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Strengthening in shear

Book Composite for Construction, L. C. Bank, Chapter 10
General

FRP sheet or fabric

(a)

A  t_f  B  C
To avoid stress concentrations, allow for a minimum radius of 15 mm.
External strengthening with FRPs:

- **Flexural failure**: Generally fairly ductile
- **Shear failure**: Sudden and brittle

Undesirable failure mode

*Control shear deformation* to avoid sudden failure

**Beam/One-Way Slab Strengthening**

**Design Principles**

**Shear Strengthening**

Externally Bonded FRP: Shear and Torsion

Fibre Composites, FS18

Masoud Motavalli
Design model for Ultimate Limit State (ULS)

Members of rectangular, T and double-T cross section

Assuming that at the ultimate limit state in shear (concrete diagonal tension) the FRP develops an effective strain in the principal material direction, \( \varepsilon_{f,e} \). The effective strain \( \varepsilon_{f,e} \) is, in general, less than the tensile failure strain, \( \varepsilon_{fu} \).
\[ V_{Rd} = V_{cd} + V_{wd} + V_{fd} \]
\[ V_{fd} = 0.9 \varepsilon_{fd,e} E_{fu} \rho_f b_w d \cdot (\cot \theta + \cot \alpha) \cdot \sin \alpha \]

Where:
\( \varepsilon_{fd,e} \) : design value of effective FRP strain.
\( b_w \) : minimum width of cross section over the effective depth.
\( d \) : effective depth of cross section.
\[ \rho_f : \frac{(2 \, t_f \sin \alpha)}{b_w} \] for continuously bonded shear reinforcement of thickness \( t_f \) (\( b_w = \) minimum width of concrete cross section over the effective depth), or \( \frac{(2t_f/b_w)(b_f/s_f)}{1} \) for FRP reinforcement in the form of strips or sheets of width \( b_f \) at the spacing \( s_f \).

\[ E_{fu} : \] elastic modulus of FRP in the principal fiber orientation.

\[ \theta \] : diagonal crack angle, assumed to be 45°.

\[ \alpha \] : fiber orientation
Circular cross-sections

\[ V_{fd} = \frac{\varepsilon_{\text{max}}}{\gamma_f} E_{fu} \rho_f \left( 1 - \frac{\pi D^2}{2} \right) \cot \theta \]

Where:
\( \varepsilon_{\text{max}} : 0.006 \) (experimentally verified by Priestley et al, 1995)

D : column diameter
According to Canadian Code:

\[ V_{cd} = 0.2 \Phi_c \cdot b \cdot d \cdot \sqrt{f'_c} \]

\[ \Phi_c = 0.6 \]
\[ V_{wd} = A_s \varepsilon_s E_s \frac{z_s}{s_f} \cdot \cotg \theta \quad \text{if:} \quad \varepsilon_s \leq \frac{f_y}{E_s} \]

\[ V_{wd} = A_s f_y \frac{z_s}{s_f} \cdot \cotg \theta \quad \text{if:} \quad \varepsilon_s > \frac{f_y}{E_s} \]

Where:
- \( z_s \): internal lever arm.
- \( s_f \): spacing of stirrups.
- \( f_y \): yield strain.
- \( \theta \): crack angle, assumed to be 45°.
- Fully wrapped (or properly anchored) CFRP-FRP fracture controls:

\[
\varepsilon_{f,e} = 0.17 \left( \frac{f_{cm}^{2/3}}{E_{fu} \rho_f} \right)^{0.3} \varepsilon_{fu}
\]

- Side or U-shaped CFRP jackets:

\[
\varepsilon_{f,e} = \min \left[ 0.65 \left( \frac{f_{cm}^{2/3}}{E_{fu} \rho_f} \right)^{0.56} \times 10^{-3} \right. \& \left. 0.17 \left( \frac{f_{cm}^{2/3}}{E_{fu} \rho_f} \right)^{0.30} \varepsilon_{fu} \right]
\]

*Peeling off*  
*Fracture*

Note that in all equations \( f_{cm} \) is in \( MPa \) and \( E_{fu} \) is in \( GPa \).
Design recommendations

It should point out that the spacing of strips ($s_f$), if they are used vertically, should not exceed:

$$s_f \leq 0.9d - \frac{b_f}{2}$$  
for rectangular cross sections

$$s_f \leq d - h_f - \frac{b_f}{2}$$  
for T-beams

In which $h_f$ is slab thickness and $b_f$ is FRP width.
**Serviceability limit state**

