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Flexural strengthening

Book Composite for Construction, L. C. Bank, Chapter 9



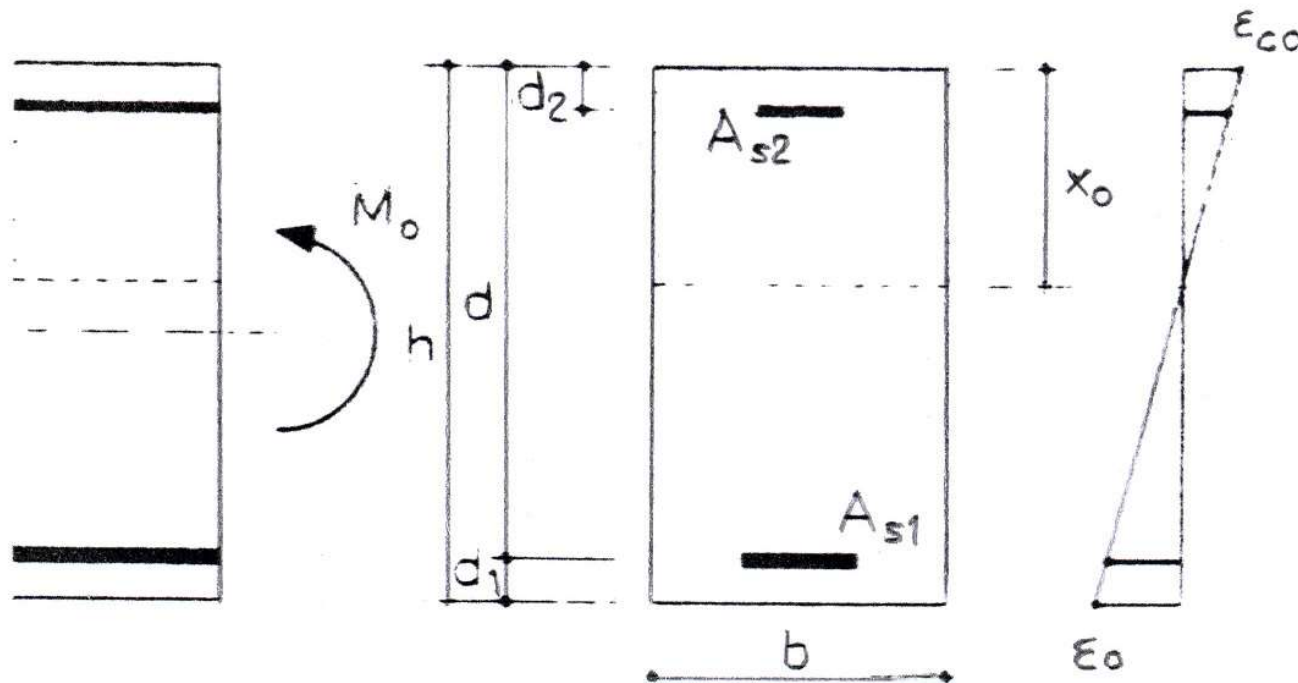
Externally Bonded FRP: Flexural

Fibre Composites, FS25

Masoud Motavalli

Initial situation prior to strengthening

The effect of the initial load prior to strengthening should be considered in the calculation of strengthened member. Based on the theory of elasticity and with M_0 the service moment (*no* load safety factors are applied) acting on the critical RC section during strengthening, the strain distribution of the member can be evaluated. As M_0 is typically larger than the cracking moment M_{cr} , the calculation is based on a cracked section.



If M_0 is smaller than M_{cr} , its influence on the calculation of the strengthened member may easily be neglected.

Based on the transformed cracked section, the neutral axis depth x_0 can be solved from:

$$\frac{1}{2}bx_0^2 + (\alpha_s - 1)A_{s2}(x_0 - d_2) = \alpha_s A_{s1}(d - x_0)$$

Where:

$$\alpha_s = \frac{E_s}{E_c}$$

The concrete strain at the top fiber can be expressed as:

$$\varepsilon_{c0} = \frac{M_0 x_0}{E_c I_{02}}$$

Where I_{02} is the moment of inertia of the transformed cracked section:

$$I_{02} = \frac{bx_0^3}{3} + (\alpha_s - 1)A_{s2}(x_0 - d_2)^2 + \alpha_s A_{s1}(d - x_0)^2$$

Based on strain compatibility, the concrete strain at the extreme tension fiber can be derived as:

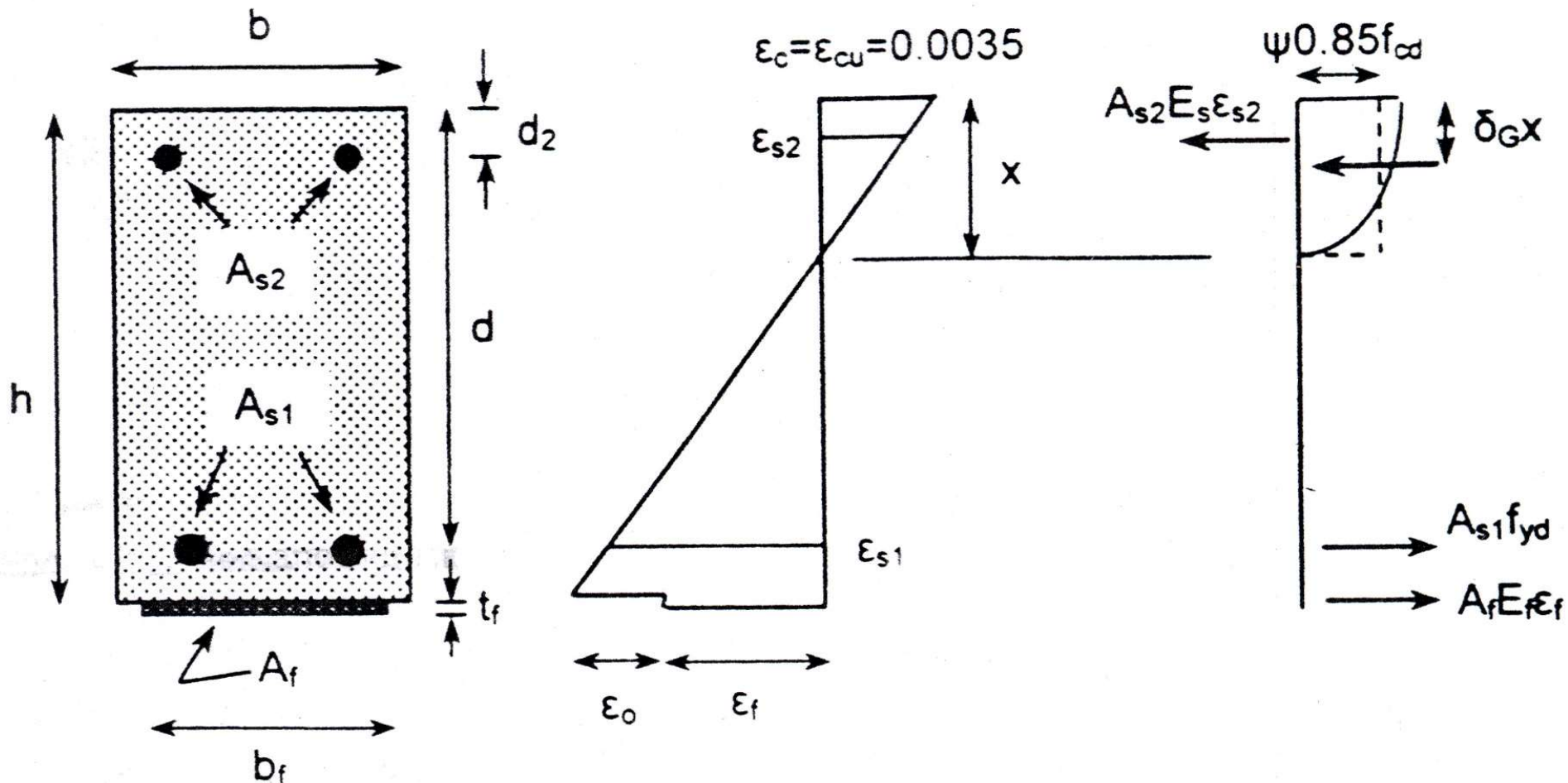
$$\varepsilon_0 = \varepsilon_{c0} \frac{h - x_0}{x_0}$$

This strain equals the initial axis strain at the level of the FRP, needed for the evaluation of the strengthened member.

Analysis of Ultimate Limit State (ULS)

Full composite action

Steel yielding followed by concrete crushing



Calculation of neutral axis depth, x :

$$0.85.\psi.f_{cd}bx + A_{s2}E_s\varepsilon_{s2} = A_{s1}f_{yd} + A_fE_{fu}\varepsilon_f$$

Where:

$$\psi = 0.8$$

and:

$$\varepsilon_{s2} = \varepsilon_{cu} \frac{x - d_2}{x} \quad (E_s\varepsilon_{s2} \text{ not to exceed } f_{yd})$$
$$\varepsilon_f = \varepsilon_{cu} \frac{h - x}{x} - \varepsilon_0$$

Design bending moment capacity:

$$M_{Rd} = A_{s1} f_{yd} (d - \delta_G x) + A_f E_f \varepsilon_f (h - \delta_G x) + A_{s2} E_s \varepsilon_{s2} (\delta_G x - d_2)$$

Where:

$$\delta_G = 0.4$$

Check if

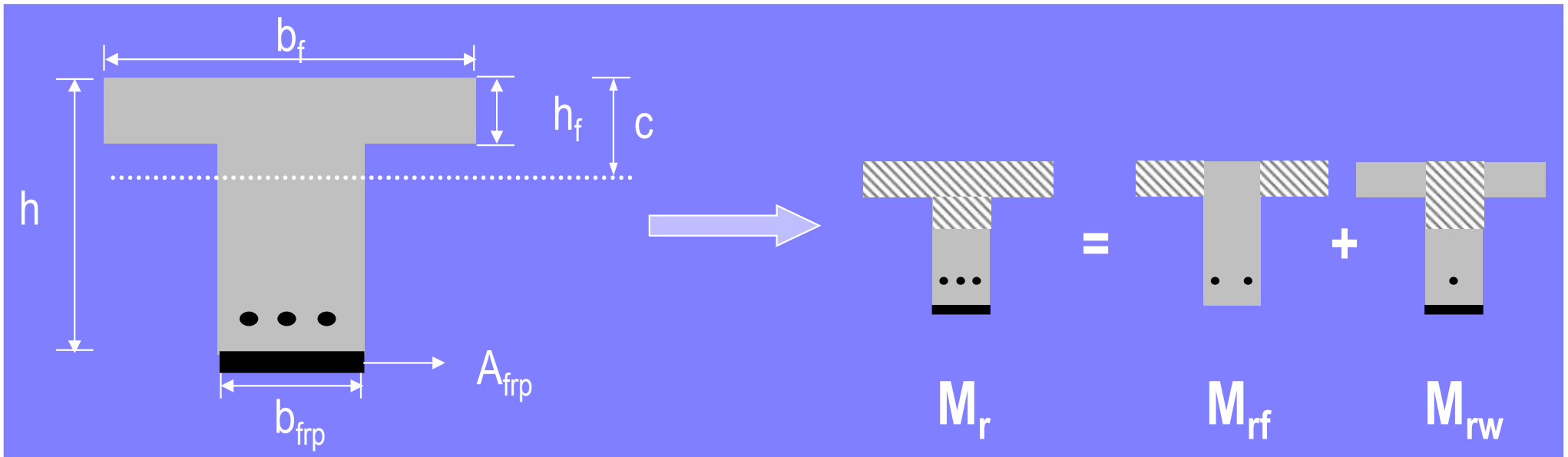
a) Yielding of tensile steel reinforcement:

$$\varepsilon_{s1} = \varepsilon_{cu} \frac{d - x}{x} \geq \frac{f_{yd}}{E_s}$$

b) Straining of the FRP is limited to the ultimate strain:

$$\varepsilon_f = \varepsilon_{cu} \frac{h - x}{x} - \varepsilon_0 \leq \varepsilon_{fud}$$

Tee Beams

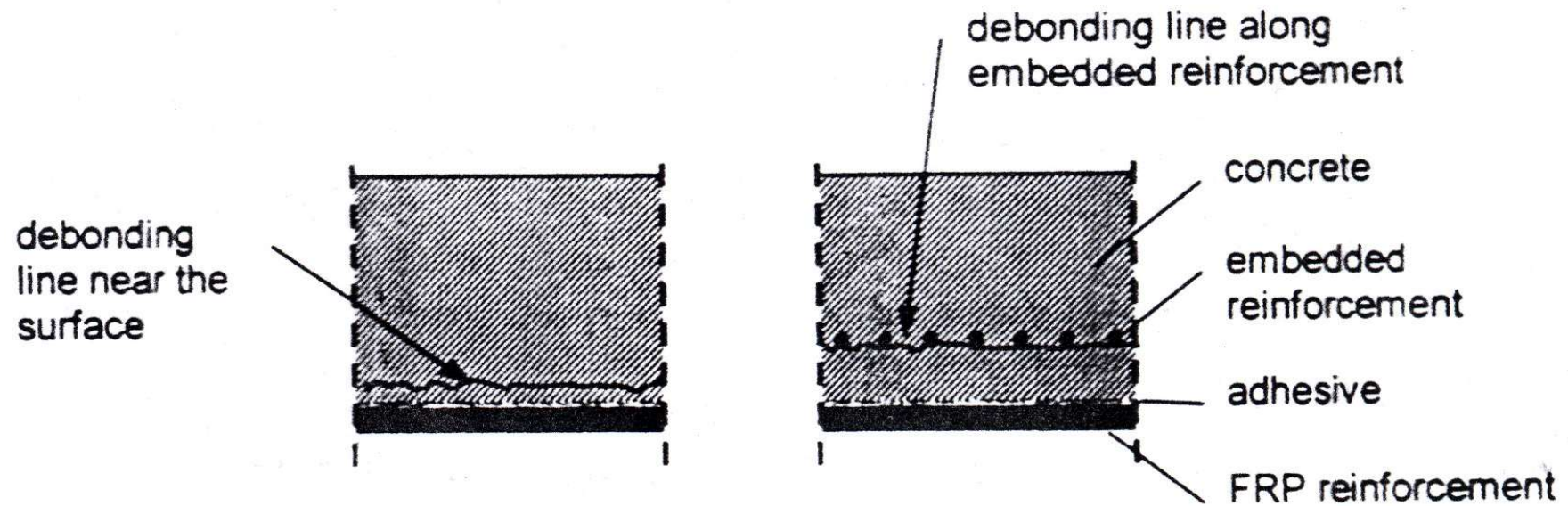
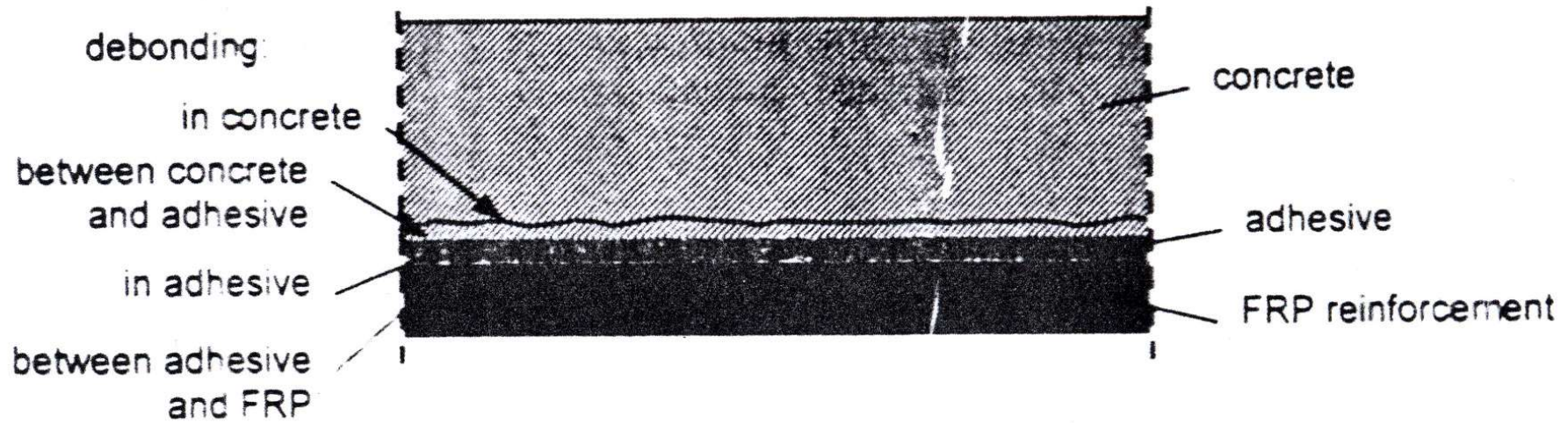


Neutral axis in flange: treat as rectangular section

Neutral axis in web: treat as tee section

Debonding and bond failure modes

- Debonding in the concrete near the surface or along a weakened layer, e.g. along the line of the embedded steel reinforcement.
- Debonding in the adhesive (cohesion failure).
- Debonding at the interfaces between concrete and adhesive or adhesive and FRP (adhesion failure).
- Debonding inside the FRP (interlaminar shear failure).



