

Another limit on effective strain in FRP, ε_{frpe}:

$$\boxed{\varepsilon_{frpe} \le \frac{\alpha k_1 k_2 L_e}{9525}}$$

0.8

- ⇨ Parameters, k_1 and k_2:

$$\boxed{k_1 = \left(\frac{f'_c}{27.65}\right)^{2/3}}$$

$$\boxed{k_2 = \left(\frac{d_{frp} - n_e L_e}{d_{frp}}\right)}$$

Beam/One-Way Slab Strengthening

⇨ Effective anchorage length, L_e:

$$\boxed{L_e = \left(\frac{25350}{\left(t_{frp} E_{frp} \right)^{0.58}} \right)}$$

Beam/One-Way Slab Strengthening

Limit on spacing of strips, s_{frp}:

$$\boxed{s_{frp} \leq w_{frp} + \frac{d}{4}}$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Design Principles

Limit on maximum allowable shear strengthening, V_{frp}:

$$V_r \leq V_c + \underbrace{0.8\lambda\phi_c\sqrt{f'_c}\,b_w d}$$

Shear contribution due to steel stirrups
<u>and</u> FRP strengthening must be less
than this term

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 2: Determine A_{frp}, ρ_{frp}, L_e for effective strain calculation

A_{frp}: ……………………

$$A_{frp} = 2\, t_{frp}\, w_{frp} = 2\,(1.3)\,(100)$$

$$A_{frp} = 260\ \text{mm}^2$$

ρ_{frp}: ……………………

$$\rho_{frp} = \left(\frac{2\, t_{frp}}{b_w}\right)\left(\frac{w_{frp}}{s_{frp}}\right) = \left(\frac{2\,(1.3)}{105}\right)\left(\frac{100}{200}\right)$$

$$\rho_{frp} = 0.0124$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 2: Determine A_{frp}, ρ_{frp}, L_e for effective strain calculation

L_e:

$$L_e = \left(\frac{25350}{\left(t_{frp} E_{frp} \right)^{0.58}} \right) = \left(\frac{25350}{\left(1.3 \times 22700 \right)^{0.58}} \right)$$

$$L_e = 64.8 \text{ mm}$$

Beam/One-Way Slab Strengthening

Solution

Step 3: Determine k_1, k_2 and effective strain, ε_{frpe} *[Limit 2]*

k_1:

$$k_1 = \left(\frac{f'_c}{27.65}\right)^{2/3} = \left(\frac{45}{27.65}\right)^{2/3} = 1.38$$

Because of u-wrap

k_2:

$$k_2 = \left(\frac{d_{frp} - n_e L_e}{d_{frp}}\right) = \left(\frac{325 - 1\,(64.8)}{325}\right) = 0.80$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 3: Determine k_1, k_2 and effective strain, ε_{frpe} *[Limit 2]*

Note: This strain is one of three limits placed on the FRP

ε_{frpe}: ....

$$\varepsilon_{frpe} \leq \frac{\alpha k_1 k_2 L_e}{9525}$$

$$\varepsilon_{frpe} = \frac{0.8\ (1.38)\ (0.80)\ (64.8)}{9525}$$

$$\varepsilon_{frpe} = 0.0060$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 4: Determine R and effective strain, ε_{frpe} *[Limit 1]*

R: ……………………
$$R = \alpha\lambda_1 \left(\frac{f'^{\,2/3}_c}{\rho_{frp} E_{frp}} \right)^{\lambda_2}$$

$$R = 0.8\,(1.23) \left(\frac{45^{\,2/3}}{0.0124\,(22700)} \right)^{0.47}$$

$$R = 0.229$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 4: Determine R and effective strain, ε_{frpe} *[Limit 1]*

Note: This strain is one of three limits placed on the FRP

ε_{frpe}: …………………… $\varepsilon_{frpe} = R\ \varepsilon_{frpu} \leq 0.004$

$$\varepsilon_{frpe} = 0.229\ (0.02)$$

$$\varepsilon_{frpe} = 0.0046$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 5: Determine governing effective strain, ε_{frpe}

For design purposes, use the smallest limiting value of:

$$\varepsilon_{frpe} = 0.0046$$

$$\boxed{\varepsilon_{frpe} = 0.0040}$$

$$\varepsilon_{frpe} = 0.0060$$

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 6: Calculate contribution of FRP to shear capacity

V_{frp}:

$$V_{frp} = \frac{\phi_{frp}\, A_{frp}\, E_{frp}\, \varepsilon_{frpe}\, d_{frp}\, (\sin\beta + \cos\beta)}{s_{frp}}$$

$$V_{frp} = \frac{0.5\,(260)\,(22700)\,(0.004)\,(325)\,(\sin 90 + \cos 90)}{200}$$

V_{frp} = 19200 N = 19.2 kN

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 7: Compute total shear resistance of beam

V_r: ....

