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References:

Outline

- Introduction
  (Pro’s and con’s of FRP / Examples)

- Materials
  (Manufacturing process / Materials / Durability)

- Design Concept
  (Concept / Basic assumptions / …)
  - Bending Beam
    (Timoshenko theory / Stresses / Deformations / Buckling …)
  - Axial Members
    (Serviceability and ultimate limit states)
  - Connections
    (Bolted joints / Glued joints)
Introduction
**Introduction: Pro’s and con’s**

### Pro’s

- **High specific strength:**

  \[ l_{\text{max}} = \frac{\sigma_{\text{max}}}{\rho \cdot g} \]

<table>
<thead>
<tr>
<th>Material</th>
<th>CFRP</th>
<th>GFRP</th>
<th>Steel S500</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>l_{\text{max}}</strong></td>
<td>138.4 km</td>
<td>27.8 km</td>
<td>6.4 km</td>
</tr>
</tbody>
</table>

- **Good in-plane mechanical properties**
- **High fatigue and environmental resistance**
- **Adjustable mechanical properties**
- **Lightweight-> ease of handling, small additional load…**
- **Quick assembly / erection**
- **Low maintenance**
- **Highly cost-effective (2-10 €/kg)**
Introduction: Pro’s and con’s

- Con’s
  - Brittle
  - High initial costs
  - Low to moderate application temperature (-20 up to 80 °C)
  - Low fire resistance (sometimes with unhealthy gases)
Introduction: Common profiles

- Structural profiles
  - Most structural profiles produced in conventional profile shapes similar to metallic materials
  - Similarity in geom. and properties, however no standard geom., mechanical and physical properties used by all manufacturers
Introduction: Examples

- Footbridges

Fiberline Bridge in Kolding, DK
1997
The bridge was installed during 18 hours over 3 nights
Span: 40 m
Cost: 0.5 mio CHF
Only Fiberline standard profiles used

http://fiberline.com/fiberline-bridge-kolding

Pontresina bridge, Switzerland
1997
Span: 2 x 12.5 m
Weight: 3.3 tons (installation by helicopter)

http://fiberline.com/pontresina-bridge-switzerland
Introduction: Examples

Footbridges

Composite pedestrian bridge in Lleida, Spain

Span: 38 m
Width: 3.0 m

http://fiberline.com/international-award-innovative-grp-footbridge

“GRP does not conduct electricity, which is also very important as it means there is no magnetic interference with the electrified railway,” continues Mr. Sobrino.
Introduction: Examples (Avançon Bridge, Switzerland, 2012)

- Road bridges

Cross section of new two-lane bridge (dimensions in [mm])

Bridge details
Location: Bex, Suisse
Installed on: 12th October, 2012
Dimensions: 12m x 7m (9 tons)
**Introduction: Examples**

- **Bridgedeck** (Footbridges)

  *Würenlos, Switzerland*

  *Loopersteg, Switzerland*
Introduction: Examples

- Buildings

Eyecatcher Building, Basel, Switzerland 1998

Height: 15 m
Storeys: 5


Project: Maagtechnic
Introduction: Examples

- Laboratory bridge

Empa Laboratory Bridge, Switzerland

Span: 19 m
Width: 1.6 m
Load capacity: 15 tons
Introduction: Examples

- Noise barrier SBB

Gösschenen, Switzerland

Project: Maagtechnic
Introduction: Examples

- Balconies

Switzerland

Project: Maagtechnic
Introduction: Examples

bogie

Introduction: Application

Applications where GFRP structures are competitive:

- Significant **corrosion** and **chemical** resistance is required
  (Food and chemical processing plants, cooling towers, offshore platforms …)

- **Electromagnetic** transparency or **electrical** insulation is required.

- **Light-weight** is cost essential
  (fast deployment …)

- **Prestige** and demonstration objects
  (e.g. Novartis Campus Entrance Building)
Material
Material: Pultrusion process

- Only pultruded GFRP profiles will be considered in this lecture

Pultrusion line

- Production of profiles with constant cross-section along the length
- High quality
- Continuous longitudinal fiber bundles and filament mats
**Pultrusion process**

Take from [https://www.youtube.com/watch?v=aXq1hrzne2k]
Material: Components

- Pultruded profiles contain three primary components:
  - Reinforcement
  - Matrix
    - polyester
    - epoxy
    - phenol
  - Supplementary constituents
    - polymerisation agents
    - fillers
    - additives

Types of roving:
- Unidirectional
- Spun
- Mock

Types of mat:
- Continuous mat
  - Random fibre orientation
- Weave
  - $0^\circ/90^\circ$
- Complex mat
  - $0^\circ/90^\circ$ membrane + random fibre orientation
- Bidirectional complex mat
  - $0^\circ/45^\circ/90^\circ$ weave + random fibre orientation
**Material: Shapes of pultruded profiles**

- **Available Profiles on Stock:**

<table>
<thead>
<tr>
<th>Name</th>
<th>Tubes</th>
<th>Flat - Profiles</th>
<th>Square Tubes</th>
<th>T - Profile</th>
<th>Angle</th>
<th>U / UL - Profile</th>
<th>I / IL - Profile</th>
<th>Plank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td>Ra = 37.5 / 45</td>
<td>B = 30 - 1220</td>
<td>H = 50 - 240</td>
<td>H = 60 / 90</td>
<td>H = 50 - 150</td>
<td>H = 120 - 360</td>
<td>H = 120 - 360</td>
<td>B = 500</td>
</tr>
<tr>
<td>[mm]</td>
<td>T = 5</td>
<td>B = 60 / 72</td>
<td>T = 6 / 10</td>
<td>B = 60 / 180</td>
<td>T = 6 - 18</td>
<td>B = 60 / 180</td>
<td>H = 40</td>
<td></td>
</tr>
</tbody>
</table>

Length up to 12 m (for transportation reasons)!