The externally bonded reinforcement should not debond at serviceability limit state. This is important, so that problems related to moisture penetration, crack propagation, noise from debonding etc. may be avoided. To verify this, the strain in the FRP in the serviceability state, $\varepsilon_{fk,e}$, should be limited to:

$$\varepsilon_{fk,e} \leq 0.8 \frac{f_{yk}}{E_s}$$
Strengthening in torsion

General

Strengthening for increased torsional capacity may be required in conventional beams and columns, as well as in bridge box girders.

The principles applied to strengthening in shear are also valid in the case of torsion, with a few minor differences.
Design model in the ULS – rectangular cross sections

Torsional cracking

Shear cracking
These forces are calculated below for the case \( \alpha = 90^\circ \) where \( \alpha \) is fiber direction with respect to member axis and \( \theta \) is diagonal crack angle with respect to member axis (\( \sim 45^\circ \)).

\[
F_{fd,v} = \varepsilon_{fd,e} E_{fu} \frac{t_f b_f}{s_f} h.\cotg \theta
\]

\[
F_{fd,h} = \varepsilon_{fd,e} E_{fu} \frac{t_f b_f}{s_f} b.\cotg \theta
\]
Hence the contribution of FRP to torsional capacity, $T_{fd}$, is given by the following equation:

$$T_{fd} = F_{fd,v}b + F_{fd,h}h = 2\varepsilon_{fd,e}E_{fu} \frac{t_fb_f}{s_f} bh.\cotg\theta$$
The design value of the effective FRP strain in the principal material direction is given as in the preceding analysis of shear of the case of fully wrapped FRP:

\[ \varepsilon_{fd,e} = \frac{k \varepsilon_{f,e}}{\gamma_f} \]

Where:

\[ k = 0.8 \]
\[ \gamma_f = 1.3 \]

And:

\[ \varepsilon_{f,e} = 0.17 \left( \frac{f_{cm}^{2/3}}{E_{fu} \rho_f} \right)^{0.30} \varepsilon_{fu} \]  
CFRP

\[ \varepsilon_{f,e} = 0.048 \left( \frac{f_{cm}^{2/3}}{E_{fu} \rho_f} \right)^{0.47} \varepsilon_{fu} \]  
GFRP
Serviceability limit state

\[ \varepsilon_{fk,e} \leq 0.8 \frac{f_{yk}}{E_s} \]
Problem statement

Calculate the shear capacity \( (V_r) \) for an FRP-strengthened concrete section

Section information

- **Section**
  - \( b = 105 \text{ mm} \)
  - \( h = 350 \text{ mm} \)
  - \( d = 325 \text{ mm} \)
  - 3-10M bars
  - GFRP wrap

- **Elevation**
  - \( \lambda = 1.0 \)
  - \( f'_c = 45 \text{ MPa} \)
  - \( \varepsilon_{frp} = 2.0 \% \)
  - \( f_y = 400 \text{ MPa (rebar)} \)
  - \( f_y = 400 \text{ MPa (stirrup)} \)
  - \( E_{frp} = 22.7 \text{ GPa} \)
  - \( s_s = 225 \text{ mm c/c} \)
  - \( t_{frp} = 1.3 \text{ mm} \)
  - \( w_{frp} = 100 \text{ mm} \)
  - \( s_{frp} = 200 \text{ mm} \)

- **Material Properties**
  - **Concrete**
    - \( f'_c = 45 \text{ MPa} \)
  - **Steel**
    - \( f_y = 400 \text{ MPa (rebar)} \)
    - \( f_y = 400 \text{ MPa (stirrup)} \)
  - **FRP**
    - \( E_{frp} = 22.7 \text{ GPa} \)
Beam/One-Way Slab Strengthening

Solution

Step 1: Calculate concrete and steel contributions

Concrete: \( V_c = 0.2 \phi_c \sqrt{f'_c} b_w d \)

\[ V_c = 0.2 (0.6) \sqrt{45} (105) (325) \]

\[ V_c = 27470 \text{ N} = 27.47 \text{ kN} \]