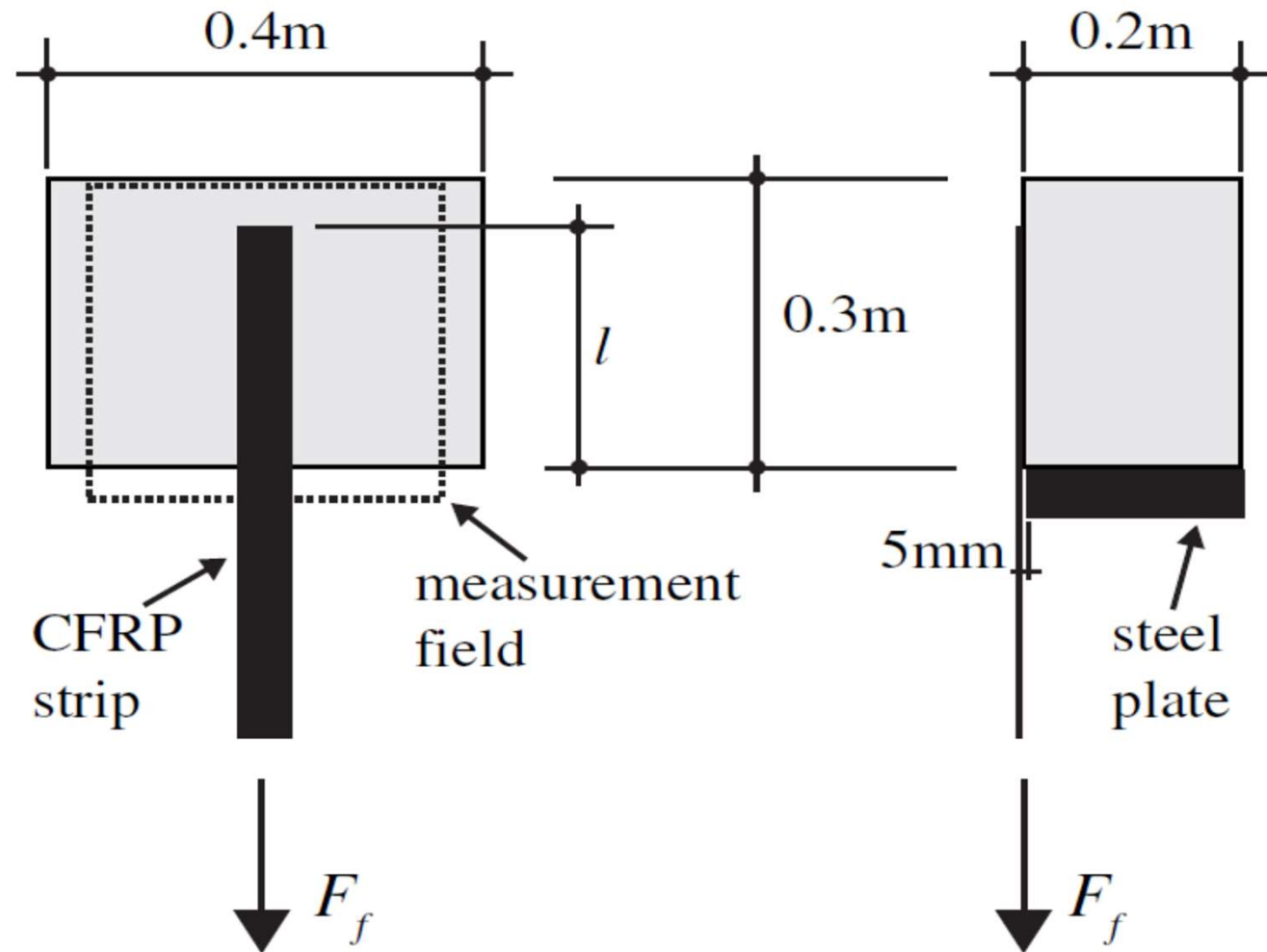
Debonding Video Clips

Non Prestressed CFRP

Prestressed CFRP

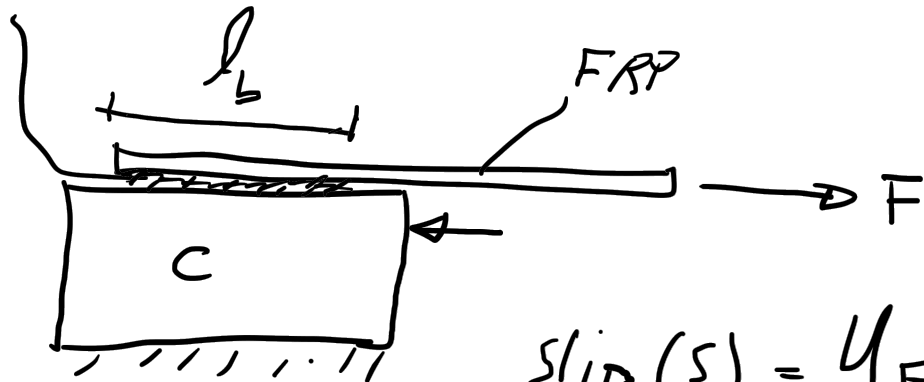


Lap Shear Test



Lap shear test (Bond test)

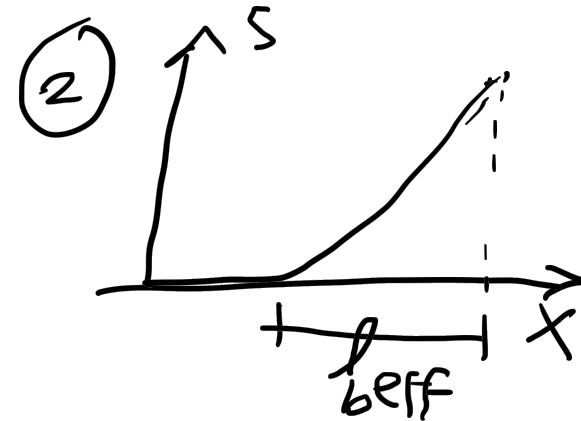
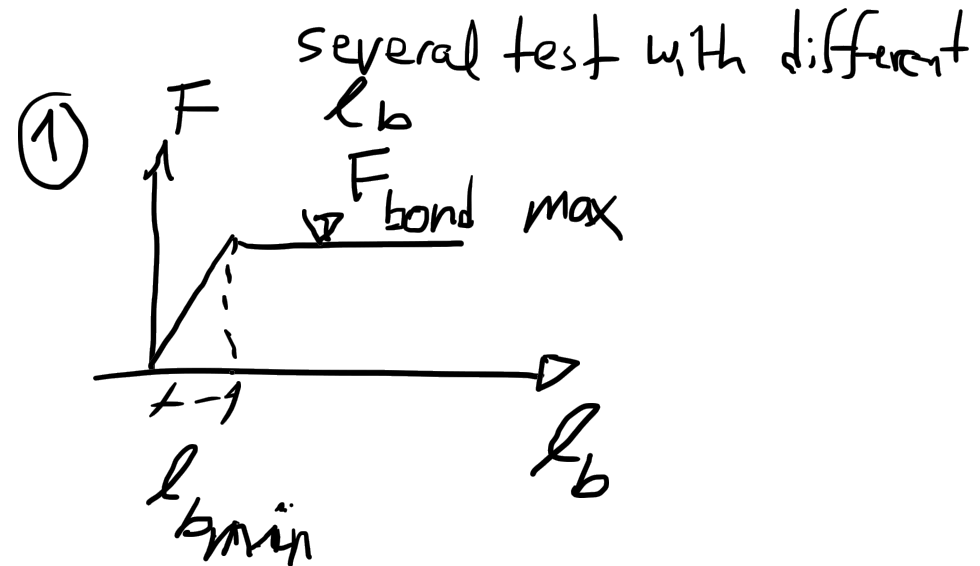
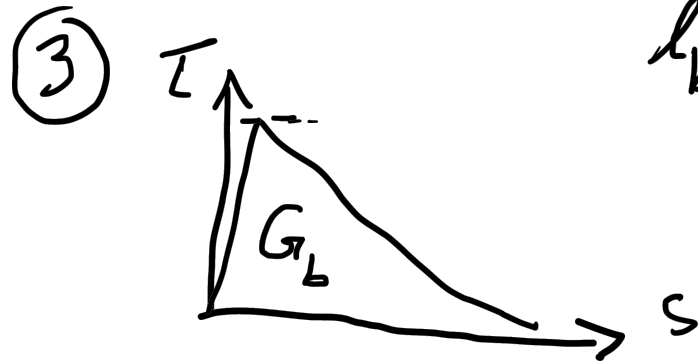
adhesive