$$V_r = V_c + V_s + V_{frp}$$

$$V_r = 27.5 + 17.7 + 19.2$$

$$\boxed{V_r = 64.4 \text{ kN}}$$

Beam/One-Way Slab Strengthening

Solution

Step 8: Check maximum shear strengthening limits

$$V_r \leq V_c + 0.8\lambda\phi_c f'_c b_w d$$

$$64400 \leq 27500 + 0.8\ (1)\ (0.6)\ (45)\ (105)\ (325)$$

$$64400 \leq 137400$$

⇨ OK

Beam/One-Way Slab Strengthening

Shear Strengthening

Example

Solution

Step 9: Check maximum band spacing

$$s_{frp} \leq w_{frp} + \frac{d}{4}$$

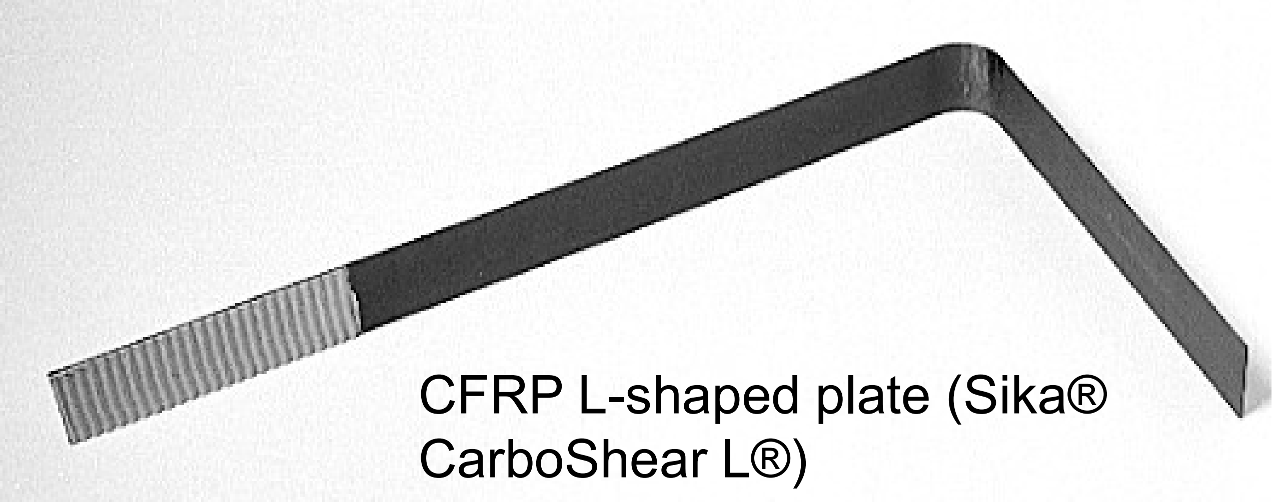
$$200 \leq 100 + \frac{325}{4}$$

$$200 \leq 181$$

⇨ Not true, therefore use 180 mm spacing

Shear Strengthening of Reinforced Concrete with CFRP: Empa-Project

Material

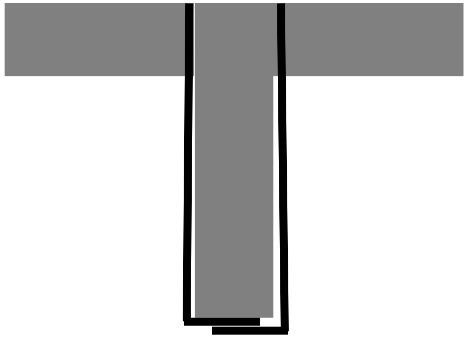


CFRP L-shaped plate (Sika® CarboShear L®)

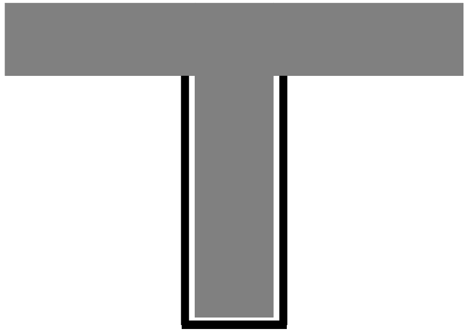
CFRP fabric (SikaWrap® Hex-230C)

Application

CFRP L-shaped plate



CFRP fabric



Referenzproject

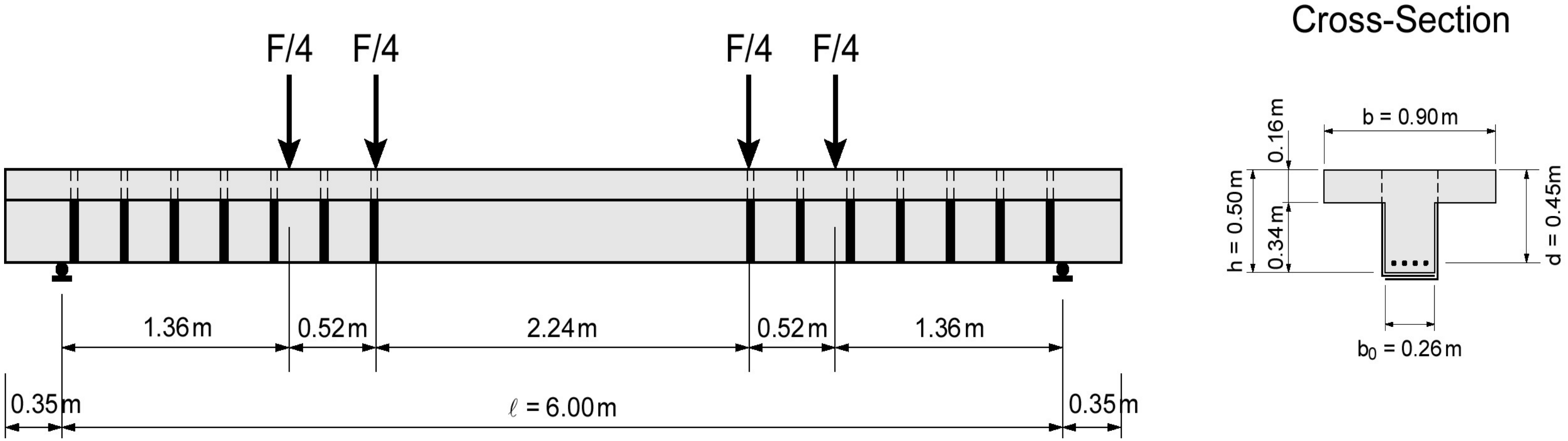
Ponte Brogeda (Chiasso, Switzerland)

Photos from Sika®

Referenzproject (3)

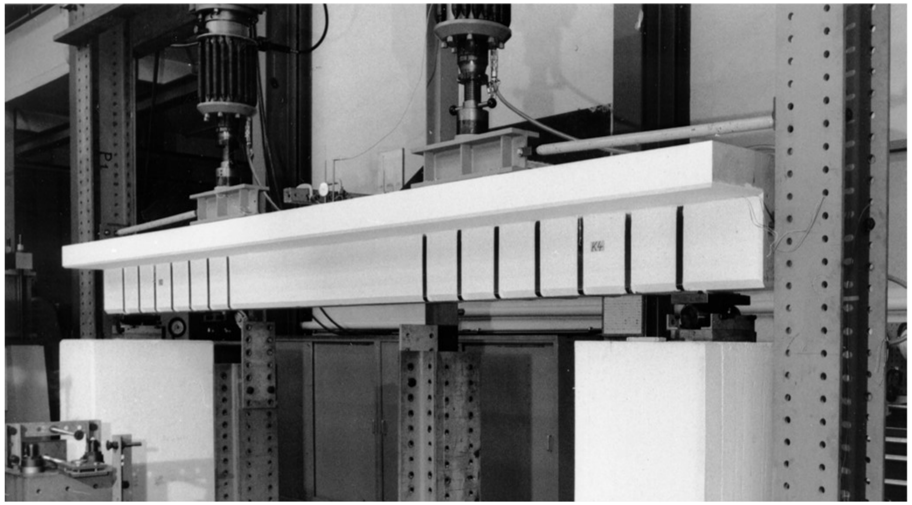
Abutment of the Duttweiler-bridge, Zurich