- Special cross-sections can be designed and ordered (several kilometres are necessary → special tools have to be designed)
Material: Durability

- Various environmental and load conditions that affect durability of (G)FRPs in terms of strength, stiffness, fiber/matrix interface integrity, cracking:
  - water/sea water
  - chemical solutions
  - prolonged freezing
  - thermal cycling (freeze-thaw)
  - elevated temperature exposure
  - UV radiation
  - creep and relaxation
  - fatigue
  - fire...
**Material: Durability**

- Environmental reduction factor for different FRP systems and exposure conditions

<table>
<thead>
<tr>
<th>Exposure condition</th>
<th>Fiber / resin type</th>
<th>Environ. reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior exposure</td>
<td>Carbon/epoxy</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>Glass/epoxy</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Aramid/epoxy</td>
<td>0.85</td>
</tr>
<tr>
<td>Exterior exposure (bridges,</td>
<td>Carbon/epoxy</td>
<td>0.85</td>
</tr>
<tr>
<td>piers parking garages)</td>
<td>Glass/epoxy</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Aramid/epoxy</td>
<td>0.75</td>
</tr>
<tr>
<td>Agressive environ. (chemical</td>
<td>Carbon/epoxy</td>
<td>0.85</td>
</tr>
<tr>
<td>plants)</td>
<td>Glass/epoxy</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>Aramid/epoxy</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Material: Manufacturers

- GFRP profiles available on stock

In Europe two companies pultrude FRP-Profiles:

- **Fiberline Composites, Denmark**
  - [www.fiberline.com](http://www.fiberline.com)
  - **Fiberline Design Manual** ([www.fiberline.dk](http://www.fiberline.dk))
  - helpful tool to design structures
    - (material properties, geometries, connections, ...)

- **Top Glass, Italy**
  - [www.topglass.it](http://www.topglass.it)
Material: Manufacturers

- GFRP profiles available on stock

  In North America:
  
  - Strongwell, USA
    www.strongwell.com
  
  - Creative Pultrusions, USA
    www.creativepultrusions.com
  
  - Bedford Reinforced Plastics, USA
    www.bedfordplastics.com
Design Concept
Design Concept: Basic Assumptions

- Definitions and directions

90° → perpendicular
0° → parallel to pultrusion direction
Design Concept

- Codes
  - Every manufacturer has its own profile design → No European Design Code is available! (only EN13706, about testing and notation)
  - Fiberline Design Manual is based on Eurocomp 1996.
    - Design concept (according to Eurocodes and Swisscodes)
    - Partial safety factors
    - Measured material parameters
    - Rules for bolted connections
Design Concept

- Concept of **Limit State Design** (According to Euro Codes and Swiss Codes)

- **Ultimate limit stress**

\[ E_d \leq R_d \]

- \( E_d \) ... Calculated stress (including load factors) ... SIA260 / 261
- \( R_d \) ... Rated value of the resistance capability

where

\[ R_d = \frac{R_k}{\gamma_m} \]

- \( R_k \) ... the resistance capability
- \( \gamma_m \) ... the reduction coefficient / partial safety factor
### Design Concept

- **Partial safety factor**  \( \gamma_m = \gamma_{m,1} \cdot \gamma_{m,2} \cdot \gamma_{m,3} \cdot \gamma_{m,4} \)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Description</th>
<th>Max. ( \gamma_m )</th>
<th>Min. ( \gamma_m )</th>
<th>Fiberline</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_{m,1} )</td>
<td>Derivation of mat. properties</td>
<td>2.25</td>
<td>1.15</td>
<td>1.15</td>
</tr>
<tr>
<td>( \gamma_{m,2} )</td>
<td>Degree of postcuring</td>
<td>1.6</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>( \gamma_{m,3} )</td>
<td>Production process</td>
<td>2.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \gamma_{m,4} )</td>
<td>Operating temperature</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operating temperature °C</th>
<th>( \gamma_{m,4} )</th>
<th>( \gamma_{m,4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term load</td>
<td>Long-term load</td>
<td></td>
</tr>
<tr>
<td>-20 ... +60</td>
<td>1.0</td>
<td>2.5</td>
</tr>
<tr>
<td>80</td>
<td>1.25</td>
<td>3.13</td>
</tr>
</tbody>
</table>
**Design Concept**

- **Serviceability limit states**

  \[ E_d \leq C_d \]

- \( E_d \) … the crucial action effect due to the load cases considered in the investigated dimensioning situation. Typically maximal deflection response of the structure.