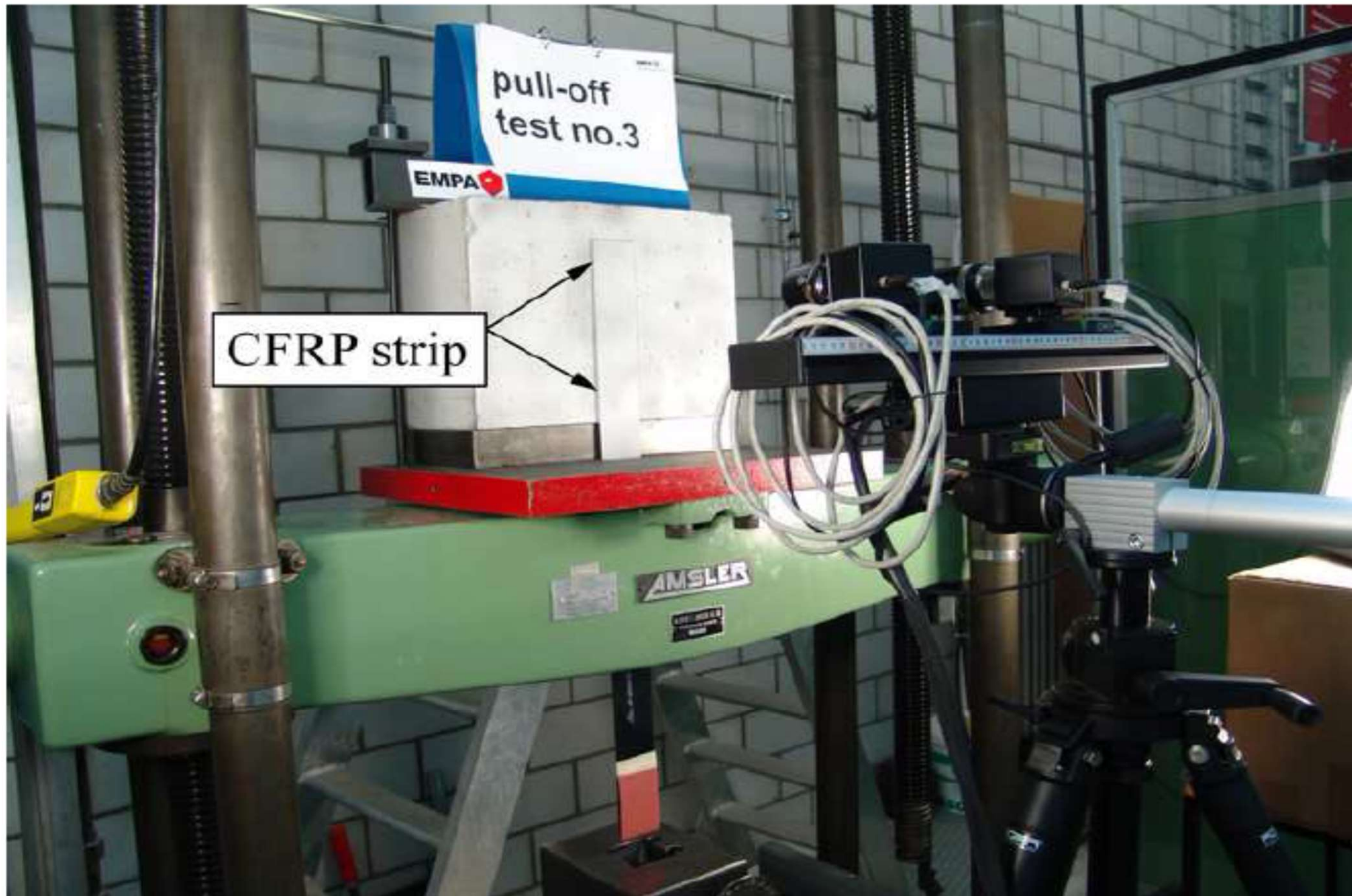


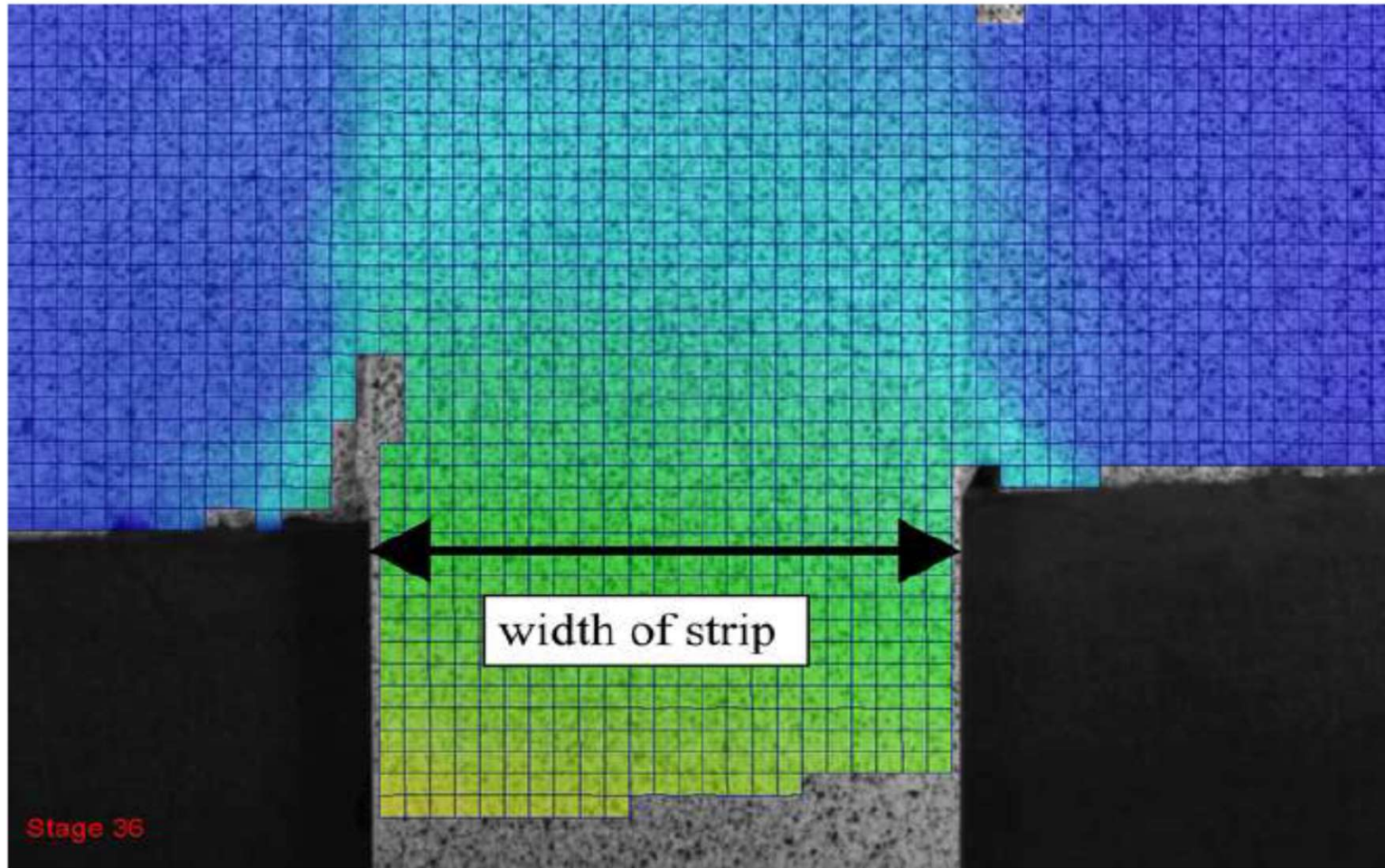
x

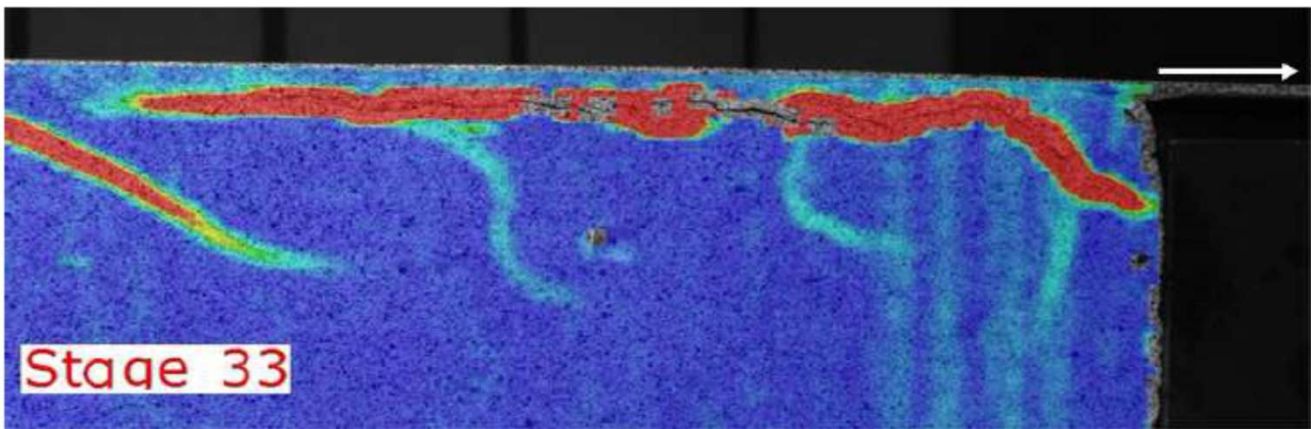
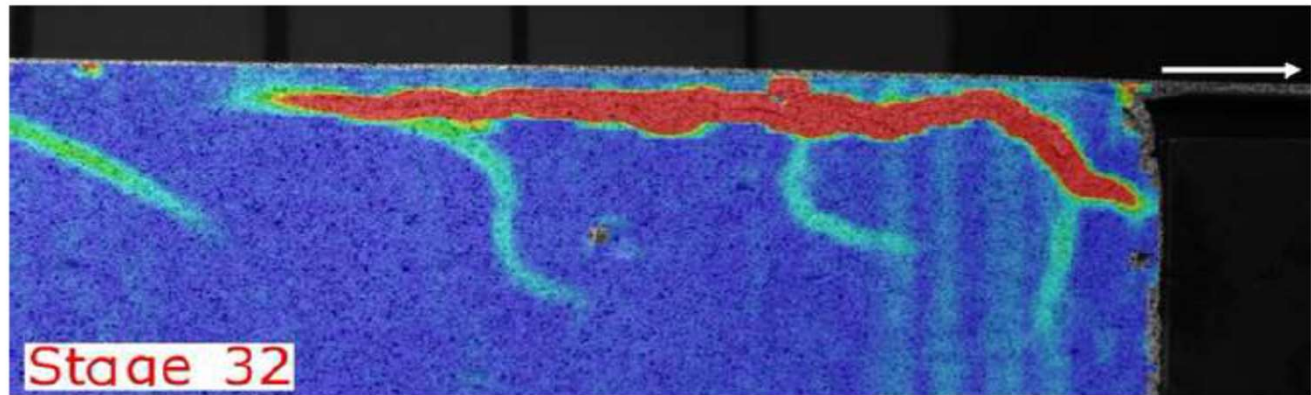
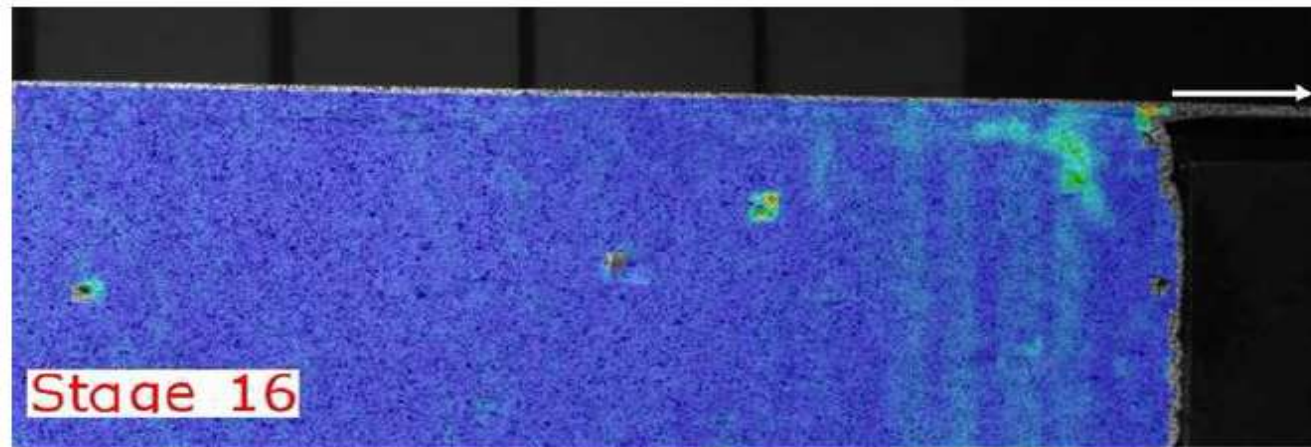
$$\text{slip}(s) = u_{\text{FRP}} - u_c$$