Beam T1

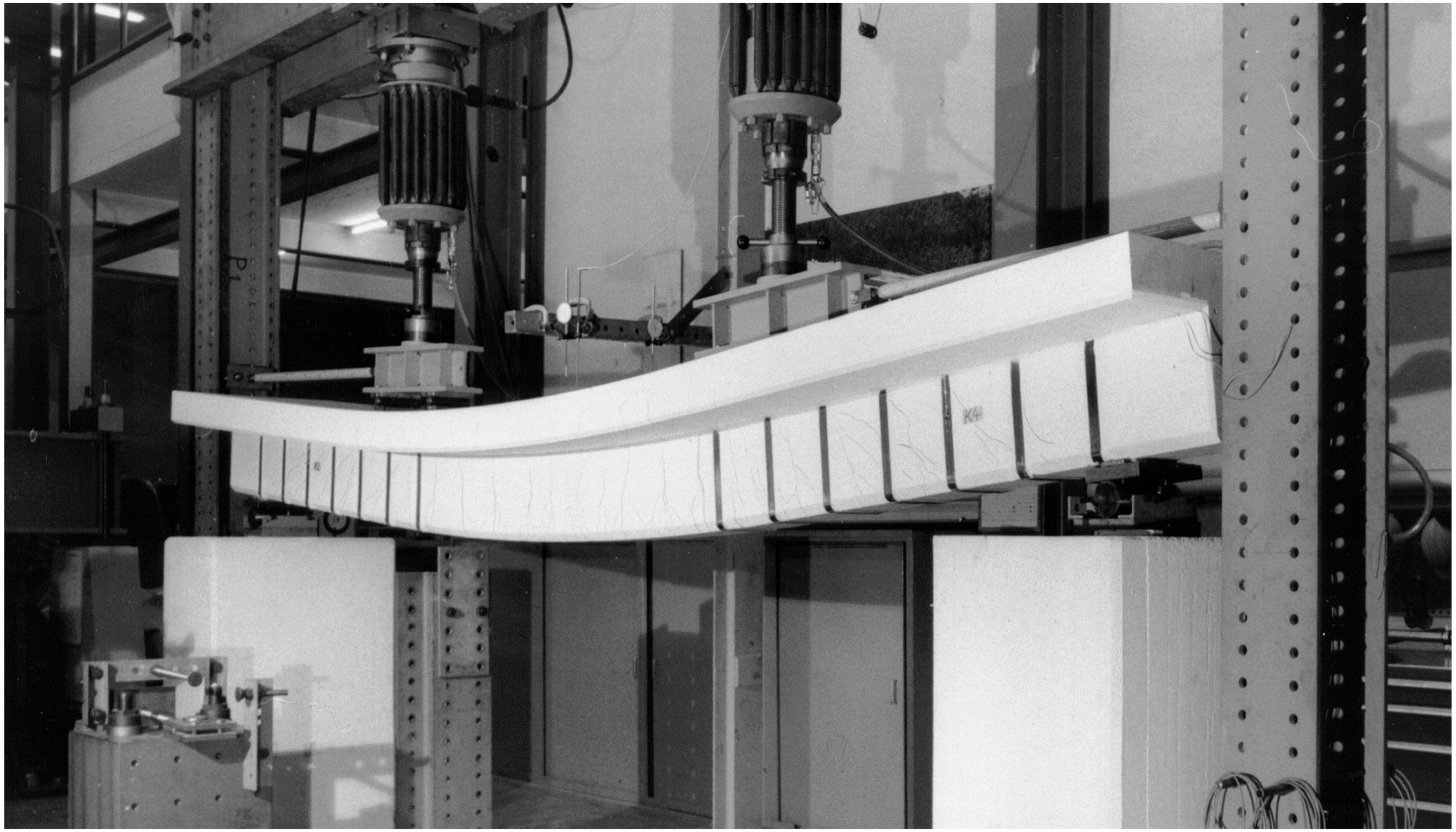


$\ell/h = 12:1$

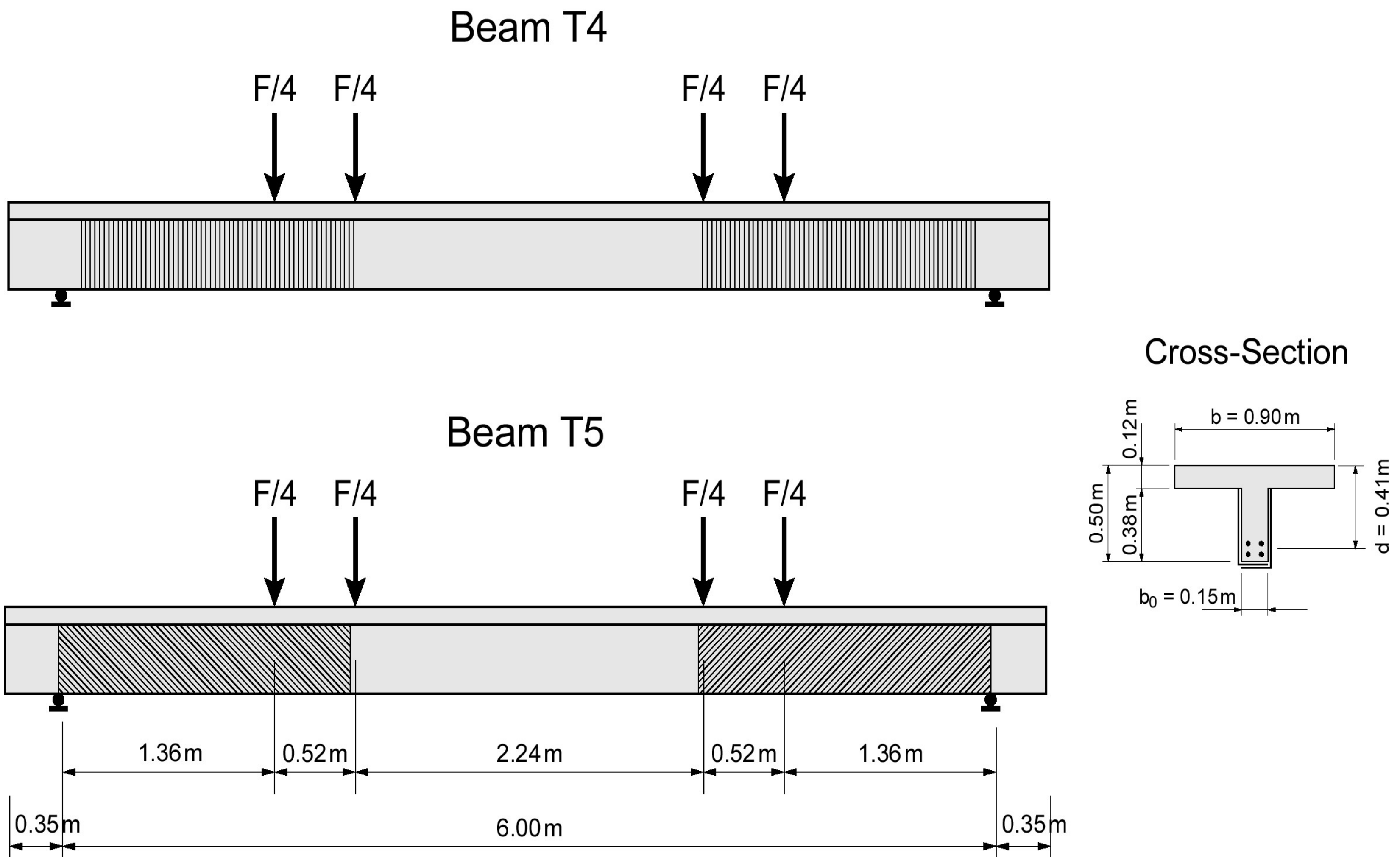
$a/d = 3.6$



Beam T1 after Test

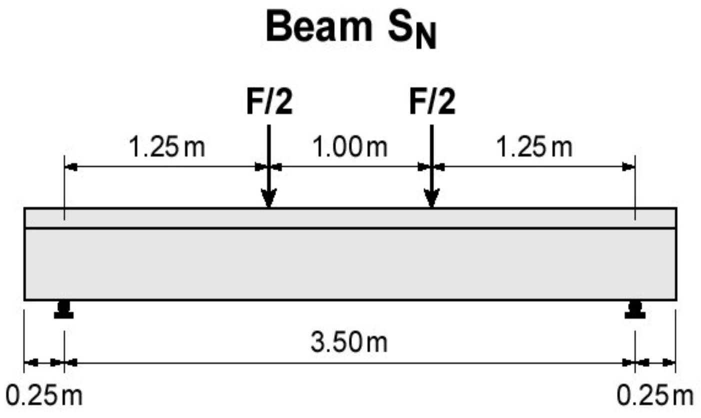


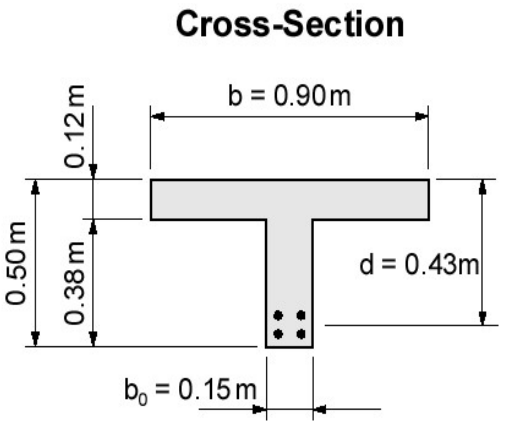
Strengthening with CFRP Fabric