Steel: \( V_s = \frac{\phi_s f_y A_v d}{s} \)

\[ V_s = \frac{0.85 (400) (36) (325)}{225} \]

\[ V_s = 17680 \text{ N} = 17.68 \text{ kN} \]
Shear resistance of a beam:

\[ V_{frp} = \phi_{frp} A_{frp} E_{frp} \varepsilon_{frp e} d_{frp} \left( \sin \beta + \cos \beta \right) \]

\[ s_{frp} \]

\[ \Rightarrow A_{frp} = 2 t_{frp} w_{frp} \]

\[ \Rightarrow d_{frp}: \text{distance from free end of FRP to bottom of internal steel stirrups} \]
Effective strain in FRP, $\varepsilon_{frpe}$:

$$\varepsilon_{frpe} = R \varepsilon_{frpu} \leq 0.004$$

Reduction factor, R:

$$R = \alpha \lambda_1 \left( \frac{f'_c}{\rho_{frp} E_{frp}} \right)^{2/3} \lambda_2$$

Carbon: $\lambda_1 = 1.35$, $\lambda_2 = 0.30$
Glass: $\lambda_1 = 1.23$, $\lambda_2 = 0.47$

Prevents shear cracks from widening beyond acceptable limits.
Ensures aggregate interlock!

0.8
FRP shear reinforcement ratio, $\rho_{\text{frp}}$:

$$\rho_{\text{frp}} = \frac{2 t_{\text{frp}}}{b_w} \left( \frac{W_{\text{frp}}}{s_{\text{frp}}} \right)$$
Another limit on effective strain in FRP, $\varepsilon_{frpe}$:

$$\varepsilon_{frpe} \leq \frac{\alpha k_1 k_2 L_e}{9525}$$

$\Rightarrow$ Parameters, $k_1$ and $k_2$:

$$k_1 = \left( \frac{f' c}{27.65} \right)^{2/3}$$

$$k_2 = \left( \frac{d_{frp} - n_e L_e}{d_{frp}} \right)$$
Effective anchorage length, $L_e$:

$$L_e = \frac{25350}{t_{frp} E_{frp}^{0.58}}$$
Limit on spacing of strips, $s_{frp}$:

$$s_{frp} \leq w_{frp} + \frac{d}{4}$$
Limit on maximum allowable shear strengthening, $V_{frp}$:

$$V_r \leq V_c + 0.8\lambda \phi_c \sqrt{f'c} b_w d$$

Shear contribution due to steel stirrups and FRP strengthening must be less than this term.
Solution

Step 2: Determine $A_{frp}$, $\rho_{frp}$, $L_e$ for effective strain calculation

$A_{frp}$: $A_{frp} = 2 \ t_{frp} \ w_{frp} = 2 \times (1.3) \times (100)$

$A_{frp} = 260 \ mm^2$

$\rho_{frp}$: $\rho_{frp} = \frac{2 \ t_{frp}}{b_w} \times \frac{w_{frp}}{s_{frp}} = \frac{2 \times (1.3)}{105} \times \frac{100}{200}$

$\rho_{frp} = 0.0124$
Solution

Step 2: Determine $A_{frp}$, $\rho_{frp}$, $L_e$ for effective strain calculation

$$L_e = \sqrt[0.58]{\frac{25350}{t_{frp}E_{frp}}} = \sqrt[0.58]{\frac{25350}{1.3 \times 22700}}$$

$L_e = 64.8$ mm
Solution

Step 3: Determine \( k_1 \), \( k_2 \) and effective strain, \( \varepsilon_{frpe} \) [Limit 2]

\[
k_1 = \left( \frac{f'c}{27.65} \right)^{2/3} = \left( \frac{45}{27.65} \right)^{2/3} = 1.38
\]

\[
k_2 = \left( \frac{d_{frp} - n_e L_e}{d_{frp}} \right) = \left( \frac{325 - 1 (64.8)}{325} \right) = 0.80
\]
Solution

Step 3: Determine $k_1$, $k_2$ and effective strain, $\varepsilon_{frpe}$ [Limit 2]