- \( C_d \) … corresponding serviceability limit. SIA 261
### Design Concept: Basic Assumptions

- Material Properties, strength values \((\text{Fiberline Profiles})\)

<table>
<thead>
<tr>
<th>All values given in [MPa]</th>
<th>Typical strength values</th>
<th>Reduced strength values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>short-term</td>
</tr>
<tr>
<td>Flexural strength, 0°</td>
<td>(f_{b,0°})</td>
<td>240</td>
</tr>
<tr>
<td>Flexural strength, 90°</td>
<td>(f_{b,90°})</td>
<td>100</td>
</tr>
<tr>
<td>Tensile strength, 0°</td>
<td>(f_{t,0°})</td>
<td>240</td>
</tr>
<tr>
<td>Tensile strength, 90°</td>
<td>(f_{t,90°})</td>
<td>50</td>
</tr>
<tr>
<td>Compressive strength, 0°</td>
<td>(f_{c,0°})</td>
<td>240</td>
</tr>
<tr>
<td>Compressive strength, 90°</td>
<td>(f_{c,90°})</td>
<td>70</td>
</tr>
<tr>
<td>Shear strength (in-plane)</td>
<td>(f_t)</td>
<td>25</td>
</tr>
</tbody>
</table>

\(\gamma_m = 1.3\)

\(\gamma_m = 3.2\)
### Design Concept: Basic Assumptions

- **Material Properties, stiffness values** *(Fiberline Profiles)*

<table>
<thead>
<tr>
<th>Typical stiffness values</th>
<th>[ GPa ]</th>
<th>[ -- ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity</td>
<td>$E_{0^\circ}$</td>
<td>23 / 28</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>$E_{90^\circ}$</td>
<td>8.5</td>
</tr>
<tr>
<td>Modulus in shear</td>
<td>$G$</td>
<td>3.0</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>$\nu_{0^\circ,90^\circ}$</td>
<td>0.23</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>$\nu_{90^\circ,0^\circ}$</td>
<td>0.09</td>
</tr>
</tbody>
</table>
**Design Concept: Basic Assumptions**

- **Typical data sheet of a profile** (Fiberline I-Profile)

<table>
<thead>
<tr>
<th>I-profile</th>
<th>H</th>
<th>B</th>
<th>T₁</th>
<th>T₂</th>
<th>R</th>
<th>A</th>
<th>Aₖᵧ</th>
<th>Aₖₓ</th>
<th>g</th>
<th>Iₓₓ</th>
<th>Wₓₓ</th>
<th>Iᵧᵧ</th>
<th>Wᵧᵧ</th>
<th>E₀PKG₀</th>
<th>E₀PKG₀-Iₓₓ</th>
</tr>
</thead>
<tbody>
<tr>
<td>HxBxT¹</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm²</td>
<td>mm²</td>
<td>mm²</td>
<td>kg/m</td>
<td>mm⁴</td>
<td>mm³</td>
<td>mm³</td>
<td>mm³</td>
<td>MPa</td>
<td>Nmm²</td>
</tr>
<tr>
<td>factor</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10³</td>
<td>10³</td>
<td>10³</td>
<td>1</td>
<td>10⁶</td>
<td>10³</td>
<td>10³</td>
<td>10³</td>
<td>10⁹</td>
<td></td>
</tr>
<tr>
<td>1120x60x6</td>
<td>120</td>
<td>60</td>
<td>6</td>
<td>6</td>
<td>7.5</td>
<td>1.42</td>
<td>0.68</td>
<td>0.58</td>
<td>2.55</td>
<td>3.10</td>
<td>51.7</td>
<td>0.22</td>
<td>7.30</td>
<td>23</td>
<td>71.30</td>
</tr>
<tr>
<td>1160x80x8</td>
<td>160</td>
<td>80</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>2.49</td>
<td>1.22</td>
<td>1.02</td>
<td>4.48</td>
<td>9.66</td>
<td>121</td>
<td>0.69</td>
<td>17.3</td>
<td>28</td>
<td>270.5</td>
</tr>
<tr>
<td>1200x100x10</td>
<td>200</td>
<td>100</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>3.89</td>
<td>1.90</td>
<td>1.60</td>
<td>6.99</td>
<td>23.6</td>
<td>236</td>
<td>1.69</td>
<td>33.7</td>
<td>28</td>
<td>660.8</td>
</tr>
<tr>
<td>1240x120x12</td>
<td>240</td>
<td>120</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>5.60</td>
<td>2.74</td>
<td>2.30</td>
<td>10.1</td>
<td>48.9</td>
<td>408</td>
<td>3.50</td>
<td>58.3</td>
<td>28</td>
<td>1369</td>
</tr>
<tr>
<td>1300x150x15</td>
<td>300</td>
<td>150</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>8.74</td>
<td>4.28</td>
<td>3.60</td>
<td>15.7</td>
<td>119</td>
<td>796</td>
<td>8.54</td>
<td>114</td>
<td>28</td>
<td>3332</td>
</tr>
<tr>
<td>1360x180x18</td>
<td>360</td>
<td>180</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>12.6</td>
<td>6.16</td>
<td>5.18</td>
<td>22.7</td>
<td>248</td>
<td>1376</td>
<td>17.7</td>
<td>197</td>
<td>28</td>
<td>6944</td>
</tr>
</tbody>
</table>
Bending Beam
Bending Beam: Design of ...

