# Table of content:

- Introduction
- Materials and Properties of Polymer Matrix Composites
- Mechanics of a Lamina
- Laminate Theory
- Ply by Ply Failure Analysis
- Externally Bonded FRP Reinforcement for RC Structures: Overview
  - Flexural Strengthening: Basics
  - Strengthening in Shear
  - Column Confinement
  - CFRP Strengthening of Metallic Structures
  - FRP Strengthening of Timber Structures
- Design of FRP Profiles and all FRP Structures
  - An Introduction to FRP Reinforced Concrete
  - Monitoring and Testing of Civil Engineering Structures
  - Composite Manufacturing
  - Testing Methods
Flexural strengthening

Book Composite for Construction, L. C. Bank, Chapter 9
Initial situation prior to strengthening

The effect of the initial load prior to strengthening should be considered in the calculation of strengthened member. Based on the theory of elasticity and with $M_0$ the service moment (*no load safety factors are applied*) acting on the critical RC section during strengthening, the strain distribution of the member can be evaluated. As $M_0$ is typically larger than the cracking moment $M_{cr}$, the calculation is based on a cracked section.
If $M_0$ is smaller than $M_{cr}$, its influence on the calculation of the strengthened member may easily be neglected.
Based on the transformed cracked section, the neutral axis depth $x_0$ can be solved from:

$$\frac{1}{2} b x_0^2 + (\alpha_s - 1) A_{s2} (x_0 - d_2) = \alpha_s A_{s1} (d - x_0)$$

Where:

$$\alpha_s = \frac{E_s}{E_c}$$
The concrete strain at the top fiber can be expressed as:

\[ \varepsilon_{c0} = \frac{M_0 x_0}{E_c I_{02}} \]

Where \( I_{02} \) is the moment of inertia of the transformed cracked section:

\[ I_{02} = \frac{b x_0^3}{3} + (\alpha_s - 1) A_{s2} (x_0 - d_2)^2 + \alpha_s A_{s1} (d - x_0)^2 \]

Based on strain compatibility, the concrete strain at the extreme tension fiber can be derived as:

\[ \varepsilon_0 = \varepsilon_{c0} \frac{h - x_0}{x_0} \]

This strain equals the initial axis strain at the level of the FRP, needed for the evaluation of the strengthened member.
Analysis of Ultimate Limit State (ULS)

**Full composite action**
Steel yielding followed by concrete crushing
Calculation of neutral axis depth, $x$:

$$0.85 \cdot \psi \cdot f_{cd} bx + A_{s2} E_s \varepsilon_{s2} = A_{s1} f_{yd} + A_f E_{fu} \varepsilon_f$$

Where:

$$\psi = 0.8$$

and:

$$\varepsilon_{s2} = \varepsilon_{cu} \left( \frac{x - d_2}{x} \right)$$

$$(E_s \varepsilon_{s2} \text{ not to exceed } f_{yd})$$

$$\varepsilon_f = \varepsilon_{cu} \left( \frac{h - x}{x} \right) - \varepsilon_0$$
Design bending moment capacity:

\[ M_{Rd} = A_s f_y (d - \delta_G x) + A_f E_f \varepsilon_f (h - \delta_G x) + A_s E_s \varepsilon_s (\delta_G x - d_2) \]

Where:

\[ \delta_G = 0.4 \]
Check if

a) Yielding of tensile steel reinforcement:

\[ \varepsilon_{s1} = \varepsilon_{cu} \frac{d - x}{x} \geq \frac{f_{yd}}{E_s} \]

b) Straining of the FRP is limited to the ultimate strain:

\[ \varepsilon_f = \varepsilon_{cu} \frac{h - x}{x} - \varepsilon_0 \leq \varepsilon_{fud} \]
Tee Beams

Neutral axis in flange: treat as rectangular section

Neutral axis in web: treat as tee section
Debonding and bond failure modes

- Debonding in the concrete near the surface or along a weakened layer, e.g. along the line of the embedded steel reinforcement.
- Debonding in the adhesive (cohesion failure).
- Debonding at the interfaces between concrete and adhesive or adhesive and FRP (adhesion failure).
- Debonding inside the FRP (interlaminar shear failure).
Debonding Video Clips

Non Prestressed CFRP

Prestressed CFRP
Lap Shear Test
Pull-off Test No. 3

Displacement in z-direction [mm]

failure
initiation of debonding
Stage 61

x-coordinate [mm]