Note: This strain is one of three limits placed on the FRP

\[
\varepsilon_{frpe} \leq \frac{\alpha k_1 k_2 L_e}{9525}
\]

\[
\varepsilon_{frpe} = \frac{0.8 \times (1.38) \times (0.80) \times (64.8)}{9525}
\]

\[
\varepsilon_{frpe} = 0.0060
\]
Solution

Step 4: Determine R and effective strain, $\varepsilon_{frpe}$ [Limit 1]

$$R = \alpha \lambda_1 \left( \frac{f'_c 2/3}{\rho_{frp} E_{frp}} \right) \lambda_2$$

$$R = 0.8 \ (1.23) \left( \frac{45^{2/3}}{0.0124 \ (22700)} \right) 0.47$$

$$R = 0.229$$
Step 4: Determine R and effective strain, $\varepsilon_{frpe}$ [Limit 1]

Note: This strain is one of three limits placed on the FRP

$$\varepsilon_{frpe} = R \, \varepsilon_{frpu} \leq 0.004$$

$$\varepsilon_{frpe} = 0.229 \ (0.02)$$

$$\varepsilon_{frpe} = 0.0046$$
Solution

Step 5: Determine governing effective strain, $\varepsilon_{frpe}$

For design purposes, use the smallest limiting value of:

$\varepsilon_{frpe} = 0.0046$

$\varepsilon_{frpe} = 0.0040$

$\varepsilon_{frpe} = 0.0060$
Step 6: Calculate contribution of FRP to shear capacity

\[ V_{frp} = \phi_{frp} A_{frp} E_{frp} \varepsilon_{frpe} d_{frp} (\sin \beta + \cos \beta) \]

\[ V_{frp} = \frac{0.5 \times 260 \times 22700 \times 0.004 \times 325 \times (\sin 90 + \cos 90)}{200} \]

\[ V_{frp} = 19200 \text{ N} = 19.2 \text{ kN} \]
Step 7: Compute total shear resistance of beam

\[ V_r = V_c + V_s + V_{frp} \]

\[ V_r = 27.5 + 17.7 + 19.2 \]

\[ V_r = 64.4 \text{ kN} \]
Solution

Step 8: Check maximum shear strengthening limits

\[ V_r \leq V_c + 0.8 \lambda \phi f'_c b_w d \]

\[
64400 \leq 27500 + 0.8 (1) (0.6) (45) (105) (325)
\]

\[
64400 \leq 137400
\]

⇒ OK
Solution

Step 9: Check maximum band spacing

\[ S_{frp} \leq W_{frp} + \frac{d}{4} \]

\[ 200 \leq 100 + \frac{325}{4} \]

\[ 200 \leq 181 \]

⇒ Not true, therefore use 180 mm spacing
Shear Strengthening of Reinforced Concrete with CFRP: Empa-Project
References


Material

CFRP L-shaped plate (Sika® CarboShear L®)

CFRP fabric (SikaWrap® Hex-230C)
Application

CFRP L-shaped plate

CFRP fabric
Referenzproject

Ponte Brogeda (Chiasso, Switzerland)

Photos from Sika
Referenzproject (2)

Conversion Migrosklubschule, HB St. Gallen
Referenzproject (3)

Abutment of the Duttweiler-bridge, Zurich
Beam T1

\[ \frac{\ell}{h} = 12:1 \]

\[ a/d = 3.6 \]
Beam T1 after Test
Strengthening with CFRP Fabric

Beam T4

Beam T5

Cross-Section

Externally Bonded FRP: Shear and Torsion  Fibre Composites, FS18  Masoud Motavalli
Debonding of CFRP-Fabric
**Systematic Test Program**

![Beam SN](image)

<table>
<thead>
<tr>
<th>Typ of Test</th>
<th>Internal Reinforcement</th>
<th>External Reinforcement</th>
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<tbody>
<tr>
<td>S1 Static Loading</td>
<td>$\varnothing 8 \text{ s} = 150 \text{ mm}$</td>
<td>Without</td>
</tr>
<tr>
<td>S2 Static Loading</td>
<td>Without</td>
<td>Without</td>
</tr>
<tr>
<td>S3 Static Loading</td>
<td>Without</td>
<td>CFRP L- Plates $s = 300 \text{ mm}$</td>
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<tr>
<td>S4 Static Loading</td>
<td>$\varnothing 8 \text{ s} = 150 \text{ mm}$</td>
<td>CFRP L- Plates $s = 300 \text{ mm}$</td>
</tr>
<tr>
<td>S5 Pre Loading</td>
<td>$\varnothing 8 \text{ s} = 150 \text{ mm}$</td>
<td>CFRP L- Plates $s = 300 \text{ mm}$</td>
</tr>
<tr>
<td>S6 Fatigue</td>
<td>$\varnothing 8 \text{ s} = 150 \text{ mm}$</td>
<td>CFRP L- Plates $s = 300 \text{ mm}$</td>
</tr>
</tbody>
</table>

$$a/d = 2.9 \quad b/b_0 = 6.0$$
Test Beams

![Diagram of test beams with labeled forces and dimensions.]