Debonding of CFRP-Fabric

Systematic Test Program



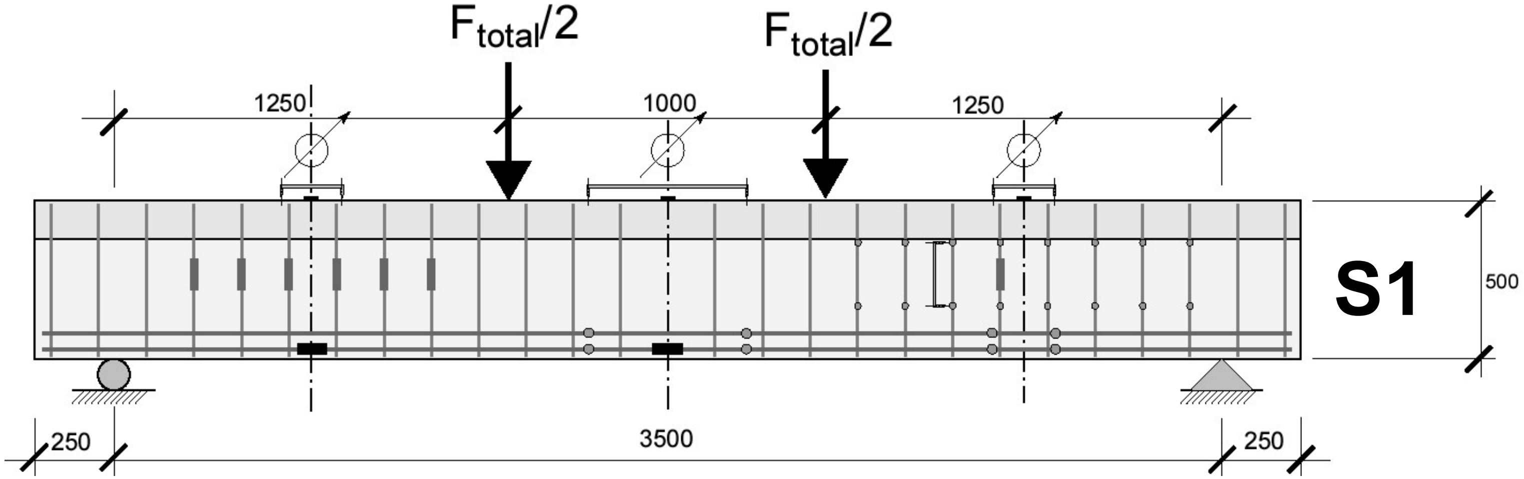


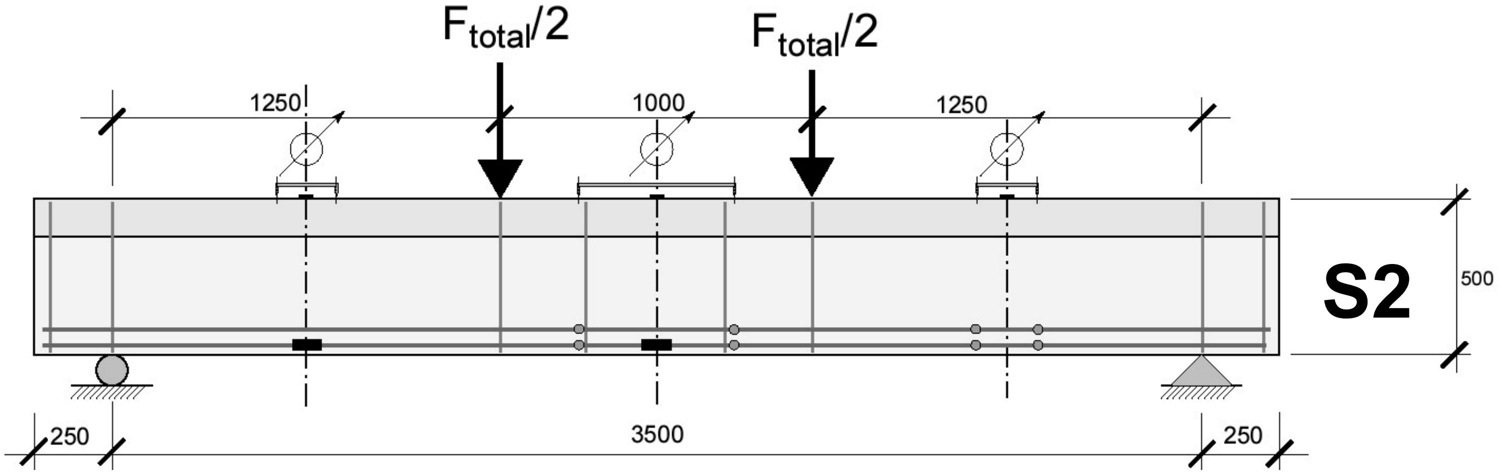
$a/d = 2.9$

$b/b_0 = 6.0$

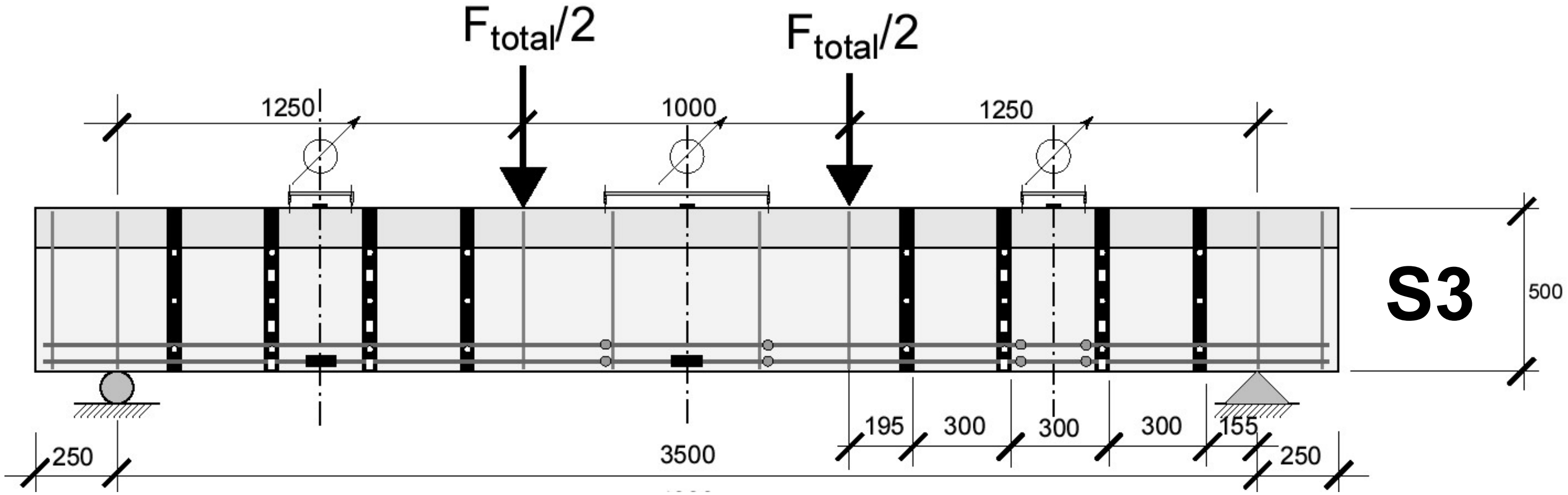
	Typ of Test	Internal Reinforcement	External Reinforcement
S1	Static Loading	ø8 s = 150 mm	Without
S2	Static Loading	Without	Without
S3	Static Loading	Without	CFRP L- Plates s = 300 mm
S4	Static Loading	ø8 s = 150 mm	CFRP L- Plates s = 300 mm
S5	Pre Loading	ø8 s = 150 mm	CFRP L- Plates s = 300 mm
S6	Fatigue	ø8 s = 150 mm	CFRP L- Plates s = 300 mm