0  50  100  150  200  250

0  0.05  0.1  0.15  0.2  0.25  0.3

-0.05
Externally Bonded FRP: Flexural Fibre Composites, FS20  
Masoud Motavalli
Bond failure of RC members strengthened with FRP:

See next lecture given by Dr. Christoph Czaderski
Summary of the three Swiss Code (SIA 166) verifications
See next lecture given by Dr. Christoph Czaderski

1. End strip debonding failure at the last crack
   \[ F_{fcr} \leq F_{b,R} \]

2. Debonding at strong strain increase in strip
   \[ \left( \frac{\Delta F_f}{\Delta x} \right) \leq \left( \frac{\Delta F_f}{\Delta x} \right)_R \]

3. Debonding at flexural cracks
   \[ \varepsilon_f \leq \varepsilon_{f,\text{lim},d} = 8\% \]

Serviceability Limit State (SLS)

- linear elastic material behavior
- cracked section analysis
Calculation of neutral axis $x_e$:

\[
\frac{1}{2} bx_e^2 + (\alpha_s - 1) A_{s2} (x_e - d_2) = \alpha_s A_{s1} (d - x_e) + \alpha_f A_f \left[ h - (1 + \frac{\varepsilon_0}{\varepsilon_c}) x_e \right]
\]

Where:

\[
\alpha_f = \frac{E_f}{E_c}
\]

\[
\alpha_s = \frac{E_s}{E_c}
\]

And the cracking moment for rectangular beams:

\[
M_{cr} \approx f_{ctm} \cdot \frac{bh^2}{6}
\]
Stress limitation

limit stresses in the concrete, steel and FRP to prevent

- damage or excessive creep of the concrete
- steel yielding
- excessive creep or creep rupture of the FRP
\[ \sigma_c \leq 0.60 f_{ck} \] under the rare load combination

\[ \sigma_c \leq 0.45 f_{ck} \] under the quasi-permanent load combination

where: \[ \sigma_c = E_c \varepsilon_c \]
To prevent yielding of the steel at service load:

$$\sigma_s = E_s \varepsilon_c \cdot \frac{d - x_e}{x_e} \leq 0.80 f_{yk}$$

rare load combination
FRP stress under service load should be limited as:

\[ \sigma_f = E_f \cdot (\varepsilon_c \cdot \frac{h - x_e}{x_e} - \varepsilon_0) \leq \eta \cdot f_{fk} \text{ quasi-permanent load combination} \]

Where

\[ \eta = \begin{cases} 0.8 & : \text{CFRP} \\ 0.5 & : \text{AFRP} \\ 0.3 & : \text{GFRP} \end{cases} \]
Verification of deflections

The mean deflection, $a$, is calculated from:

$$a = a_1 \cdot (1 - \zeta_b) + a_2 \cdot \zeta_b$$

Where $a_1$ and $a_2$ are the deflections in the uncracked and the fully cracked state, respectively and the distribution coefficient is:

$$\zeta_b = 0 \ldots \ldots M_k < M_{cr}$$

$$\zeta_b = 1 - \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_k}\right)^{n/2} \ldots \ldots M_k > M_{cr}$$
Where $\beta_1$ is a coefficient taking into account the bond characteristics of the reinforcement and equals 0.5 and 1 for smooth and deformed steel, respectively;

$\beta_2$ is a coefficient taking into account the loading type and equals 0.5 and 1 for long-term and short term loading, respectively.

The power $n$ equals 2. For high strength concrete more accuracy is obtained with $n$ equal to 3.
The deflection in the **uncracked state**, $a_1$, and in the **fully cracked state**, $a_2$, can be calculated by classical elasticity analysis, referring to a flexural stiffness in the **uncracked state** $E_c I_1$ and in the **fully cracked state** $E_c I_2$, respectively.
Verification of crack widths

Neglecting the tension stiffening effect ($\zeta = 1$) and assuming $\varepsilon_0 \approx 0$

$$w_k = 2.1 \rho_{c,\text{eff}} \cdot \frac{M_k}{E_s d\rho_{eq}} \cdot \frac{1}{(u_s + 0.694u_f)}$$

Where the ratio of the effective area in tension is:

$$\rho_{c,\text{eff}} = \frac{A_{c,\text{eff}}}{bd}$$

$\rho_{eq}$ is the equivalent reinforcement ratio and $u_s$ and $u_f$ is the bond perimeter of the steel and FRP reinforcement.
Summary of design procedure:

- Before strengthening: check ULS and SLS (just to compare with the strengthened member!).