$$\tau = \frac{F}{l_{\text{eff}} \cdot W}$$

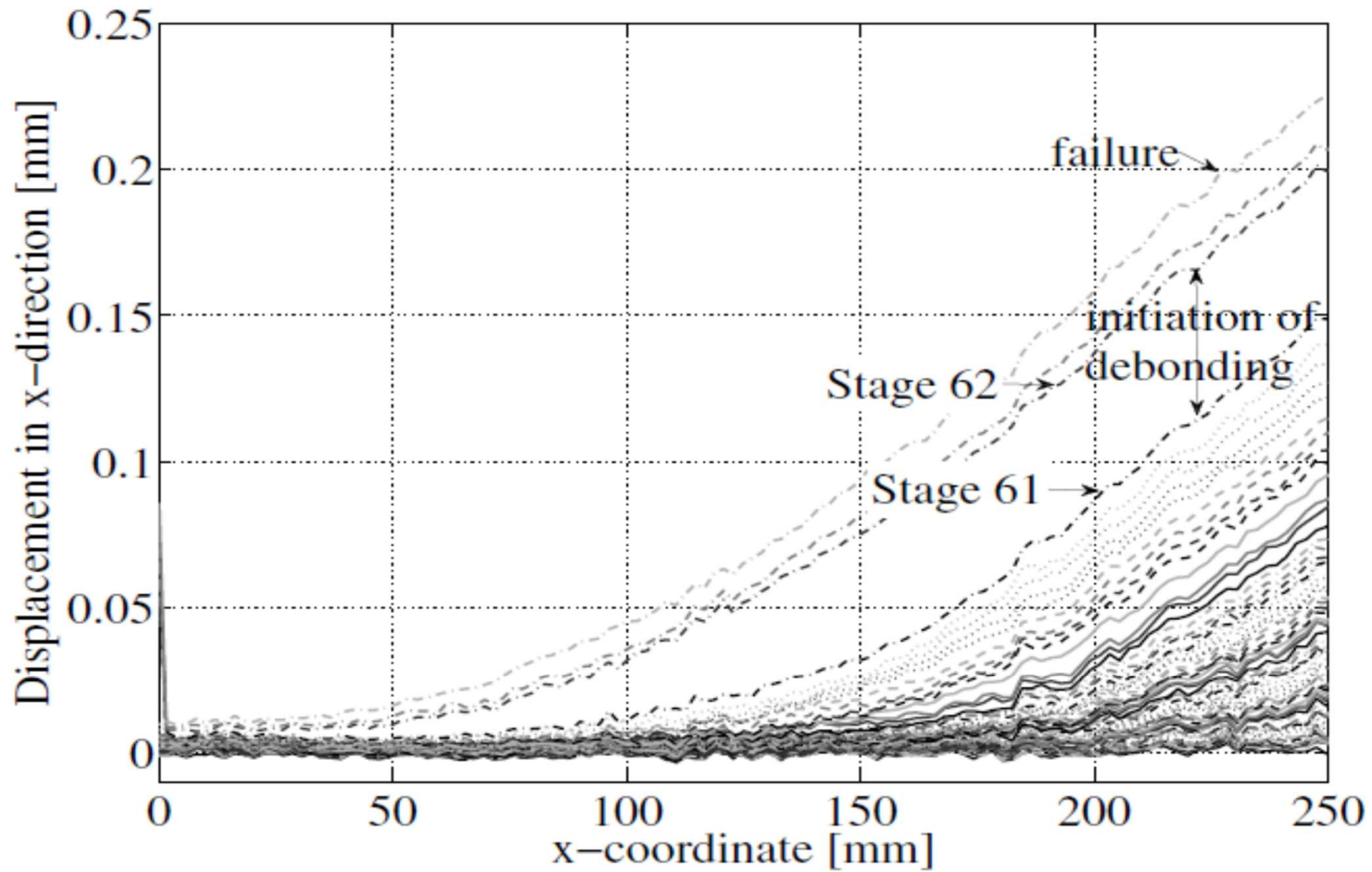




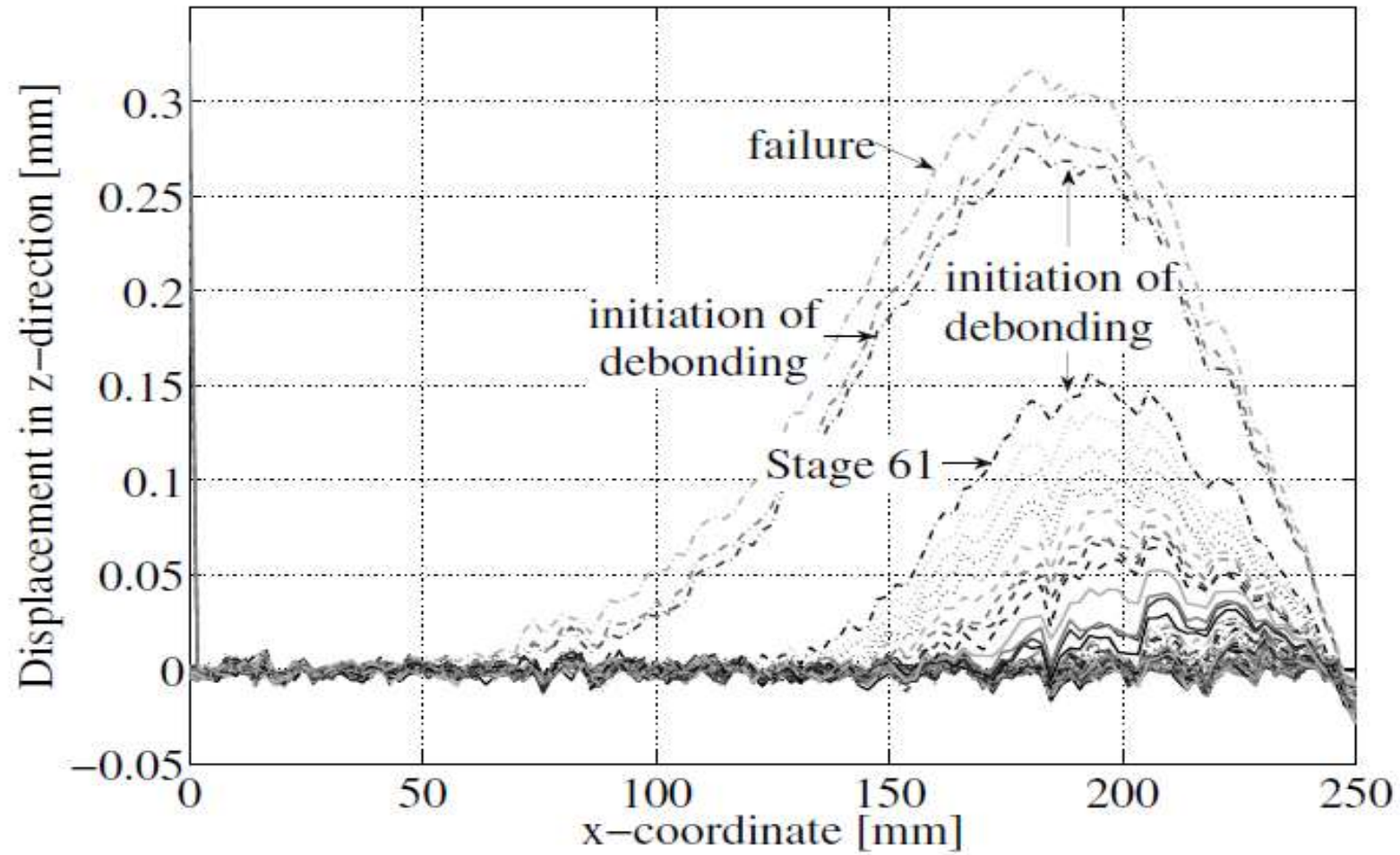




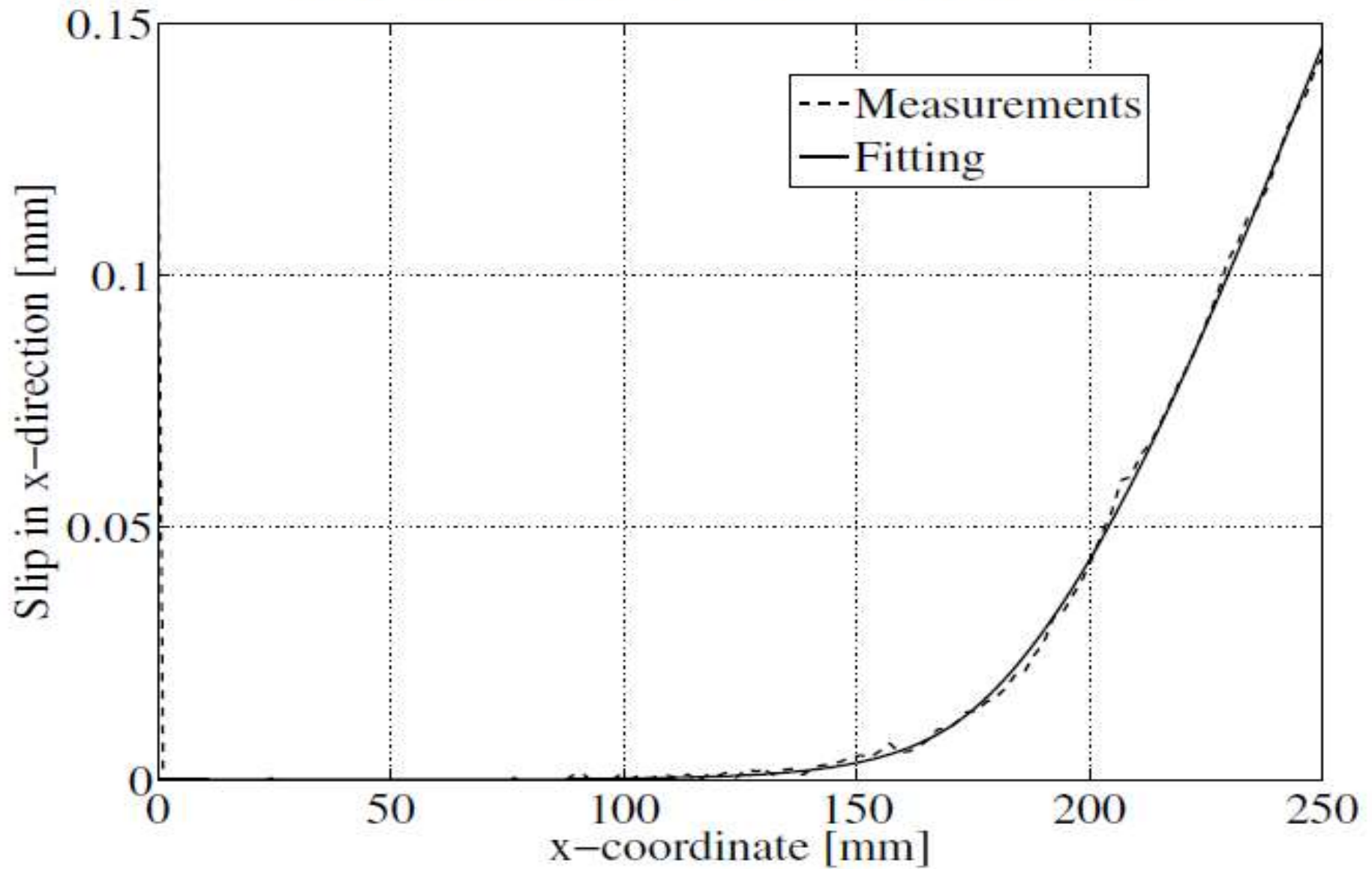
Pull-off Test No. 3

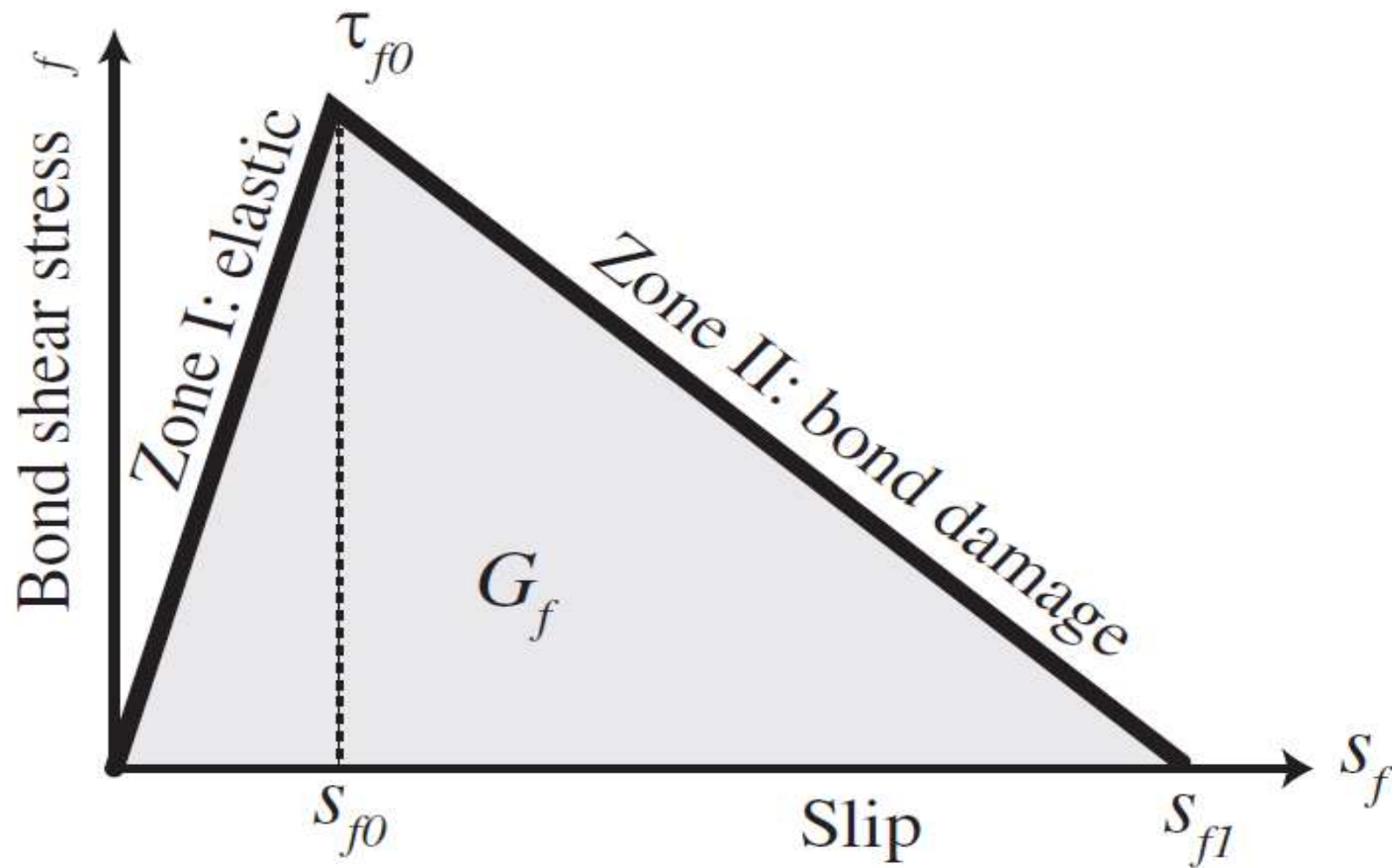


Pull-off Test No. 3



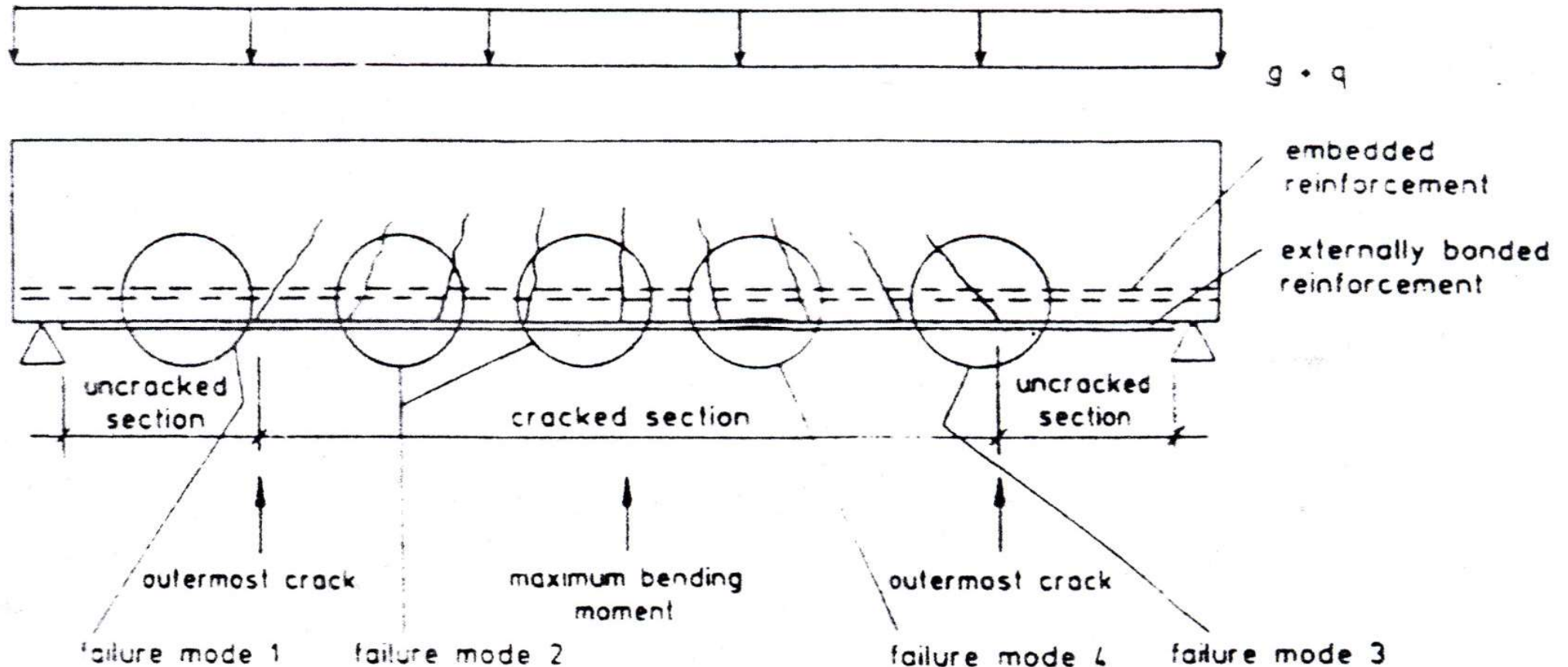
Pull-off Test No. 1, Stage 36, Load 21.1kN





Bond failure of RC members strengthened with FRP:

See next lecture given by Dr. Christoph Czaderski

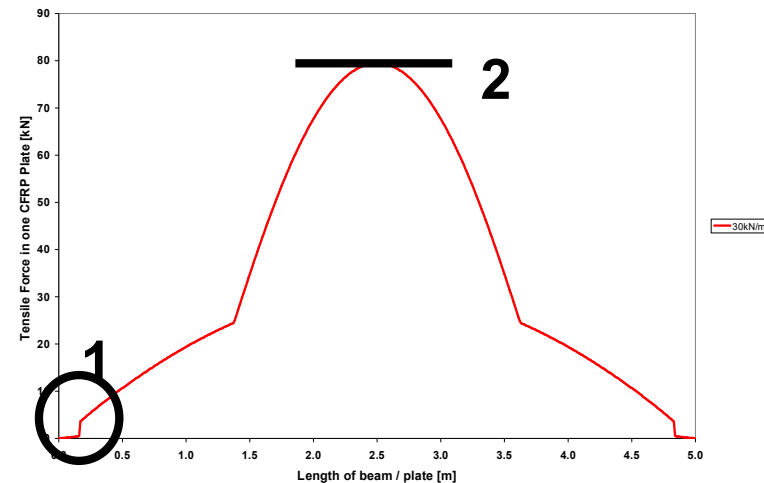


Summary of the two Swiss Code SIA 166 (2024) verifications

See next lecture given by Dr. Christoph Czaderski

1. End strip debonding failure at the last crack $F_{fcr} \leq F_{b,R}$

2. Debonding at maximum strain $\varepsilon_f \leq \varepsilon_{fd,lim}$



Summary of the Equations, Code SIA 166 (2024) verifications

See next lecture given by Dr. Christoph Czaderski

$$F_{b0,Rd} = b_f \sqrt{2G_{Fcd}E_f t_f}$$

$$G_{Fcd} = \frac{1}{8} \eta_u \eta_l \frac{f_{hk}}{\gamma_h}$$

$$\tau_{c,max,d} = \frac{4}{3} \eta_u \eta_l \frac{f_{hk}}{\gamma_h}$$

G_{Fcd} in N/mm f_{hk} in N/mm²

$$l_{b0d} = 2.5 \sqrt{\frac{G_{Fcd}E_f t_f}{\tau_{c,max,d}^2}}$$

for $l_{bd} < l_{b0d}$:

$$F_{b,Rd} = F_{b0,Rd} \frac{l_{bd}}{l_{b0d}} \left(2 - \frac{l_{bd}}{l_{b0d}} \right)$$

$$\varepsilon_{fd,lim} = c \eta_u \eta_l \sqrt{\frac{f_{hd}}{E_f t_f}} \leq \varepsilon_{fud}$$