- Calculate **bending moments** $M_d$ **and shear forces** $Q_d$ acting on the profile, using the appropriate load factors (SIA 260 / 261)

- **Ultimate limit state**
  - **Bending:**
    \[
    \sigma_{\text{max}} = \frac{M_{d,y,\max}}{W_y} + \frac{M_{d,z,\max}}{W_z} \leq \frac{f_{b,0^\circ}}{\gamma_m}
    \]
  - **Shear:**
    \[
    \tau_{\text{max}} = \frac{Q_{d,y,\max}}{A_{k,y}} \leq \frac{f_\tau}{\gamma_m}
    \]

$A_k$ ... relevant shear area
### Bending Beam: Design of …

- **Serviceability limit state**
  - **Deflection limit:** \( \frac{w_{\text{max}}}{L} < \frac{1}{\alpha} \)

  \[\alpha \quad \text{… typically selected between 200 and 400 \ given by SIA 261 or the building owner} \]

  \[w_{\text{max}} \quad \text{… calculated including shear deformations} \]

- **Vibrations**

  Light-weighted and ‘soft’ structures are susceptible to vibrations (traffic, wind, the movement of people …!!)!!
**Bending Beam: Timoshenko Theory**

- Pultruded profiles have a low shear modulus $\rightarrow$ shear deformation must be taken into account!

- Several bending theories have been published for beams:
  - Euler-Bernoulli theory (1702)
  - Timoshenko theory (1968)
  - Higher order beam theory

- A simply supported beam with a symmetric cross-section is discussed
**Bending Beam: Timoshenko Theory**

<table>
<thead>
<tr>
<th>Euler-Bernoulli</th>
<th>Timoshenko</th>
<th>Higher order</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
</tbody>
</table>

- Cross-sections plane and perpendicular
- 1 degree of freedom
- \( w \)

- Cross-sections plane but NOT perpendicular
- 2 degrees of freedom
- \( w \) and \( \psi \)

- Cross-sections do NOT remain plane
- 3+ degrees of freedom
- \( w, \psi \) and …
**Bending Beam: Euler vs. Timoshenko Theory**

Kinematic relationships

\[ u_x = -y \cdot w(x),_x \]
\[ u_y = w(x) \]

\[ \varepsilon_x = \frac{\partial u_x}{\partial x} = -y \cdot w(x),_{xx} \]
\[ 2\varepsilon_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = 0 \]

\[ u_x = -y \cdot \psi(x) \]
\[ u_y = w(x) \]

\[ \varepsilon_x = \frac{\partial u_x}{\partial x} = -y \cdot \psi(x),_x \]
\[ 2\varepsilon_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = -\psi(x) + w(x),_x \]

**Hook’s law**

\[ \sigma_x = E_0 \cdot \varepsilon_x \quad \text{and} \quad \tau_{xy} = G \cdot 2\varepsilon_{xy} \]

\[ M_z = \iint_{QS} -y \cdot \sigma_x \cdot dydz = w,_{xx} \cdot E_0 \cdot I_z \]

\[ \int_{QS} -y \cdot \sigma_x \cdot dydz = \psi,_{x} \cdot E_0 \cdot I_z \]

\[ Q_y = \kappa \int_{QS} \tau \cdot dydz = (w,_{x} - \psi) \cdot \kappa \cdot GA \]


**Bending Beam: Euler vs. Timoshenko Theory**

**Equilibrium**

In a first approximation, the deflections are calculated by direct integration of:

\[ w_{xx} = \frac{M(x)}{E_0 \cdot I_z} \]

Equilibrium on an infinitesimal beam element:

\[ q(x) = -Q_x = -(w_{xx} - \psi_{,x}) \kappa \cdot GA \]

\[ M_x - Q = \psi_{,xx} \kappa \cdot E_0 \cdot I_z + (w_x - \psi) \kappa \cdot GA = 0 \]

Coupled second order differential equation

**Solution for the simply supported beam** (distributed load)

\[ M(x) = \frac{1}{2} qLx - \frac{1}{2} qx^2, \quad w(0) = 0 \quad \text{and} \quad w(L) = 0 \]

\[ w_{xx} = \frac{1}{E_0 \cdot I_z} \left( \frac{1}{2} qLx - \frac{1}{2} qx^2 \right) \]

\[ w(x) = \frac{qx}{24 \cdot E_0 \cdot I_z} \left( L^3 - 2Lx^2 + x^3 \right) \]

\[ w(0) = 0 \quad \text{and} \quad w(L) = 0 \]

\[ M(0) = 0 \rightarrow \psi_{,x}(0) = 0 \quad \text{and} \quad M(L) = 0 \rightarrow \psi_{,x}(L) = 0 \]