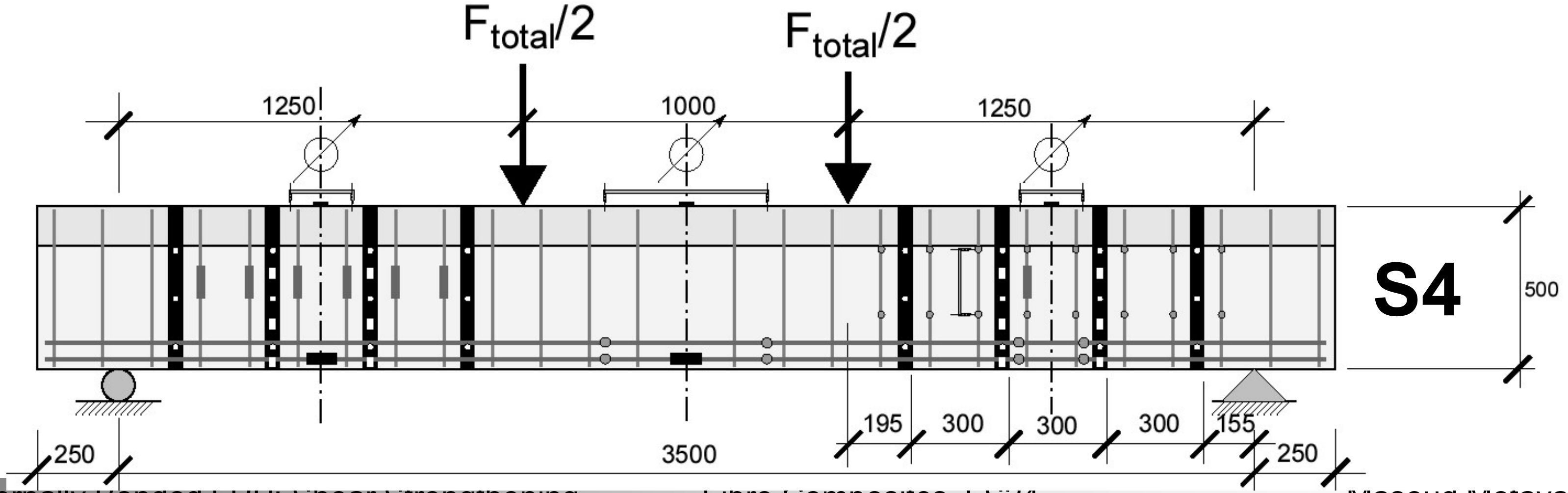
Test Beams


$F_{total}/2$
$F_{total}/2$
1250
1000
1250
S1
500
250
3500
250

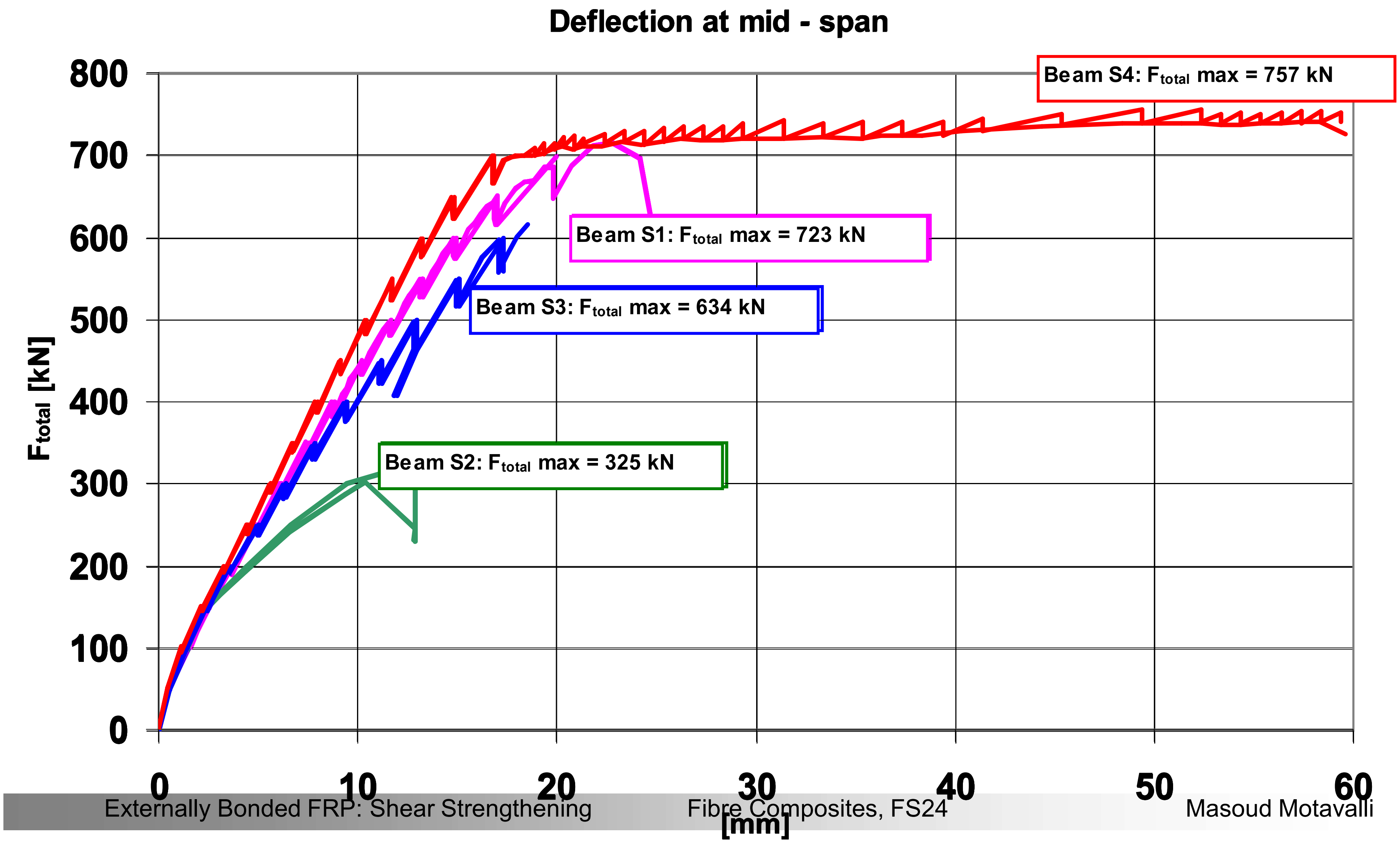

$F_{total}/2$
$F_{total}/2$
1250
1000
1250
S2
500
250
3500
250

Test Beams

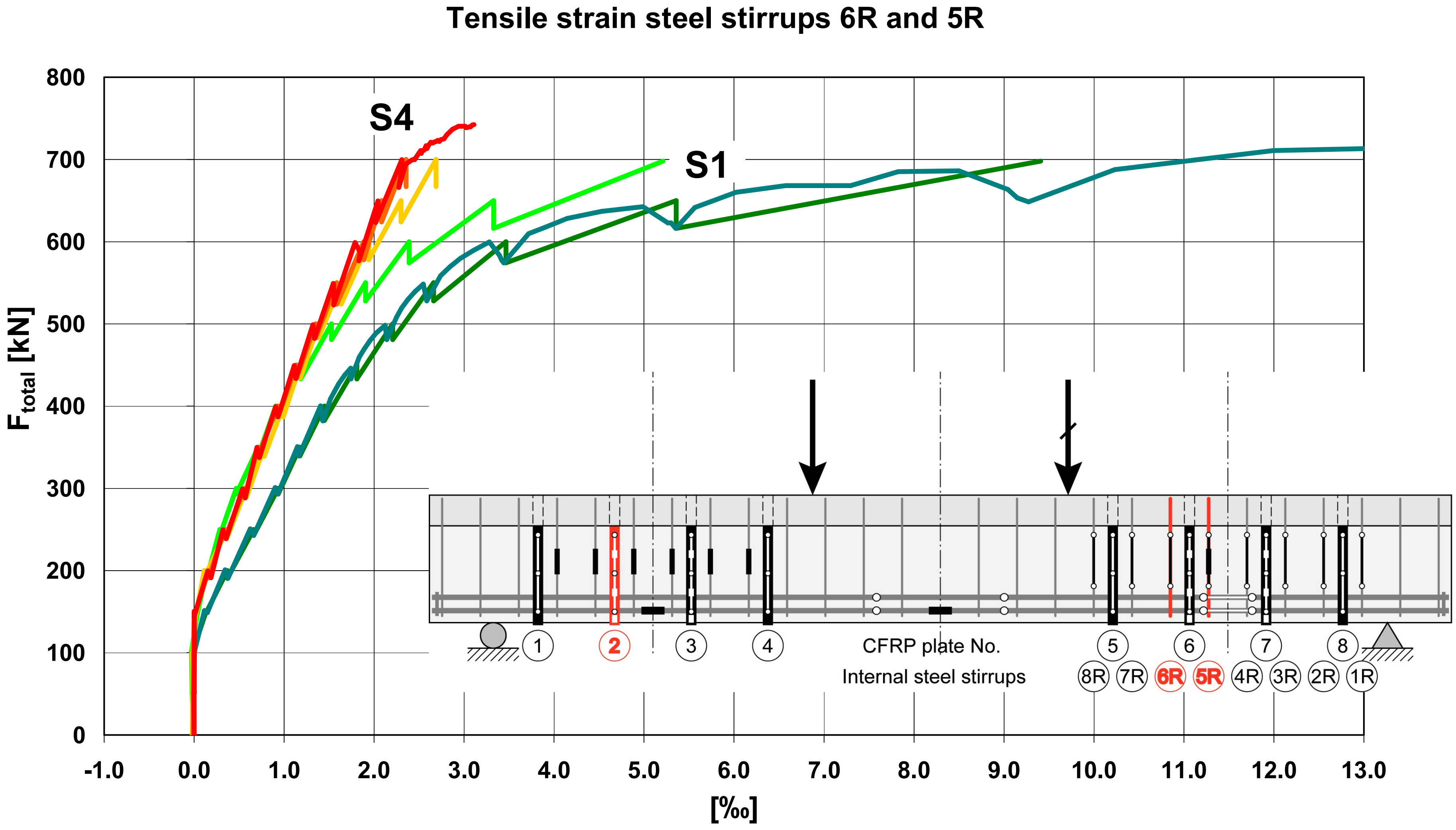

$F_{total}/2$
$F_{total}/2$
1250
1000
1250
S3
500
195
300
300
300
155
250
3500
250