$c = 2.3$ for strips, $c = 1.6$ for fabrics

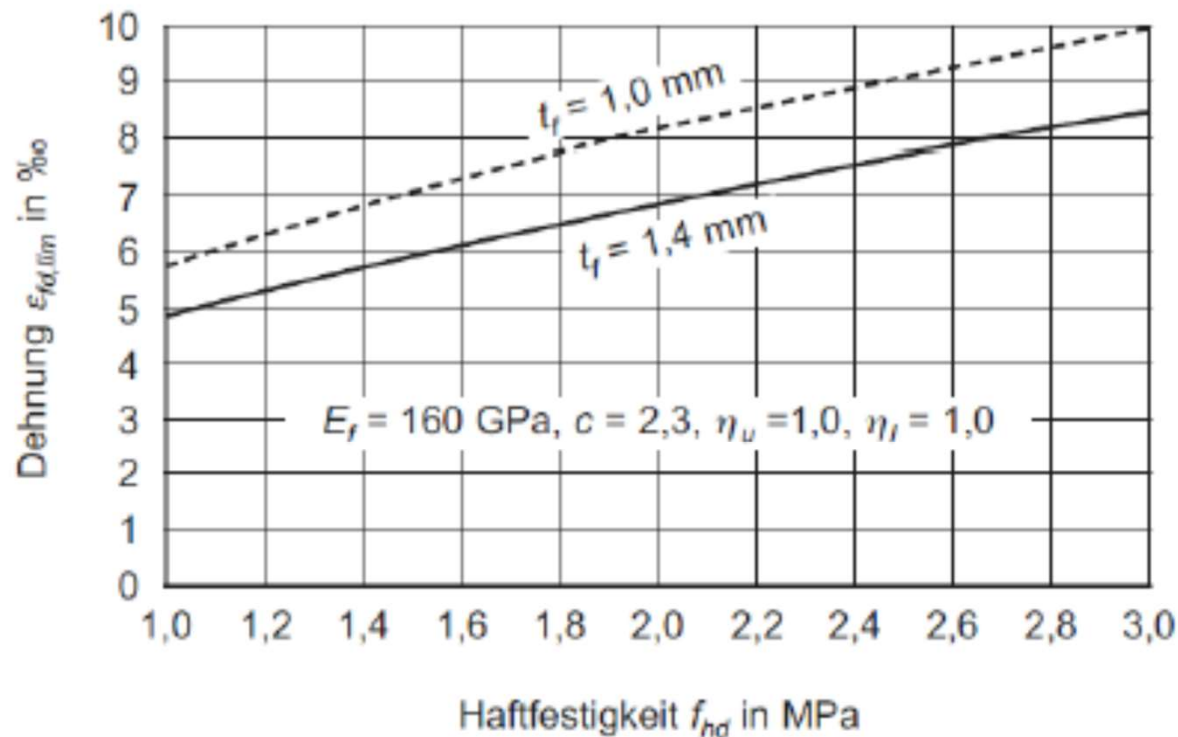
The reduction values η_u and η_l are tabulated in SIA 166(2024), they consider environment and loading conditions.

Summary of the Equations, Code SIA 166 (2024) verifications

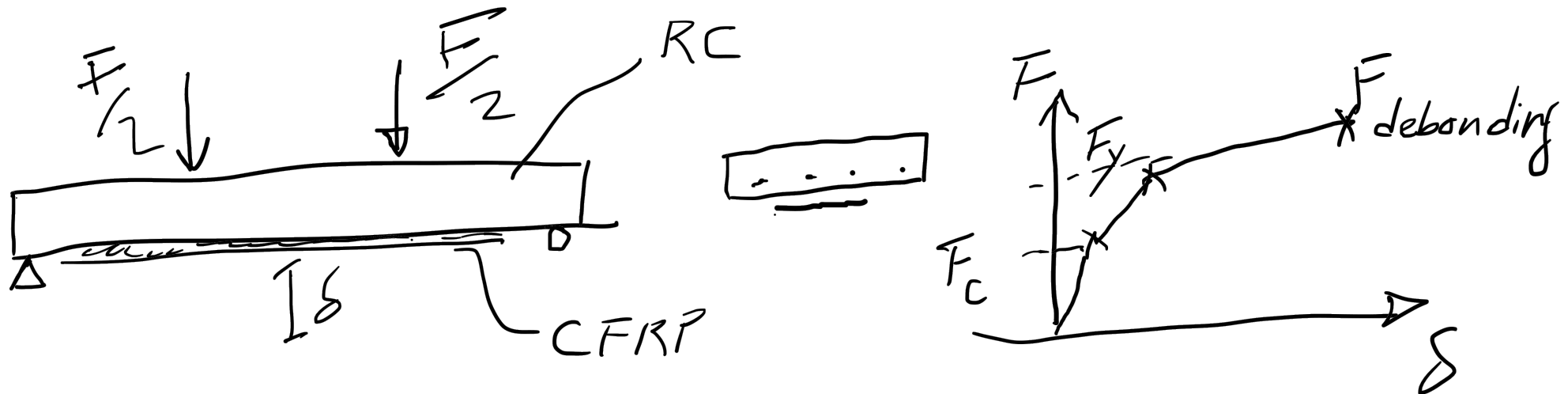
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SIA 166 (2024)

Figur 5 Beispiele für Bemessungswerte der maximalen Dehnung $\varepsilon_{fd,lim}$ für Lamellen gemäss Gleichung (33) in Abhängigkeit des Bemessungswertes der Haftfestigkeit des oberflächennahen Betons f_{hd}



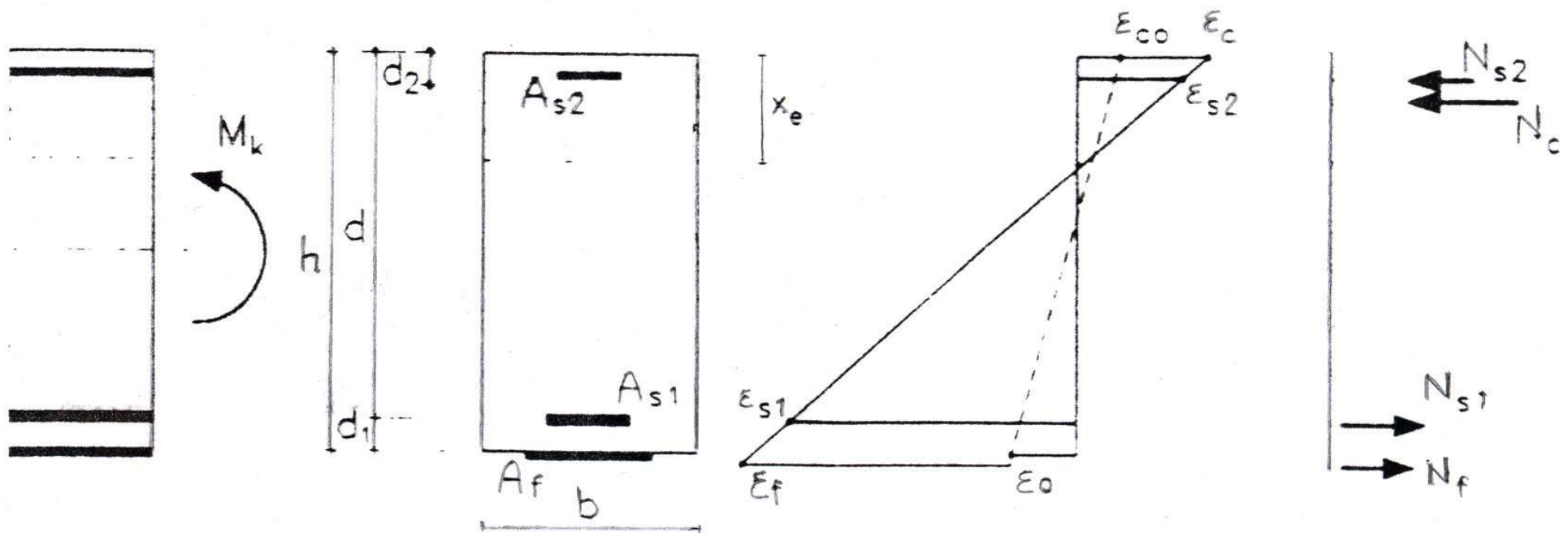
4-Point Bending test, RC beam strengthened with a CFRP strip



Debonding: Check the two criterias according to SIA 166

Serviceability Limit State (SLS)

- linear elastic material behavior
- cracked section analysis



Calculation of neutral axis x_e :

$$\frac{1}{2}bx_e^2 + (\alpha_s - 1)A_{s2}(x_e - d_2) = \alpha_s A_{s1}(d - x_e) + \alpha_f A_f \left[h - \left(1 + \frac{\varepsilon_0}{\varepsilon_c}\right)x_e \right]$$

Where: $\alpha_f = \frac{E_f}{E_c}$

$$\alpha_s = \frac{E_s}{E_c}$$

And the cracking moment for rectangular beams:

$$M_{cr} \approx f_{ctm} \cdot \frac{bh^2}{6}$$

Stress limitation

limit stresses in the concrete, steel and FRP to prevent

- damage or excessive creep of the concrete
- steel yielding
- excessive creep or creep rupture of the FRP

$$\sigma_c \leq 0.60 f_{ck} \quad \text{under the rare load combination}$$

$$\sigma_c \leq 0.45 f_{ck} \quad \text{under the quasi-permanent load combination}$$

where: $\sigma_c = E_c \varepsilon_c$

To prevent yielding of the steel at service load:

$$\sigma_s = E_s \cdot \varepsilon_c \cdot \frac{d - x_e}{x_e} \leq 0.80 f_{yk} \quad \text{rare load combination}$$

FRP stress under service load should be limited as:

$$\sigma_f = E_f \cdot \left(\varepsilon_c \cdot \frac{h - x_e}{x_e} - \varepsilon_0 \right) \leq \eta \cdot f_{fk} \quad \text{quasi-permanent load combination}$$

Where

$$\eta = \begin{cases} 0.8 : CFRP \\ 0.5 : AFRP \\ 0.3 : GFRP \end{cases}$$

Verification of deflections

The mean deflection, a , is calculated from:

$$a = a_1 \cdot (1 - \zeta_b) + a_2 \cdot \zeta_b$$

Where a_1 and a_2 are the deflections in the uncracked and the fully cracked state, respectively and the distribution coefficient is:

$$\zeta_b = 0 \dots \dots M_k < M_{cr}$$

$$\zeta_b = 1 - \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_k} \right)^{n/2} \dots \dots M_k > M_{cr}$$

- Where β_1 is a coefficient taking into account the bond characteristics of the reinforcement and equals 0.5 and 1 for smooth and deformed steel, respectively;
- β_2 is a coefficient taking into account the loading type and equals 0.5 and 1 for long-term and short term loading, respectively.
- The power n equals 2. For high strength concrete more accuracy is obtained with n equal to 3.

The deflection in the **uncracked state**, **a1**, and in the **fully cracked state**, **a2**, can be calculated by classical elasticity analysis, referring to a flexural stiffness in the **uncracked state** $E_c I_1$ and in the **fully cracked state** $E_c I_2$, respectively.

Verification of crack widths

Neglecting the tension stiffening effect ($\zeta = 1$) and assuming $\varepsilon_0 \approx 0$

$$w_k = 2.1 \rho_{c,eff} \cdot \frac{M_k}{E_s d \rho_{eq}} \cdot \frac{1}{(u_s + 0.694 u_f)}$$

Where the ratio of the effective area in tension is:

$$\rho_{c,eff} = \frac{A_{c,eff}}{bd}$$

ρ_{eq} is the equivalent reinforcement ratio and u_s and u_f is the bond perimeter of the steel and FRP reinforcement.

Summary of design procedure:

- Before strengthening: check ULS and SLS (just to compare with the strengthened member!).
- From the service moment M_0 prior to strengthening determine ε_0 at the extreme tension fiber.
- Assume full composite action and from the design moment after strengthening determine the required FRP cross section to fulfill the ULS. Verify the ductility requirements.
- Calculate the deflections in the SLS. If allowable deflection is exceeded, determine the required FRP cross section.

- Calculate the stresses in the concrete, steel and FRP and verify the allowable stresses.
- Verify that the provided FRP bond width is sufficient to control crack widths in the SLS. Increase the FRP width, if necessary, or, given a maximum width, increase the amount (thickness) of FRP.
- Verify the resisting shear force at which bond failure due to shear cracks occurs (ULS).
- Verify that bond failure at the anchorage does not occur. Otherwise mechanical anchorage should be provided.