**Functions:**

\[ w(x) = A_1 x^4 + A_2 x^3 + A_3 x^2 + A_4 x + A_5 \]

\[ \psi(x) = B_1 x^3 + B_2 x^2 + B_3 x + B_4 \]
**Bending Beam: Euler vs. Timoshenko Theory**

Put in \( \psi_{xx} \cdot E_0 I_z + (w_x - \psi) \cdot \kappa \cdot GA = 0 \) and solve for the coefficients \( \rightarrow \)

\[
B_1 = -4A_1 \quad B_2 = -2A_3 - \frac{24A_1 \cdot E_0 I_z}{\kappa \cdot GA}
\]

\[
B_2 = -3A_2 \quad B_4 = \frac{6A_2 \cdot E_0 I_z}{\kappa \cdot GA} - A_4
\]

Use the boundary conditions and the second differential eq. to calculate \( A_1 - A_5 \):

\[
w(x) = \frac{q x (L - x)}{2 \cdot \kappa \cdot GA} + \frac{q x (L - x) (L^2 + Lx - x^2)}{24 \cdot E_0 I_z}
\]

**Deflection at midspan**

\[
w(\frac{L}{2}) = \frac{5 \cdot q L^4}{384 \cdot E_0 I_z}
\]

\[
w(\frac{L}{2}) = \frac{5 \cdot q L^4}{384 \cdot E_0 I_z} + \frac{q \cdot L^2}{8 \cdot \kappa \cdot GA}
\]
**Bending Beam: Euler vs. Timoshenko Theory**

- General expression for the total beam deflection as a sum of the deflection due to bending and shear:

\[
w(x) = \frac{f_1(x)}{E_0 I_z} + \frac{f_2(x)}{\kappa \cdot GA}
\]

<table>
<thead>
<tr>
<th>Beam</th>
<th>( f_1(\text{w}_{\text{max}}) )</th>
<th>( f_2(\text{w}_{\text{max}}) )</th>
<th>( x(\text{w}_{\text{max}}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Simply supported</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniformly distr. load (q)</td>
<td>( \frac{5 \cdot qL^4}{384} )</td>
<td>( \frac{qL^2}{8} )</td>
<td>( \frac{L}{2} )</td>
</tr>
<tr>
<td>Concentrated load (P)</td>
<td>( \frac{PL^3}{48} )</td>
<td>( \frac{PL}{4} )</td>
<td>( \frac{L}{2} )</td>
</tr>
<tr>
<td><strong>Cantilever beam</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniformly distr. load (q)</td>
<td>( \frac{qL^4}{8} )</td>
<td>( \frac{qL^2}{2} )</td>
<td>( L )</td>
</tr>
<tr>
<td>Concentrated load (P)</td>
<td>( \frac{PL^3}{3} )</td>
<td>( PL )</td>
<td>( L )</td>
</tr>
</tbody>
</table>
**Bending Beam: Euler vs. Timoshenko Theory**

- **Example:** influence of the shear deformation

  Profile: 300 x 150 mm I-beam
  
  Load: uniformly distributed

General rule of thumb for slender Beams:

for GFRP beams with span/depth > 25
shear deformation can be ignored
**Bending Beam: Example**

- Choose an appropriate Profile for the following specifications

\[
L = 3.0 \text{ m} \\
q_{d,uls} = 13 \text{ kN/m} \\
q_{d,ser} = 10 \text{ kN/m} \\
w_{\text{max}}/L = 1/300 \rightarrow w_{\text{max}} = 0.01 \text{ m}
\]

1. **Deflections and loading**

\[
w_{\text{max}} = \frac{5 \cdot q_{d,ser} L^4}{384 \cdot E_0 I_z} + \frac{q_{d,ser} \cdot L^2}{8 \cdot \kappa \cdot GA}
\]

\[
\sigma_{\text{max}} = \frac{q_{d,uls} L^2}{8} \cdot \frac{h}{2 I_z}
\]

\[
\tau_{\text{max}} = \frac{q_{d,uls} L}{2 \cdot A_{k,y}}
\]
Bending Beam: Example

2. Find a profile with sufficient bending stiffness (SLS).

Shear deformations are neglected in a first step:

\[ E_{0z} I_z \geq \frac{5 \cdot q_{d,ser} L^4}{384 \cdot w_{max}} = 1.054 \cdot 10^6 \text{ Nm}^2 \]

\[ \rightarrow \text{ from specification table: choose } I_{240 \times 120 \times 12} \rightarrow E_{0z} I_z = 1.369 \cdot 10^6 \text{ Nm}^2 \]

3. Check the bending and shear stresses (ULS)

\[ \sigma_{max} = \frac{q_{d,uls} L^2}{8} \cdot \frac{h}{2 \cdot I_z} = 35.8 \text{ MPa} \leq f_{b,0^\circ,d} = 185 \text{ MPa} \] (short term)

\[ \tau_{max} = \frac{q_{d,uls} L}{2} \cdot \frac{1}{A_{k,y}} = 7.1 \text{ MPa} \leq f_{\tau,d} = 20 \text{ MPa} \] (short term)