$F_{total}/2$
$F_{total}/2$
1250
1000
1250
S4
500
195
300
300
300
155
250
3500
250

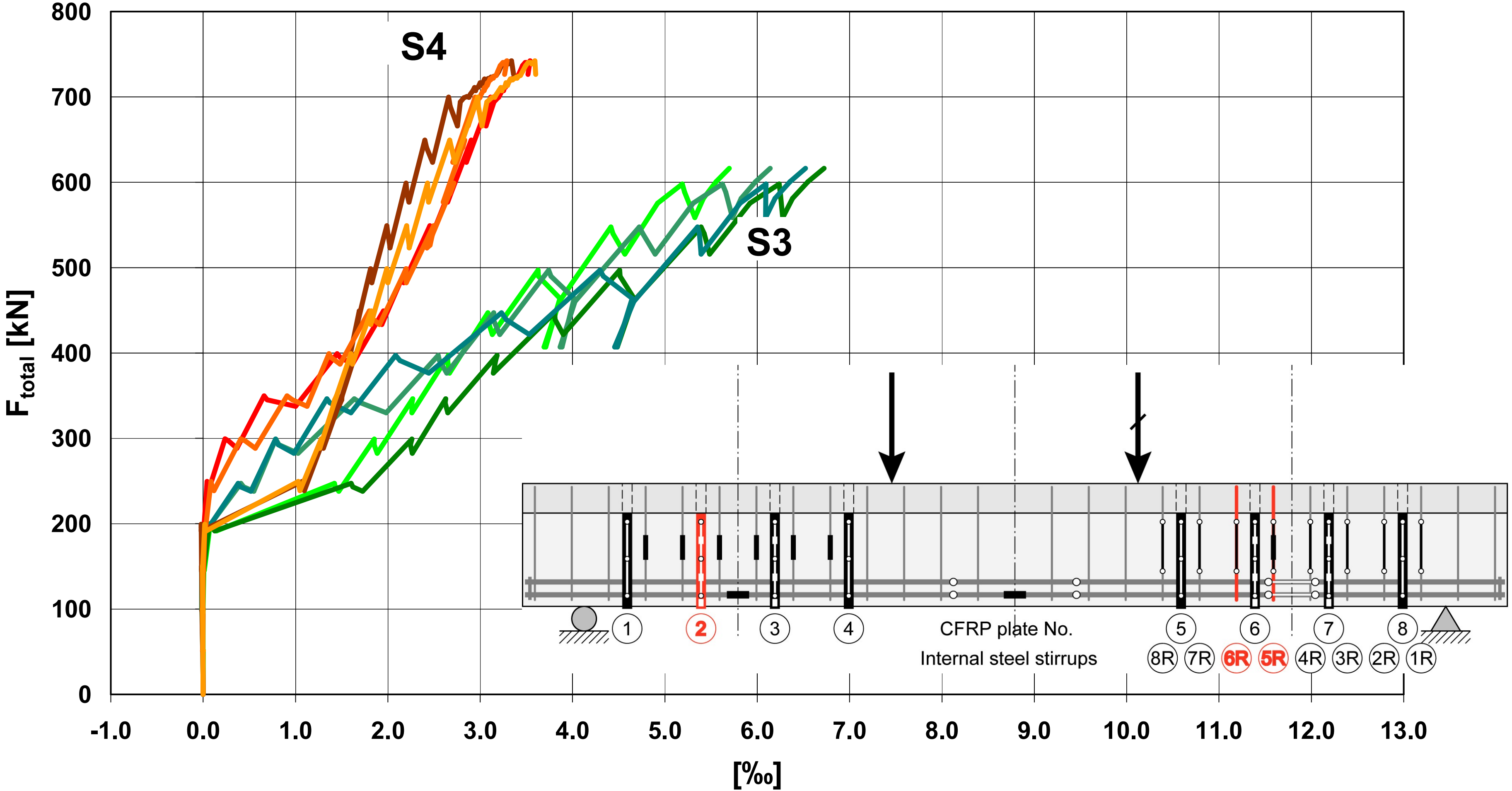
Deflection at mid-span



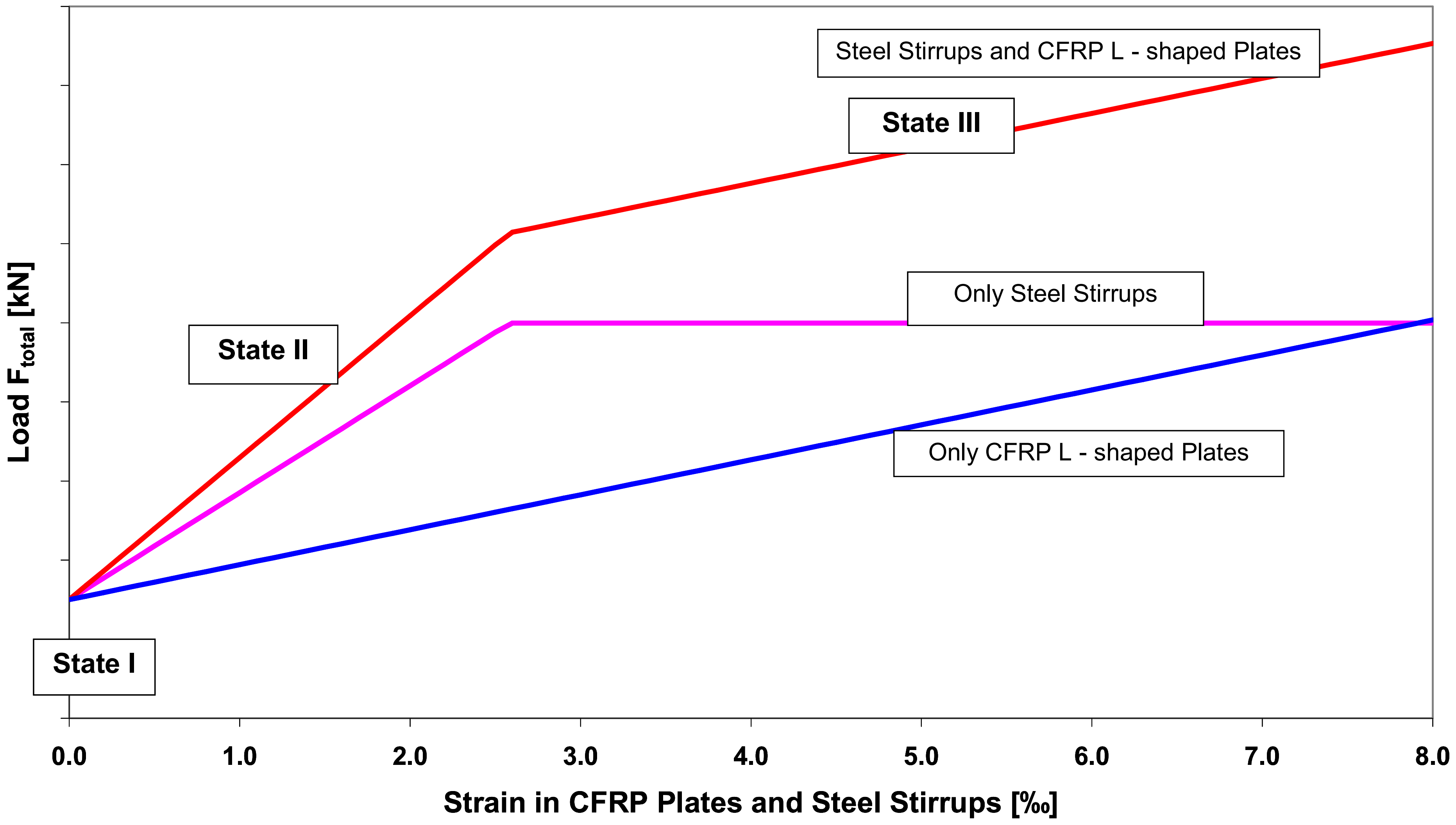
Strain in steel stirrup



Strain in CFRP L-shaped plate



Design

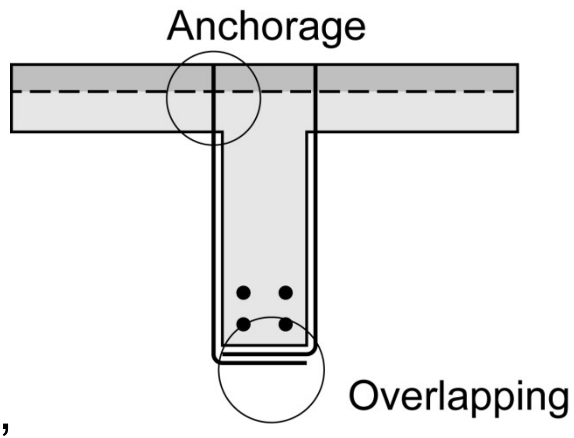

Steel Stirrups and CFRP L - shaped Plates
State III
Only Steel Stirrups
State II
Only CFRP L - shaped Plates
State I
Load F_{total} [kN]
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
Strain in CFRP Plates and Steel Stirrups [‰]

Design

$$V_R = V_C + V_S + V_F$$

V_R = Shear resistance of RC member
V_C = Concrete contribution (first shear crack)
V_S = Contribution of internal steel stirrups
V_F = Contribution of external CFRP L-shaped plates

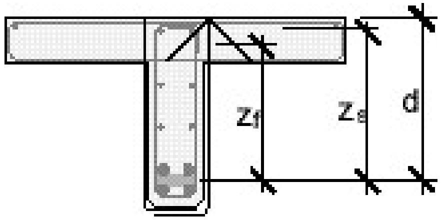
Contribution of CFRP L-shaped plates



- Ultimate limit state (ULS)
  - Failure mode “opening of the overlapping”
  - Failure of the anchorage
- Serviceability limit state (SLS)
- Analysis of unstrengthened section
  - After plate failure remaining safety factor > 1

Failure of anchorage

- Should be prevented if:
  - Anchor length → whole height of flange (if possible)



  - Carefully filling of the anchorage holes
  - Anchor length > 100 mm (see reference [2])

Design equations for Sika® CarboShear L®

Equations without any safety factors!

Verification ULS:

$$V_R = A_s \cdot f_y \cdot \frac{z_s}{s_s} \cdot \cot\alpha + F_f \cdot \frac{z_f}{s_f} \cdot \cot\alpha$$

with α ≥ 45°

Verification SLS:

$$V_{ser} = V_C + A_s \cdot 0.8 \cdot f_y \cdot \frac{z_s}{s_s} \cdot \cot\alpha + A_f \cdot (0.8 \cdot \frac{f_y}{E_s}) \cdot E_f \cdot \frac{z_f}{s_f} \cdot \cot\alpha$$