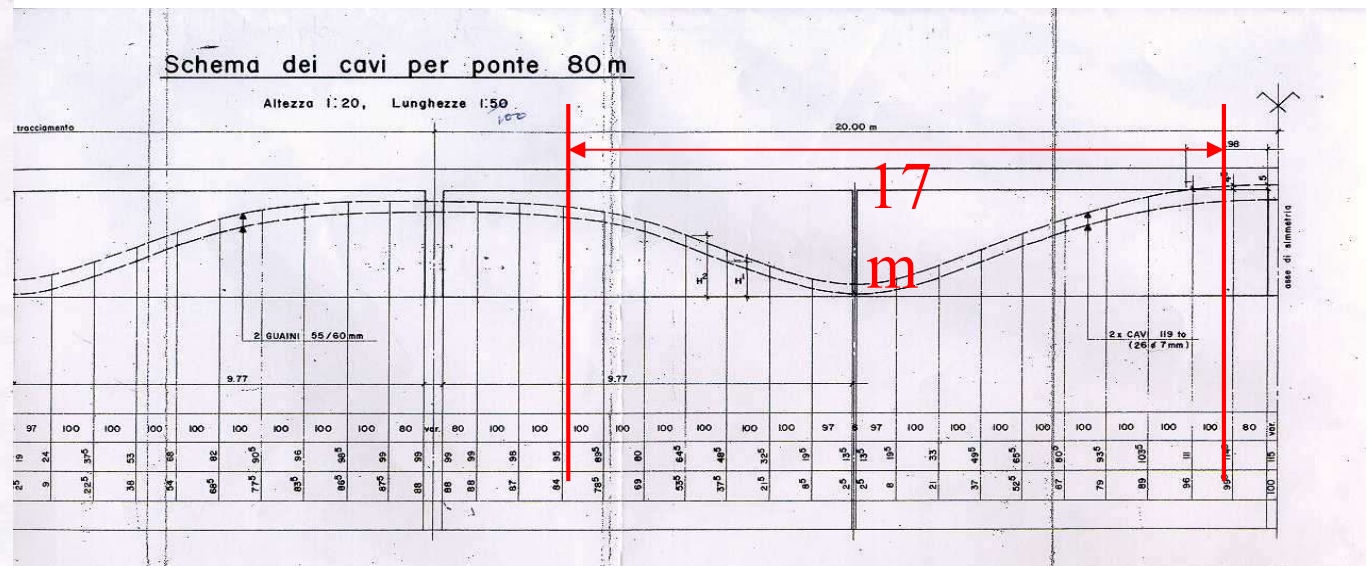
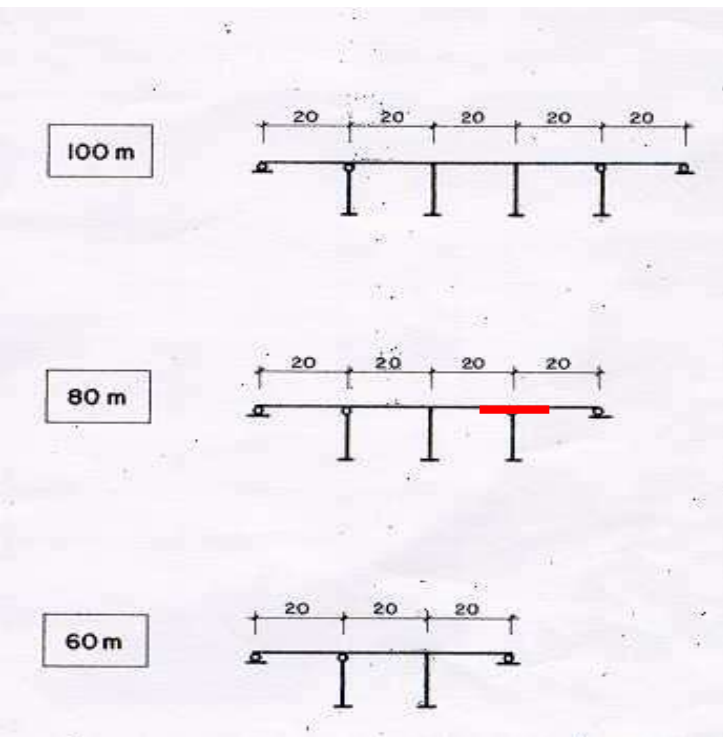
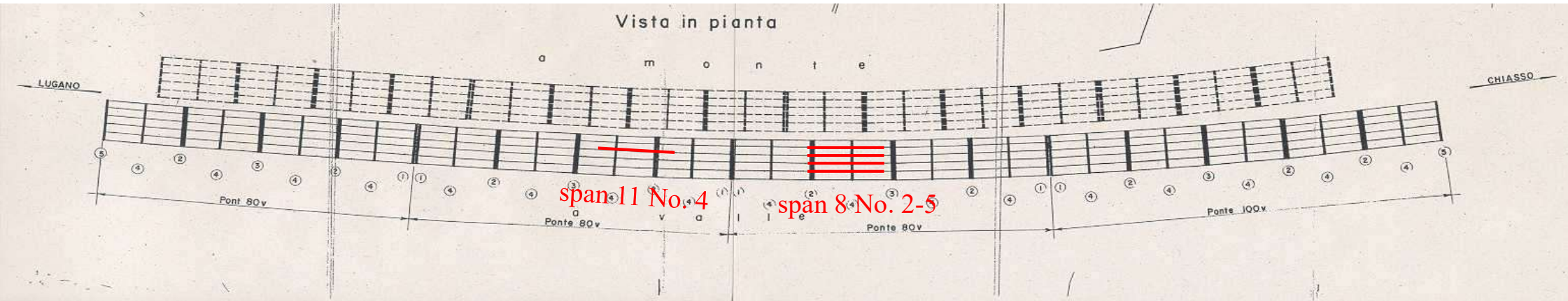
- Verify that FRP end shear failure is avoided. Provide shear strengthening at the ends if required.
- Verify the accidental situation.
- Verify the shear design resistance of the strengthened member. If needed shear strengthening should be provided.

Strengthening of a Large Scale Pre-Stressed Bridge Girder Using Carbon Fibre Reinforced Polymers:

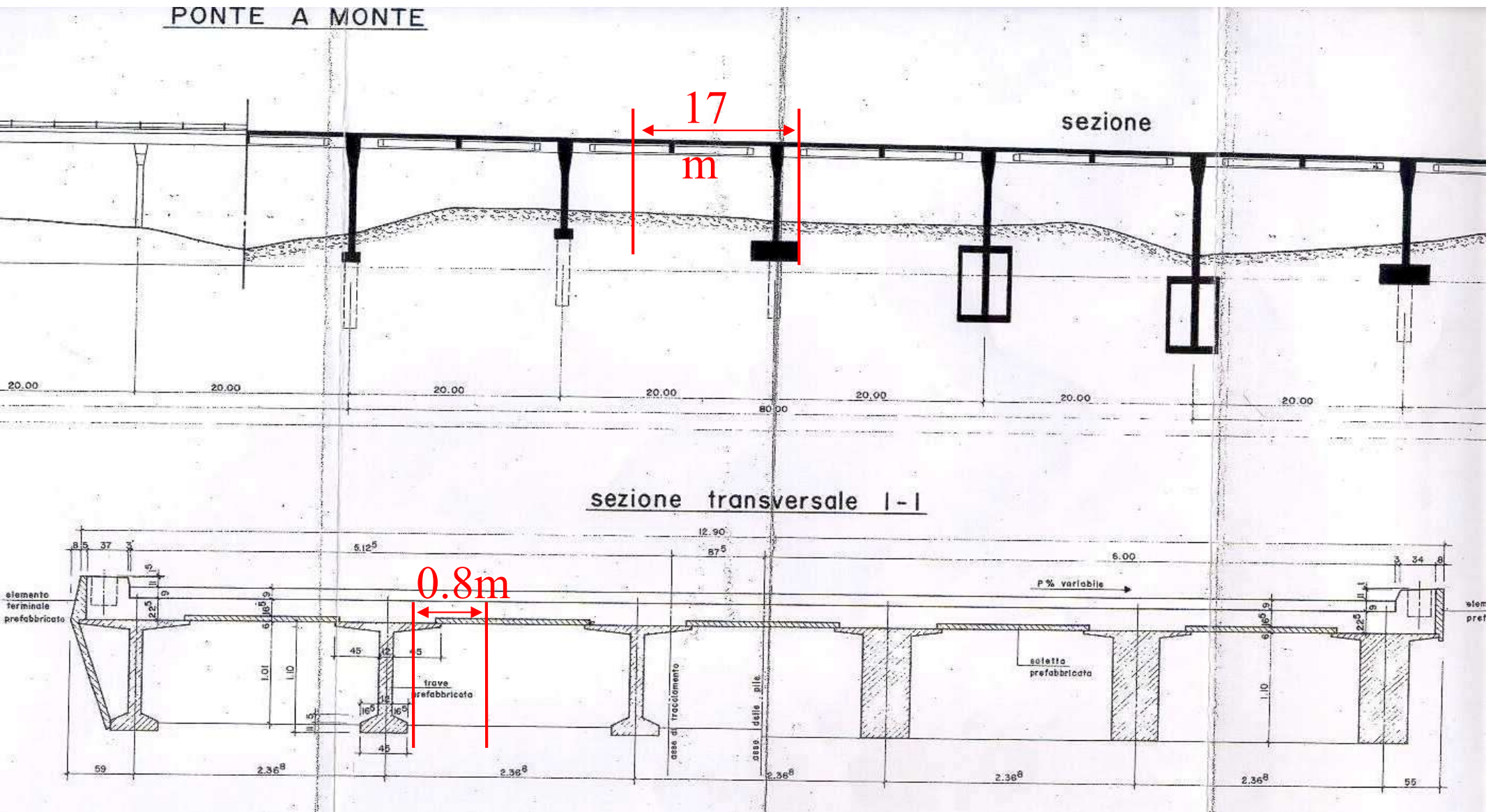
Comparision between Non Prestressed and Prestressed CFRP Plates



Bridge „Viadotto delle Cantine a Capolago“



Bridge „Viadotto delle Cantine a Capolago“



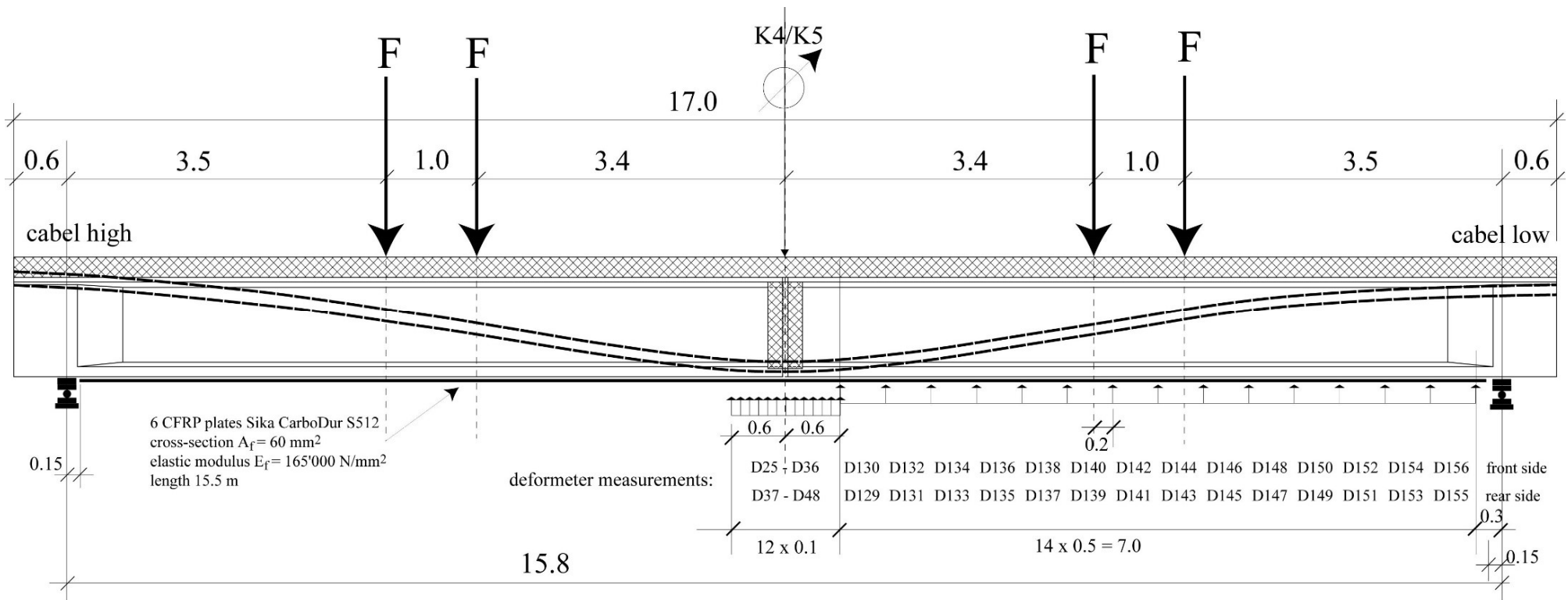


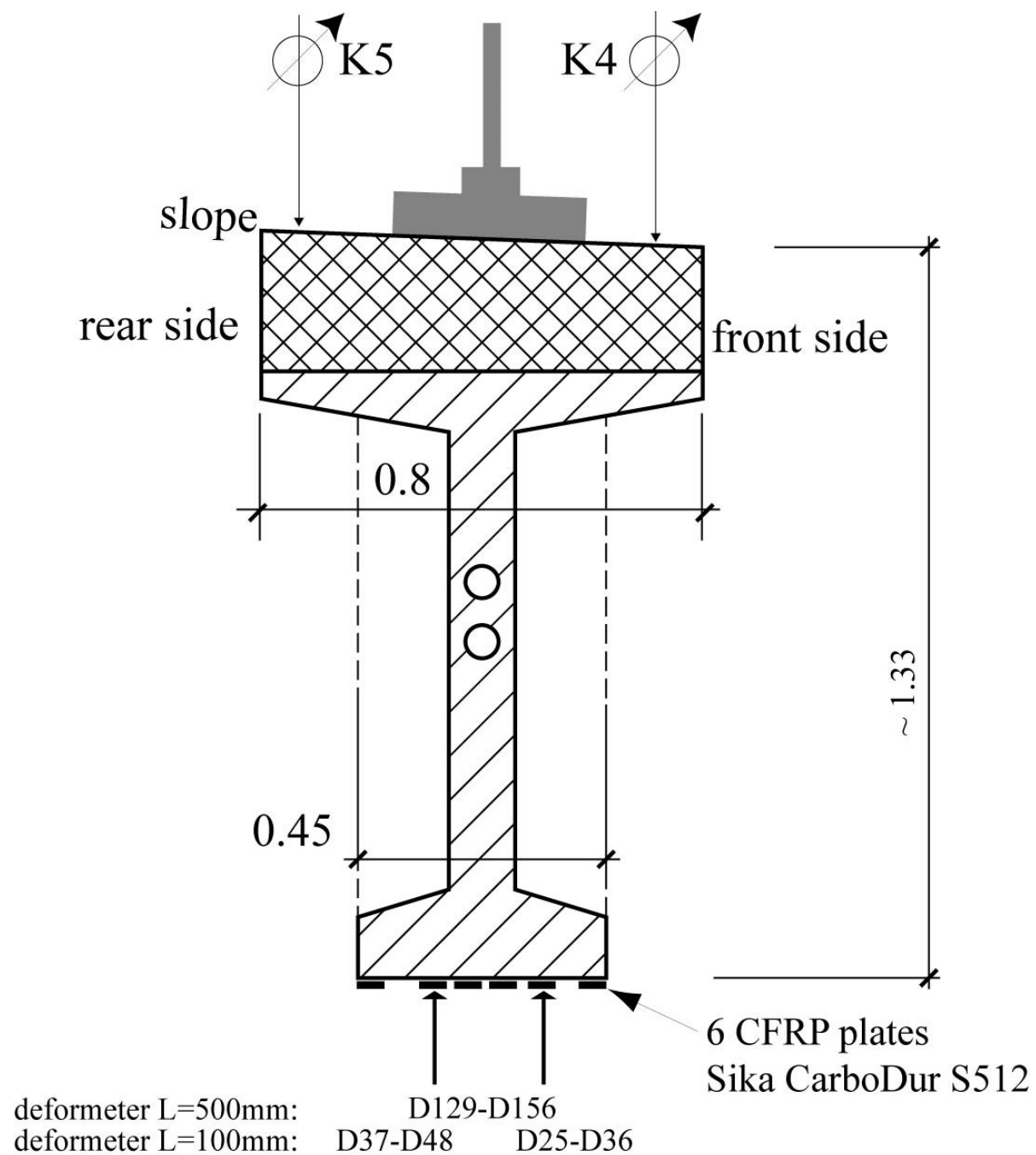


Overview

- Reference beam
- Beam strengthened with *non* prestressed CFRP plates
 - 6 Sika CarboDur 512 plates, each 15.5 m long
- Beam strengthened with *pre*stressed CFRP plates
 - the same type and number of plates
 - each plate prestressed approx. 1000 MPa (60 kN)
 - anchorage: Empa gradient method