Do not forget to check also the long term!
Bending Beam: Example

4. Check deflection (including shear deformation)

\[ w_{\text{max}} = \frac{5 \cdot q_{d,\text{ser}} L^4}{384 \cdot E_0 I_z} + \frac{q_{d,\text{ser}} \cdot L^2}{8 \cdot \kappa \cdot GA} = 9.3 \cdot 10^{-3} \text{ m} \leq w_{\text{max}} = 0.01 \text{ m} \]

\[ (7.7 \text{ mm}) \quad (1.6 \text{ mm}) \quad (\kappa=0.42) \]

\[ w_{\text{max}} = \frac{5 \cdot q_{d,\text{ser}} L^4}{384 \cdot E_0 I_z} + \frac{q_{d,\text{ser}} \cdot L^2}{8 \cdot GA_{\text{web}}} = 9.1 \cdot 10^{-3} \text{ m} \]

5. Remarks:

- Simplification: use area of the web (conventionally manufactured GFRP I and \( \square \)-profiles)
- The design of GFRP-profiles is mostly driven by serviceability criteria.
- Start the design iteration procedure using the maximal deflection criterion.
**Bending Beam: Stability problems**

- **Lateral-torsional buckling**
  - Flange (compressive) displace laterally to the transverse load direction.
  - Torsional stiffness is too low (especially for open section profiles)
  - Theoretical calculations or design measures.
  
  → see e.g. L.P. Kollár 2003, *Mechanics of composite structures*.

- **Example:**

  Compressive flanges are kept in place by connection to the bridge deck.
Bending Beam: Stability problems

- Local buckling of walls due to in-plane compression
  - Flange (compressive) displaces in the direction of the transverse load.
  - Low bending stiffness perpendicular to the pultrusion direction.
  - Weak fiber mats.

- Local buckling of walls due to in-plane shear

- Web crushing and web buckling in transverse direction
Axial Members
Axial Members: Tension

- Ultimate limit state under axial tension \( N_d \)
  \[
  \frac{N_d}{A} \leq \frac{f_{t,0^\circ}}{\gamma_m}
  \]
  \( A \) can be either gross or net area

- Serviceability limit state
  \[
  \delta_x = \frac{N \cdot L}{E_{0^\circ} \cdot A}
  \]
  \( A = \) gross area

- Remark: The critical aspect of axial members in tension are neither the serviceability nor the ultimate limit state. Critical is the load transfer to the GFRP profile!
Axial Members: Compression

- Ultimate limit state under axial compression $N_d$

$$N_d \leq \frac{F_c}{1 + \frac{N_{Euler}}{F_c}}$$

$$F_c = \frac{A \cdot f_{c,0^\circ}}{\gamma_m}$$  … maximal compressive load

$$N_{Euler} = \frac{\pi^2 \cdot E_{0^\circ} \cdot I}{\gamma_{m,E} \cdot L_k^2}$$  … Euler load

$L_k$  … Buckling length for columns

$\gamma_{m,E}$ … coefficient for Young’s modulus = 1.3
Axial Members: Compression

- The influence of shear deformation should be considered, but in the most cases, the influence will be small (less than 5%).

- **Local buckling** should be considered for short columns.

- For more information on the various buckling modes and effects → see L.P. Kollár 2003, Mechanics of composite structures
Connections
Joints in FRP composite structures

Eyecatcher Building: a mobile lightweight five-story GFRP building, Switzerland, 1998

Three GFRP frames composed of adhesively-bonded built-up sections as the main load-bearing structure

Adhesively-bonded sections built up from pultruded profiles

Bolted joints
Joints in FRP composite structures

FRP bridge deck panels
[Thesis Dr. Gürtler, CCLab 2004]

Overview of the experimental set-up

Failure of adhesive bond
### Connections: Introduction

#### Table 5.2 Typical features of different connections between FRP members

<table>
<thead>
<tr>
<th>Mechanical connections</th>
<th>Bonded connections</th>
<th>Combined connections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Advantages</strong></td>
<td><strong>Disadvantages</strong></td>
<td><strong>Advantages</strong></td>
</tr>
<tr>
<td>• Requires no special surface preparation</td>
<td>• Low strength to stress concentrations</td>
<td>• Bolts provide support and pressure during assembly and curing</td>
</tr>
<tr>
<td>• Can be disassembled</td>
<td>• Special practices required in assembly; results in time consuming assembly</td>
<td>• Structurally bolts act as backup elements - in an intact joint, bolts carry no load</td>
</tr>
<tr>
<td>• Ease of inspection</td>
<td>• Fluid and weather tightness normally requires special gaskets or sealants</td>
<td>• Growth of bondline defects is hindered by bolts</td>
</tr>
<tr>
<td>• Quasi ductile behaviour</td>
<td>• Corrosion of metallic fasteners</td>
<td></td>
</tr>
</tbody>
</table>