- From the service moment $M_0$ prior to strengthening determine $\varepsilon_0$ at the extreme tension fiber.

- Assume full composite action and from the design moment after strengthening determine the required FRP cross section to fulfill the ULS. Verify the ductility requirements.

- Calculate the deflections in the SLS. If allowable deflection is exceeded, determine the required FRP cross section.
- Calculate the stresses in the concrete, steel and FRP and verify the allowable stresses.

- Verify that the provided FRP bond width is sufficient to control crack widths in the SLS. Increase the FRP width, if necessary, or, given a maximum width, increase the amount (thickness) of FRP.

- Verify the resisting shear force at which bond failure due to shear cracks occurs (ULS).

- Verify that bond failure at the anchorage does not occur. Otherwise mechanical anchorage should be provided.
- Verify that FRP end shear failure is avoided. Provide shear strengthening at the ends if required.

- Verify the accidental situation.

- Verify the shear design resistance of the strengthened member. If needed shear strengthening should be provided.
Strengthening of a Large Scale Pre-Stressed Bridge Girder Using Carbon Fibre Reinforced Polymers: Comparison between Non Prestressed and Prestressed CFRP Plates
Bridge „Viadotto delle Cantine a Capolago“
Bridge „Viadotto delle Cantine a Capolago“

17 m

0.8 m
Overview

- Reference beam

- Beam strengthened with *non* prestressed CFRP plates
  - 6 Sika CarboDur 512 plates, each 15.5 m long

- Beam strengthened with *pre*stressed CFRP plates
  - the same type and number of plates
  - each plate prestressed approx. 1000 MPa (60 kN)
  - anchorage: Empa gradient method
Strengthened with non prestressed CFRP plates

6 CFRP plates Sika CarboDur S512
cross-section $A_f = 60 \text{ mm}^2$
elastic modulus $E_f = 165\,000 \text{ N/mm}^2$
length 15.5 m

deforometer measurements:
12 x 0.1
14 x 0.5 = 7.0

0.15
0.6
3.5
1.0
3.4
17.0

kabel high

K4/K5

kabel low

0.6
3.5
1.0
3.4
1.0
3.5

0.6

D25, D36, D130, D132, D134, D136, D138, D140, D142, D144, D146, D148, D150, D152, D154, D156

D129, D131, D133, D135, D137, D139, D141, D143, D145, D147, D149, D151, D153, D155

0.2

0.6

front side
rear side
Beam length [m]

CFRP plate strain, front side [%]

- Beam no. 4: F= 100 kN, CFRP strain
- Beam no. 4: F= 200 kN, CFRP strain
- Beam no. 4: F= 250 kN, CFRP strain
- Beam no. 4: F= 300 kN, CFRP strain
- Beam no. 4: F= 367 kN, CFRP strain
- Beam no. 4: F= 353 kN, CFRP strain

Swiss code anchorage failure
Swiss code shear failure
Joint at mid-span

CFRP plate front side
Shear stress from Deformeter-measurement

\[ \tau_1 = \frac{\Delta F}{b_1 \cdot \Delta l} = \frac{\Delta \varepsilon \cdot E_1 \cdot A_1}{b_1 \cdot \Delta l} \]
Behavior during loading

- Beam no. 4: F = 100 kN, shear stress
- Beam no. 4: F = 200 kN, shear stress
- Beam no. 4: F = 250 kN, shear stress
- Beam no. 4: F = 300 kN, shear stress
- Beam no. 4: F = 300 kN, shear stress
- Beam no. 4: F = 353 kN, shear stress

CFRP plate front side

Joint at mid-span
Summary of the three SIA 166 verifications
See next lecture given by Dr. Christoph Czaderski

1. End strip debonding failure at the last crack

\[ F_{fcr} \leq F_{b,R} \]

2. Debonding at strong strain increase in strip

\[ \left( \frac{\Delta F_f}{\Delta x} \right) \leq \left( \frac{\Delta F_f}{\Delta x} \right)_R \]