Strengthened with non prestressed CFRP plates

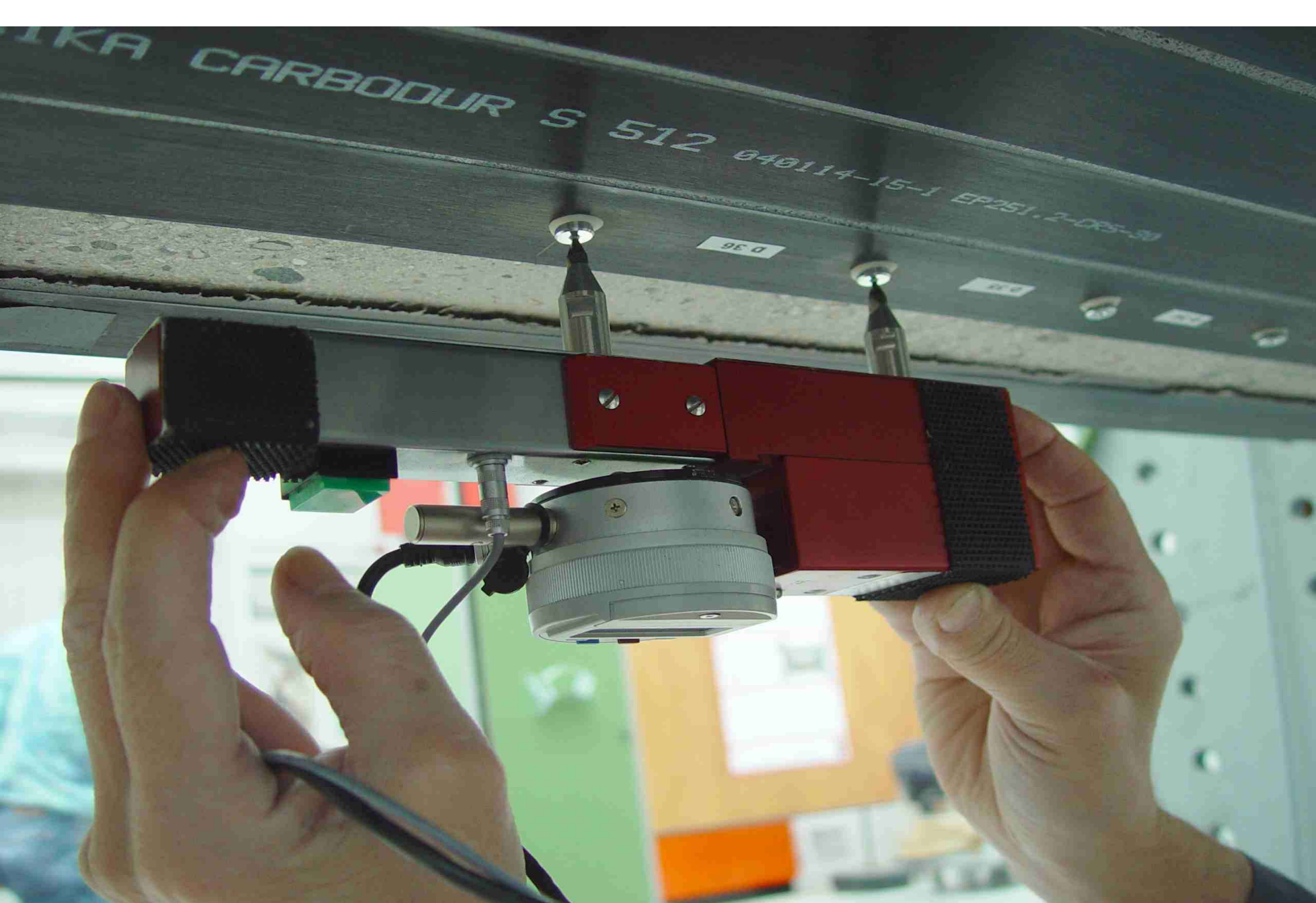


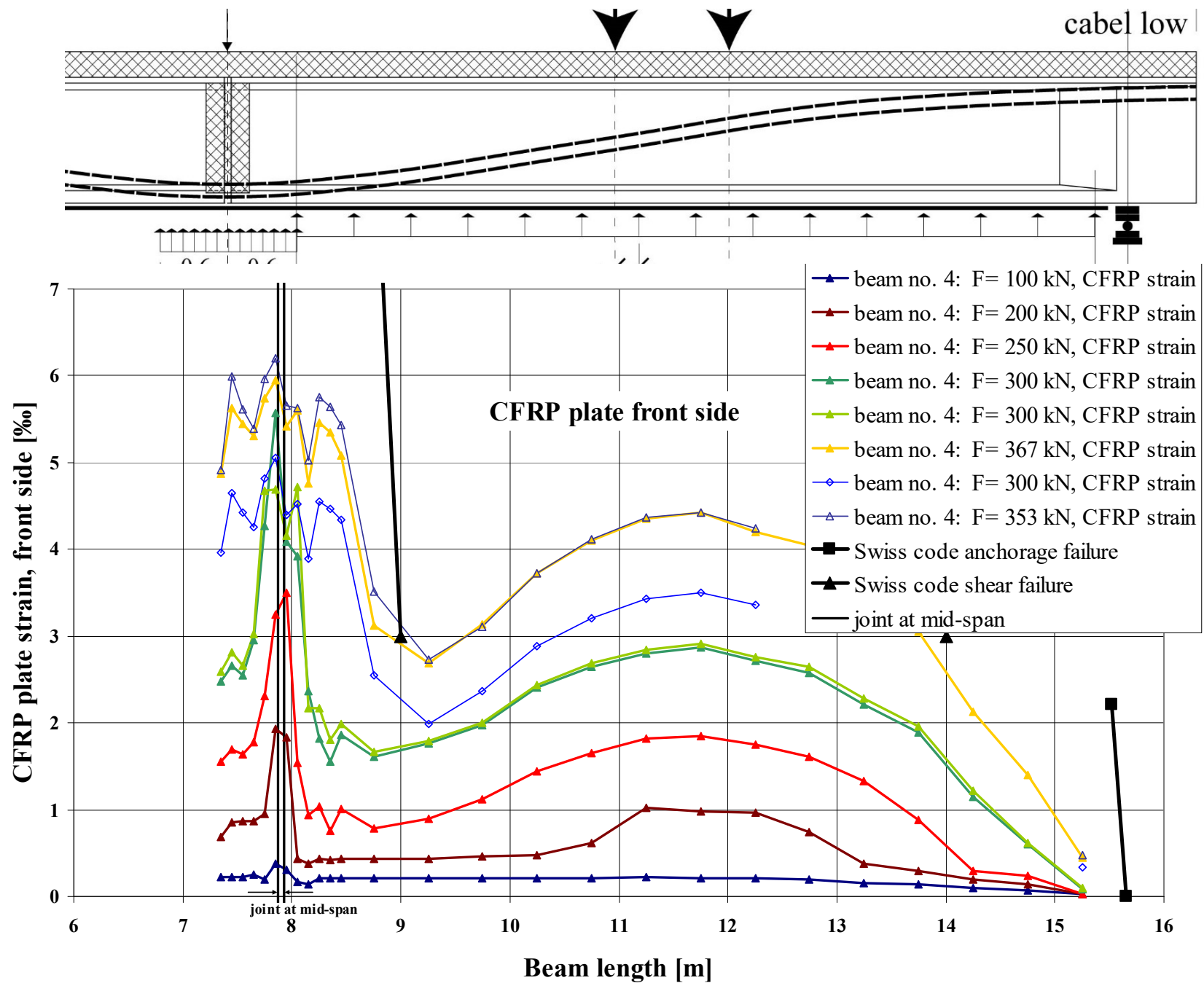




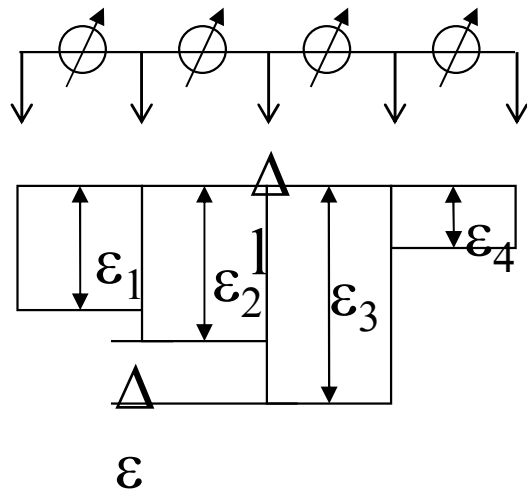
Deformeter Measurements





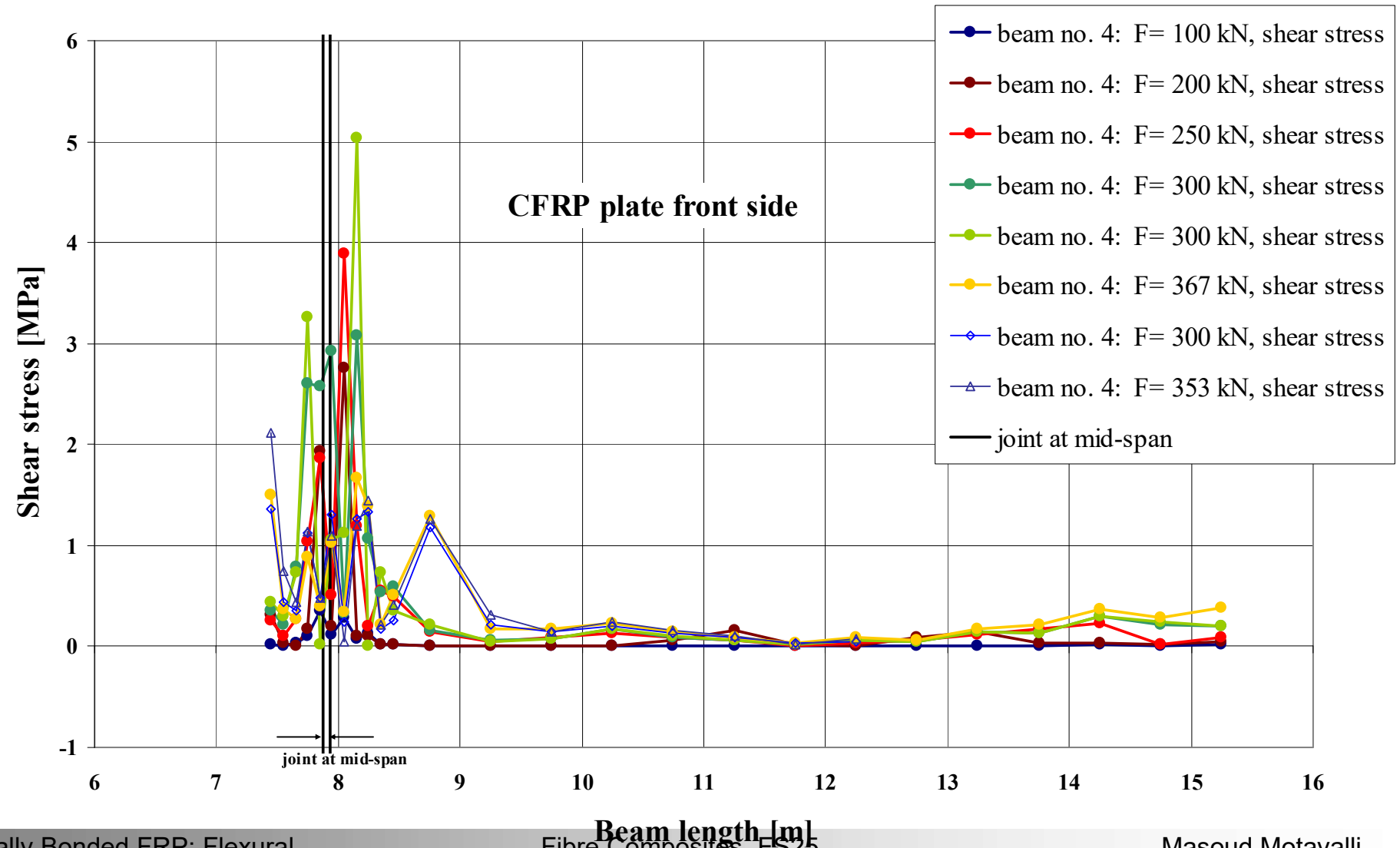


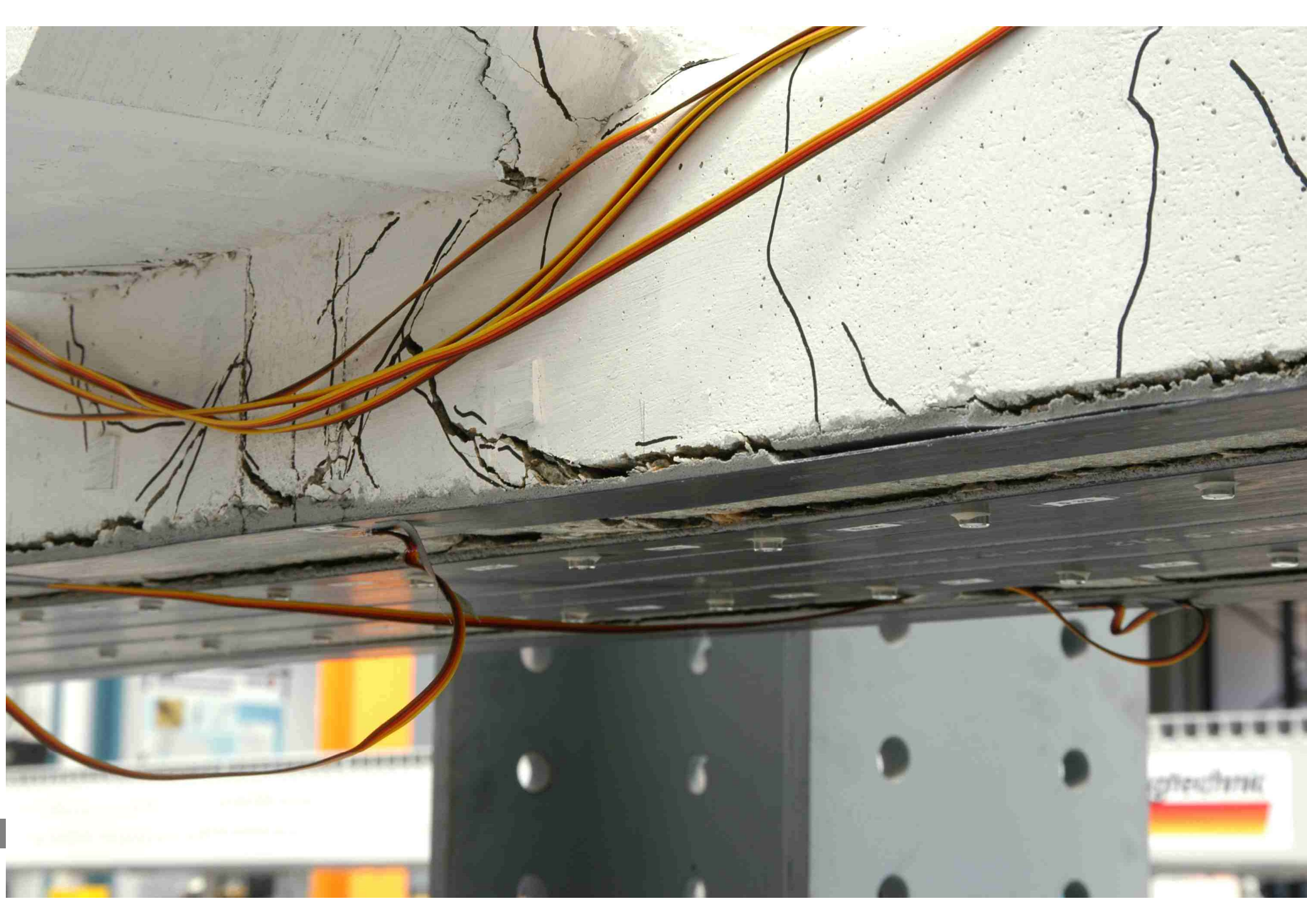
Shear stress from Deformeter-measurement



$$\tau_1 = \frac{\Delta F}{b_1 \cdot \Delta l} = \frac{\Delta \epsilon \cdot E_1 \cdot A_1}{b_1 \cdot \Delta l}$$