---

Design of FRP-Profiles and All-FRP-Structures, 14.10.2020

Fiber Composites, FS20

Prof. Dr. M. Shahverdi
### Connections: Introduction

#### Table 5.1 Characteristics of different joint categories. (from Eurocomp Design Code 1996)

<table>
<thead>
<tr>
<th></th>
<th>Mechanical</th>
<th>Bonded</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress concentration at joint</td>
<td>high</td>
<td>medium</td>
<td>medium</td>
</tr>
<tr>
<td>Strength/weight ratio</td>
<td>low</td>
<td>medium</td>
<td>medium</td>
</tr>
<tr>
<td>Seal (water tightness)</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Thermal insulation</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Electrical insulation</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Aesthetics (smooth joints)</td>
<td>bad</td>
<td>good</td>
<td>bad</td>
</tr>
<tr>
<td>Fatigue endurance</td>
<td>bad</td>
<td>good</td>
<td>good</td>
</tr>
<tr>
<td>Sensitive to peel loading</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Disassembly</td>
<td>possible</td>
<td>impossible</td>
<td>impossible</td>
</tr>
<tr>
<td>Inspection</td>
<td>easy</td>
<td>difficult</td>
<td>difficult</td>
</tr>
<tr>
<td>Heat or pressure required</td>
<td>no</td>
<td>yes/no¹</td>
<td>yes/no¹</td>
</tr>
<tr>
<td>Tooling costs</td>
<td>low</td>
<td>high</td>
<td>low</td>
</tr>
<tr>
<td>Time to develop full strength</td>
<td>immediate</td>
<td>long</td>
<td>long</td>
</tr>
</tbody>
</table>

¹ no if cold curing two-part adhesives are used in an appropriate environment
Connections: Bolted joints

- Bolts = Stress concentration in the profile and the bolt.

- It is necessary to ensure that the bolts and the profile can withstand this concentrated local stress compression.

- It is necessary to ensure that the region surrounding a group of bolts will not be torn out of the profile.

- Basic failure modes in bolted shear connections:

  ![Net-section failure](image1.png)
  ![Bearing failure](image2.png)
  ![Shear-out failure](image3.png)
  ![Bolt shear failure](image4.png)

  Prof. Keller, EPFL
Connections: Bolted joints

- The design procedure is comparable to the one for steel connections, but since there exist no standard GFRP material → each manufacturer has its own design rules for bolted joints.

**IMPORTANT REMARKS:**

- The direction of pultrusion and the direction of the force is RELEVANT!!! (anisotropic material)
- Use stainless or galvanised steel
- Do not cut threads in the composite material!
- Use screws with shafts
**Connections: Bolted joints** *(Fiberline recommendations)*

- Calculation of load bearing capacity of bolts
  - Shear in longitudinal direction (0°)
  - Shear in transverse direction (90°)
  - Tensile force

- Minimum distances
**Connections: Bolted joints (Fiberline recommendations)**

- Joint capacity tables, available for shear and tension

<table>
<thead>
<tr>
<th>Bolt</th>
<th>1 cut</th>
<th>2 cuts</th>
<th>Thickness of laminate in mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>M 6</td>
<td>2.7</td>
<td>5.4</td>
<td>3.5</td>
</tr>
<tr>
<td>M 8</td>
<td>4.8</td>
<td>9.5</td>
<td>4.6</td>
</tr>
<tr>
<td>M 10</td>
<td>7.4</td>
<td>14.9</td>
<td>5.8</td>
</tr>
<tr>
<td>M 12</td>
<td>10.7</td>
<td>21.4</td>
<td>6.9</td>
</tr>
<tr>
<td>M 14</td>
<td>14.6</td>
<td>29.2</td>
<td>8.1</td>
</tr>
<tr>
<td>M 16</td>
<td>19.0</td>
<td>38.1</td>
<td>9.2</td>
</tr>
<tr>
<td>M 20</td>
<td>30</td>
<td>59</td>
<td>11.5</td>
</tr>
<tr>
<td>M 22</td>
<td>36</td>
<td>72</td>
<td>12.7</td>
</tr>
<tr>
<td>M 24</td>
<td>43</td>
<td>86</td>
<td>13.8</td>
</tr>
<tr>
<td>M 27</td>
<td>54</td>
<td>109</td>
<td>15.6</td>
</tr>
<tr>
<td>M 30</td>
<td>67</td>
<td>134</td>
<td>17.3</td>
</tr>
<tr>
<td>M 36</td>
<td>96</td>
<td>193</td>
<td>20.8</td>
</tr>
<tr>
<td>M 42</td>
<td>131</td>
<td>262</td>
<td>24.2</td>
</tr>
<tr>
<td>M 48</td>
<td>171</td>
<td>343</td>
<td>27.7</td>
</tr>
</tbody>
</table>

**Shear in longitudinal direction 0°**

\[
P_{B,d} = \frac{d \cdot t \cdot 150 \text{ MPa}}{\gamma_m(=1.3)}
\]

**Shear in transverse direction 90°**

\[
P_{B,d} = \frac{d \cdot t \cdot 70 \text{ MPa}}{\gamma_m(=1.3)}
\]
Connections: Bolted joints (Fiberline recommendations)

- Bolted connection in shear: e.g. shear in longitudinal direction

Simplified model

The values are based on

\[
\begin{align*}
  a &= 3.5 \cdot d \\
  b &= 1.0 \cdot d \\
  c &= 2.0 \cdot d \\
  f_{t,0} &= 240 \text{ MPa} \\
  f_{t,0} &= 240 \text{ MPa} \\
  f_{t,0} &= 50 \text{ MPa} \\
  f_{t,0} &= 70 \text{ MPa} \\
  f_{t,0} &= 25 \text{ MPa}
\end{align*}
\]