3. Debonding at flexural cracks

\[ \varepsilon_f \leq \varepsilon_{f,\text{lim},d} = 8\% \]
SIA166 “Externally bonded reinforcement”

<table>
<thead>
<tr>
<th>Shear failure</th>
<th>measurement in the experiment</th>
<th>Swiss code SIA166</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.0 MPa (maximum value)</td>
<td>5.0 MPa</td>
</tr>
</tbody>
</table>

\[ \tau_{l,\text{lim}} = 2.5 \cdot \tau_c = 2.5 \cdot 2.0 = 5.0 \text{MPa} \]
Externally Bonded FRP: Flexural Fibre Composites, FS20
Masoud Motavalli

Strengthened with prestressed CFRP plates

CFRP plates prestressed approx. 1000 MPa (60 kN)
Prestressing using Gradient-method

Force in CFRP plate
Fmax = 511 kN (145%)
Fmax = 435 kN (124%)
Fmax = 352 kN (100%)

Load F [kN]

deflection at mid-span [mm]

beam 2: strengthened with prestressed CFRP plates
beam 4: strengthened with unstressed CFRP plates
reference beam

beam no. 3 (reference beam)
beam no. 3, phase 1
beam no. 3, phase 3
beam no. 3, phase 4
beam no. 4, CFRP strengthened
beam no. 4, phase 1
beam no. 4, phase 2
beam no. 4, phase 3
beam no. 4, phase 4
beam no. 2, prestressed CFRP strengthened
beam no. 2, phase 1
beam no. 2, phase 5
beam no. 2, phase 6

Fmax = 435 kN (124%)
Fmax = 352 kN (100%)
Fmax = 511 kN (145%)
List of Symbols

\( \Delta_0 \): service moment
\( M_{cr} \): cracking moment
\( A_s^1 \): steel cross-section at (tensile reinforcement)
\( A_s^2 \): steel cross-section (compression reinforcement)
\( x_0 \): position of the neutral axis prior to strengthening
\( b \): cross-section width
\( d \): depth; \( d = d_1 + d_2 \)
\( \alpha_s \): steel stress
\( \sigma_c : \) concrete stress
\( E_s : \) steel modulus of elasticity
\( E_c : \) concrete modulus of elasticity
\( I_{02} \): moment of inertia of the transformed cracked section prior to strengthening
\( l_b \): bond length
\( s_k \): slip (\( s_k = s_b \))
\( Y \approx 0.8 \) if "steel yielding followed by concrete crushing"
\( A_f \): FRP cross-section, or \( (A_{FRP}) \)
\( x \) (also \( C \)): position of the neutral axis after strengthening
\( \delta_{c0} \): position of the concrete compression force at ULS, \( \delta_{c0} = 0.4 \) if "steel yielding followed by concrete crushing"
\( h_f \): height of the flange
\( b_f \): width of the flange
\( b_{FRP} \): width of the FRP
\( S_{eq} = \frac{A_s + A_f \frac{E_c}{E_s}}{b d} \): equivalent reinforcement ratio
\( V_R = T_R b d \): \( V_R \): shear resistance; \( T_R \): shear strength
\( S_s \): steel reinforcement ratio
\( S_f \): FRP reinforcement ratio
List of Symbols

- $N_{fa, max}$: maximum FRP force, which can be anchored
- $l_{b, max}$: maximum anchorage length
- $f_{c}\text{em}$: mean concrete tensile strength
- $t_f$: FRP thickness
- $x_n$: position of neutral axis at SLS
- $\sigma_f \leq 2 \cdot f_k$: FRP stress under service load, where
  \[ 2 = \begin{cases} 0.8 & \text{CFRP} \\ 0.5 & \text{AFRP} \\ 0.3 & \text{GFRP} \end{cases} \]
- $\alpha_1$, $\alpha_2$, $\alpha_3$: mean deflection, deflection in uncracked state, deflection in fully cracked state
- $E_{sb}$: the distribution coefficient to calculate the deflection
- $E_{Ic}$: flexural stiffness in the uncracked state
- $E_{Ic,\text{fr}}$: flexural stiffness in the fully cracked state
- $W_k$: crack width at SLS
- $U_s$: bond parameter of steel
- $U_f$: bond parameter of FRP
- $\tau_{f\ell}$: maximum shear stress at FRP end at SLS
- $G_{a,\ell}$: shear modulus, thickness of adhesive