with α ≥ 45°

Verification accidental situation:

$$V_{acc} = V_C + A_s \cdot f_y \cdot \frac{z_s}{s_s} \cdot \cot\alpha$$

Shear design

All the usual design verifications for RC (failure of the concrete struts, shift of moment line, etc.) have to be considered. For ductility reasons, the member should have a minimum internal shear reinforcement ratio, otherwise a strengthening is not recommended.

Fatigue

- See design concept in reference [3]

References

[1] Czaderski, C., *Nachträgliche Schubverstärkung mit CFK-Winkeln.* Schweizer Ingenieur und Architekt SI+A, 1998(43): p. Seite 822-826.

[2] Meier, H., *CFK-Schubverstärkungselemente.* Schweizer Ingenieur und Architekt SI+A, 1998(43): p. Seite 819-821.

[3] Czaderski, C. and M. Motavalli, *Fatigue behaviour of CFRP L-shaped plates for shear strengthening of RC T-beams.* Composites Part B: Engineering, 2004. **35**(4): p. 279-290.

[4] http://www.sika.ch/con-produkte-betoninstandsetzung-kleben

"List of Symbols"
⟨shear strengthening⟩

S_f : FRP spacing, or (S_{FRP})

b_f : FRP width, or (W_{FRP})

θ : diagonal crack angle, assumed to be 45°

α : fibre direction, or (β)

$\varepsilon_{fd,e}$: design value of effective FRP strain $= \frac{k}{\gamma_F}\varepsilon_{f,e}$; $k = 0.8$; $\gamma_F = 1.3$

V_{Rd} : design value of shear force capacity of the cross-section

V_{cd} : design value of shear force capacity concrete contribution

V_{wd} : " " " " steel " or (V_s)

V_{fd} : " " " " FRP " or (V_{FRP})

T_{fd} : Contribution of FRP to torsional capacity