Investigated failure modes

\[
\begin{align*}
  g_m \cdot P_{Bolt} &\leq d \cdot t \cdot 720 \text{ MPa} \\
  g_m \cdot P_{Bolt} &\leq d \cdot t \cdot 240 \text{ MPa} \\
  g_m \cdot P_{Bolt} &\leq d \cdot t \cdot 240 \text{ MPa} \\
  g_m \cdot P_{Bolt} &\leq d \cdot t \cdot 150 \text{ MPa}
\end{align*}
\]
Connections: Bolted joints (Fiberline recommendations)

- Bolted connections in tension
- Static conditions
  - **Bolt:** Tearing of bolt in threaded cross-section
  \[ P_d \leq \frac{A_s \cdot f_{yk}}{\gamma_m} \]
  - **Laminate:** Shear fracture at rim of washer
  \[ P_d \leq \frac{2 \cdot d \cdot \pi \cdot t \cdot f_{\tau}}{\gamma_m} \]

**Geometry and strength:**
- **d** ... Diameter of the bolt
- **A_s** ... Stress area of the bolt
- **t** ... Thickness of laminate
- **2d** ... Diameter of washer
- **f_{yk}** ... Tensile strength of bolt
- **f_{\tau}** ... Shear strength of laminate
**Connections: Bonded joints**

- Using an adhesive agent for joining profiles can have **advantages:**
  - Easy to use / easy to make aesthetic joints
  - Typically **more rigid** than bolted joints
  - Glued joints subjected to **dynamic loads** are good

- **But be careful …**
  - Adhesive agents have properties that **depend** on time, temperature, humidity …
  - Failure in glued joints takes place **suddenly** (brittle behaviour)
  - The load-bearing **capacity** is not proportional to the area which is glued

- **The design of bonded joints may be based on:**
  - **Analytical** models for plate-to-plate connections (see Eurocomp 1996 Design Code)
  - Design **guidelines** supplemented by testing
  - **Finite element** analysis
Connections: Bonded joints

- A bonded joint has the following three primary failure modes:
  - adhesive failure
  - cohesive failure of adhesive
  - cohesive failure of adherend

- The design of any bonded joint shall satisfy the following conditions:
  - allowable shear stress in the adhesive is not exceeded.
  - allowable tensile (peel) stress in the adhesive is not exceeded.
  - allowable through-thickness tensile stress of the adhesive is not exceeded.
  - allowable in-plane shear stress of the adherend should not be exceeded.

- The calculation of the stresses has to be done very carefully! Often calculations are supplemented by testing.
Connections: Bonded joints

- Different types of bonded joint configurations

- Research on bonded joints for structural applications
Connections: Bonded joints, fracture modes

Displacement control,
1 mm/min, 5 Hz
Ambient conditions

Connections: Bonded joints, fracture modes

Mixed-mode bending (MMB-Mode I/II)

MMB (mixed-Mode I/II)

DCB (Mode I)

ENF (Mode II)
Mode I fracture

Double cantilever beam (DCB-Mode I)

Displacement control,
1 mm/min
✓ Visual observation
Ambient conditions

Schematic of fiber bridging phenomenon

Output from experiment
Strain energy release rate calculation

\[ G = \frac{P^2}{2B} \frac{dC}{da} \]
\[ C = ka^n \]
\[ G = \frac{nP \delta}{2Ba} \]

Experimental compliance method (ECM)

Connections: **Bonded joints**, *mixed-mode quasi-static failure criterion*

Connections: Bonded joints, mixed-mode fatigue failure criterion

Fatigue crack growth (FCG) curve

Connections: Bonded joints, mixed-mode fatigue failure criterion

Connections: Bonded joints, mixed-mode fatigue failure criterion

Connections: Bonded joints, mixed-mode fatigue failure criterion

The developed failure criteria can be used to establish a progressive damage model under variable mode-mixity fatigue loading for structural joints composed of the same adherends and adhesive in which a similar failure mode occurs.

Connections: Other joints

- **Brackets for assembly** (Fiberline)

  ![Brackets for efficient assembly of profile structures.](image1)

  ![Example of joint with a Fiberline bracket. EP patent No. 0819200](image2)

- **Custom pultruded connections**
**GFRP: Some final remarks**

- **Perpendicular to the direction of pultrusion, the material is WEAK and SOFT!**
  → avoid such loadings if possible

- **In order to use pultruded GFRP-profiles economically, the design must be done in a clever way!**
  e.g.: for bridges, the railings should be used as part of the load-bearing structure

- **GFRP structures are very light** → vibration problems may occur

- **Where large stiffness is needed (where static height and deflections must remain very small)** → GFRP does not always lead to lighter structures than with steel.
Thank you for attention

any question?
Available master theses at Empa Structural Engineering Research Lab in collaboration with ETH Zurich

Self-centering and confining memory steel reinforcements

Prestressed shape memory alloy reinforcement for 3D Concrete Printing

Gosselin et al. (2016)

Schematic illustration of different steps of the proposed method