$F_{total}/2 \quad F_{total}/2$

Bearings:
- S1
- S2

Dimensions:
- Length: 3500 mm
- Width: 800 mm
- Height: 250 mm

Externally Bonded FRP: Shear and Torsion  Fibre Composites, FS18  Masoud Motavalli
Test Beams
Deflection at mid-span

Beam S1: $F_{\text{total}} \text{ max} = 723 \text{ kN}$

Beam S2: $F_{\text{total}} \text{ max} = 325 \text{ kN}$

Beam S3: $F_{\text{total}} \text{ max} = 634 \text{ kN}$

Beam S4: $F_{\text{total}} \text{ max} = 757 \text{ kN}$
Strain in steel stirrup

Tensile strain steel stirrups 6R and 5R

![Graph showing strain in steel stirrup with lines for S1 and S4.](image_url)
Strain in CFRP L-shaped plate

![Graph showing strain in CFRP L-shaped plate](image)
Design

![Diagram showing the strain in CFRP plates and steel stirrups under different states of reinforcement.](image)

- **State I**: Only Steel Stirrups
- **State II**: Only CFRP L-shaped Plates
- **State III**: Steel Stirrups and CFRP L-shaped Plates

**Axes:**
- Y-axis: Load $F_{\text{total}}$ [kN]
- X-axis: Strain in CFRP Plates and Steel Stirrups [%]
Design

\[ V_R = V_C + V_S + V_F \]

- \( V_R \) = Shear resistance of RC member
- \( V_C \) = Concrete contribution (first shear crack)
- \( V_S \) = Contribution of internal steel stirrups
- \( V_F \) = Contribution of external CFRP L-shaped plates
Contribution of CFRP L-shaped plates

- Ultimate limit state (ULS)
  - Failure mode “opening of the overlapping”
  - Failure of the anchorage

- Serviceability limit state (SLS)

- Analysis of unstrengthened section
  - After plate failure remaining safety factor > 1
Failure of anchorage

Should be prevented if:

- Anchor length → whole height of flange (if possible)

- Carefully filling of the anchorage holes
- Anchor length > 100 mm (see reference [2])
Design equations for Sika® CarboShear L®

Equations without any safety factors!

Verification ULS:
\[ V_R = A_s \cdot f_y \cdot \frac{Z_s}{S_s} \cdot \cot \alpha + F_f \cdot \frac{Z_f}{S_f} \cdot \cot \alpha \quad \text{with } \alpha \geq 45° \]

Verification SLS:
\[ V_{ser} = V_C + A_s \cdot 0.8 \cdot f_y \cdot \frac{Z_s}{S_s} \cdot \cot \alpha + A_f \cdot (0.8 \cdot \frac{f_y}{E_s}) \cdot E_f \cdot \frac{Z_f}{S_f} \cdot \cot \alpha \quad \text{with } \alpha \geq 45° \]

Verification accidental situation:
\[ V_{acc} = V_C + A_s \cdot f_y \cdot \frac{Z_s}{S_s} \cdot \cot \alpha \]
Shear design

All the usual design verifications for RC (failure of the concrete struts, shift of moment line, etc.) have to be considered. For ductility reasons, the member should have a minimum internal shear reinforcement ratio, otherwise a strengthening is not recommended.
Fatigue

- See design concept in reference [3]
List of Symbols

 shear strengthening

$S_f$: FRP spacing, or ($W_{FRP}$)

$b_f$: FRP width, or ($W_{FRP}$)

$\Theta$: diagonal crack angle, assumed to be 45°

$\alpha$: fibre direction, or ($\beta$)

$\varepsilon_{d,e}$: design value of effective FRP strain

$V_{rd}$: design value of shear force capacity of the cross-section

$V_{cd}$: design value of shear force capacity concrete contribution

$V_{wd}$ or ($V_s$): " " " steel

$V_{fd}$ or ($V_{FRP}$): " " " FRP

$T_{fd}$: contribution of FRP to torsional capacity