Behavior during loading









Reference beam
No. 3 (2/8)



F= BRUCH
4 PHASE

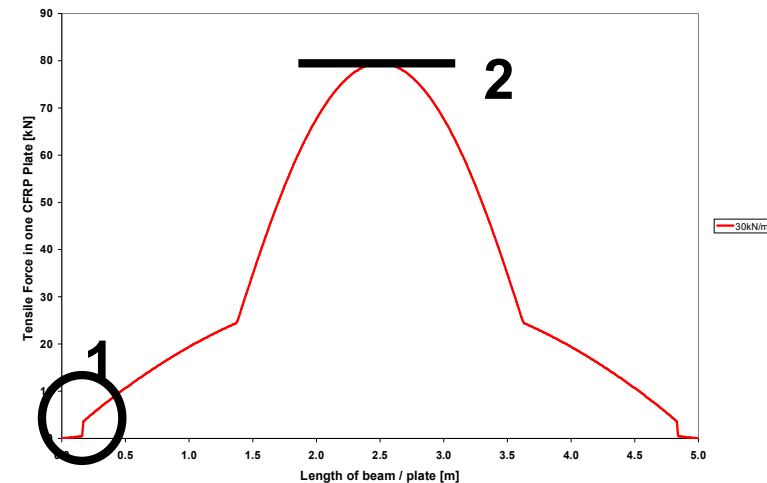


Summary of the two Swiss Code SIA 166 (2024) verifications

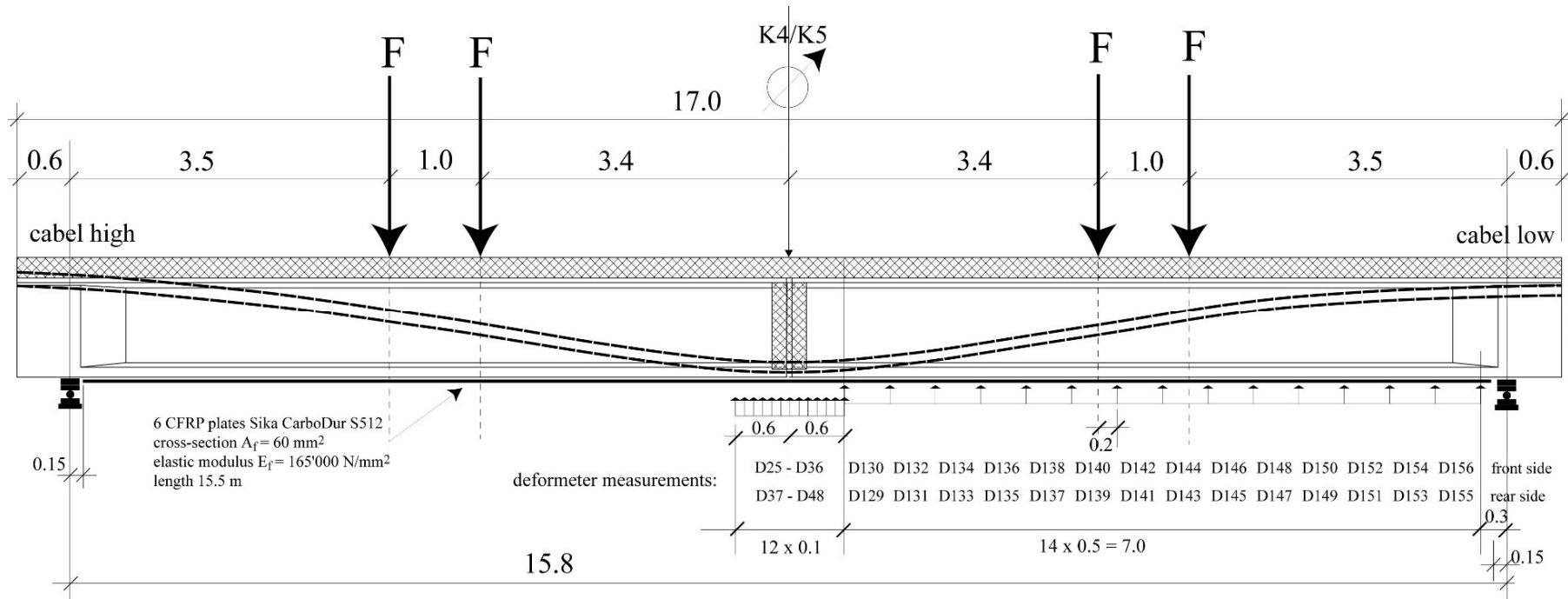
See next lecture given by Dr. Christoph Czaderski

1. End strip debonding failure at the last crack $F_{fcr} \leq F_{b,R}$

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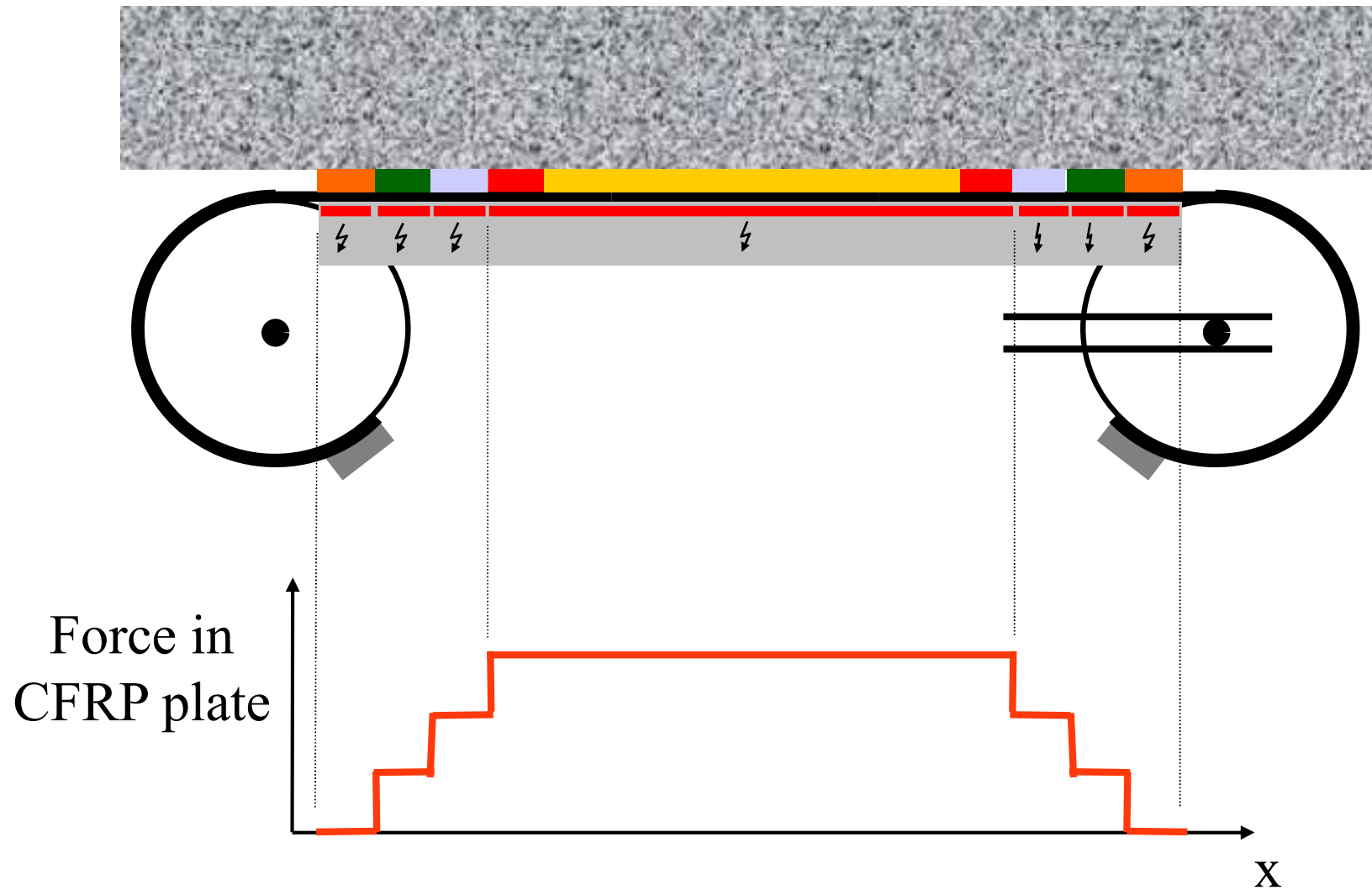


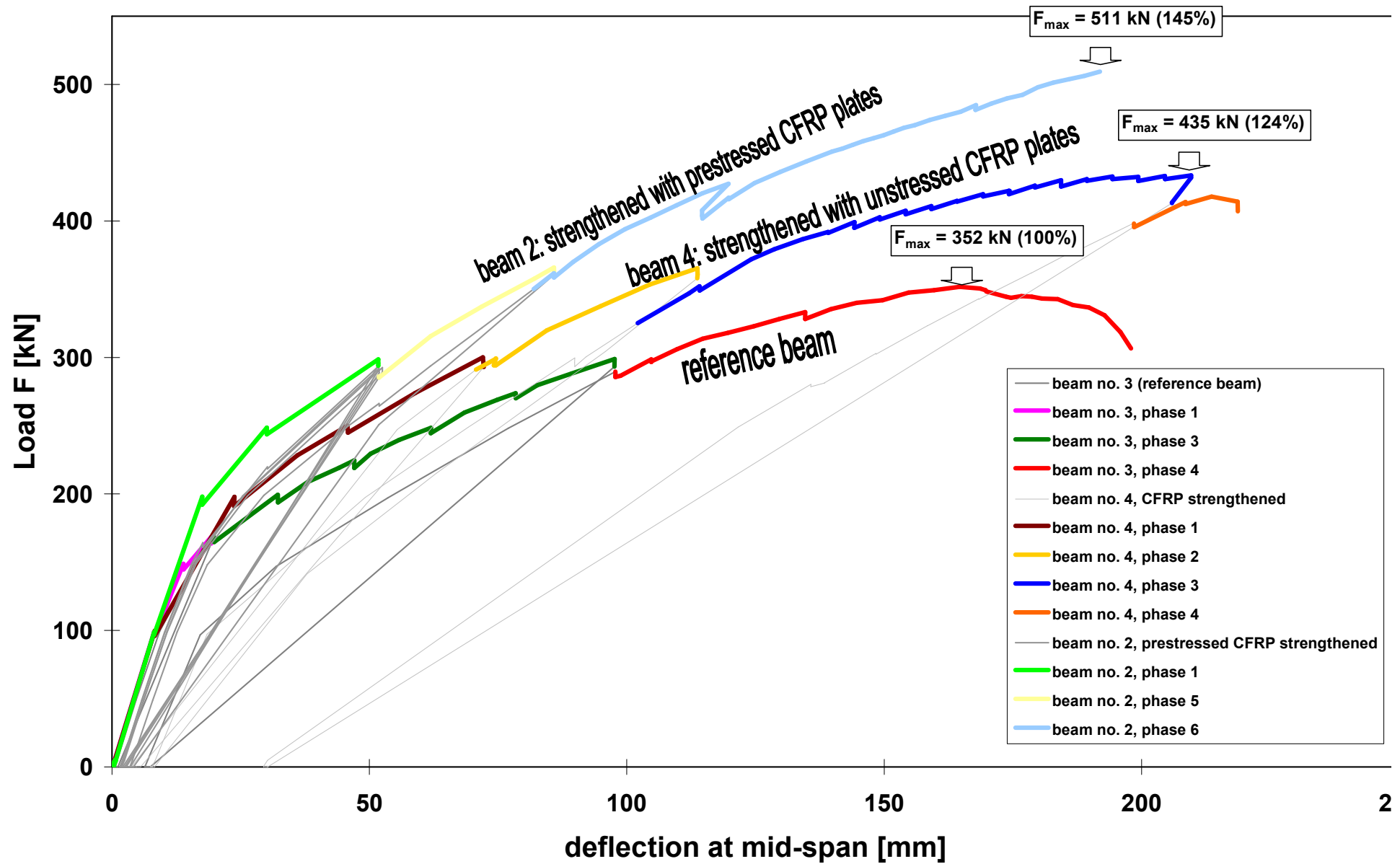
Strengthened with prestressed CFRP plates



CFRP plates prestressed approx. 1000 MPa (60 kN)

Prestressing using Gradient-method





"List of Symbols" - 1-

< Flexural strengthening >

- M_0 : service moment
 M_{cr} : cracking moment
 A_{s1} : steel cross-section ~~at~~ (tensile reinforcement)
 A_{s2} : steel cross-section (compression reinforcement)
 x_0 : Position of the neutral axis prior to strengthening
 b : cross-section width
 d : " depth ; $h = d + d_1$
 $\alpha_s = \frac{E_s}{E_c}$: $\begin{cases} E_s : \text{steel E-modulus} \\ E_c : \text{concrete "} \end{cases}$
 I_{o2} : moment of inertia of the transformed cracked section prior to strengthening
 l_b : bond length
 s_f : slip ($\approx s_d$)
 $\gamma \approx 0.8$ if "steel yielding followed by concrete crushing"
 A_f : FRP cross-section, or (A_{FRP})
 x (also c) : position of the neutral axis after strengthening
 δ_{GX} : position of the concrete compression force at ULS, ($\delta_G = 0.4$) if "steel yielding followed by concrete crushing"
 h_f : Height of the flange
 b_f : width of " "
 b_{FRP} : " " the FRP
 $\rho_{eq} = \frac{A_s + A_f \frac{E_f}{E_s}}{b d}$: equivalent reinforcement ratio
 $V_R = \tau_R \cdot b \cdot d$: V_R : shear resistance ; τ_R : shear strength
 ρ_s : steel reinforcement ratio
 ρ_f : FRP reinforcement ratio

"List of Symbols" - 2 -

< Flexural Strengthening >

- $N_{fa, max}$: maximum FRP force, which can be anchored
- $l_{b, max}$: maximum anchorage length
- f_{ctm} : mean concrete tensile strength
- t_f : FRP thickness
- x_e : position of neutral axis at SLS
- $\sigma_f \leq \lambda \cdot f_{fk}$: FRP stress under service load, where
- $$\lambda = \begin{cases} 0.8 & : \text{CFRP} \\ 0.5 & : \text{AFRP} \\ 0.3 & : \text{GFRP} \end{cases}$$
- $a; a_1; a_2$: mean deflection; deflection in uncracked state;
deflection in fully cracked state
- ξ_b : the distribution coefficient to calculate the deflection
- $E_c I$: flexural stiffness in the uncracked state
- $E_c I_2$: " " " fully cracked state
- W_k : crack width at SLS
- u_s : bond perimeter of steel
- u_f : " " of FRP
- τ_{fl} : maximum shear stress at FRP end at SLS
- G_a, t_a : shear modulus, thickness of adhesive