"As our Island of Knowledge Grows, So Does the Shore of our Ignorance"

Yakov Ben-Haim

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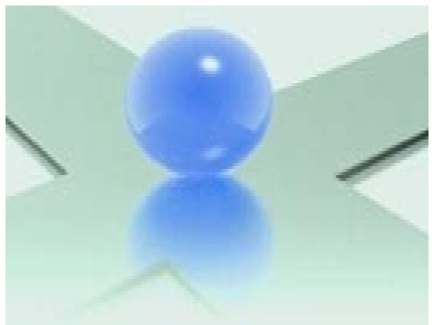

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Contents

1 *Highlights*

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§ **Questions:**

Are experiments irrelevant? Could they become so? Why?

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Highlights

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Are experiments irrelevant? Could they become so? Why?

§ **3 claims:**

- **We cannot know the extent of our ignorance.**
- **Simulation useful in exploring the known or possible. Less useful in exploring the unknown or 'impossible'.**
- **Info-gap theory for exploring the 'impossible'.**

2 *First Claim*

§ We cannot know the extent of our ignorance.

We cannot know *today,* what will be invented *tomorrow.*

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§ Can we *bet* about the unknown?

- **John Maynard Keynes:**

  Part of our knowledge we obtain direct; and part by argument. The Theory of Probability is concerned with that part which we obtain by argument, and it treats of the different degrees in which the results so obtained are conclusive or inconclusive.

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§ Probability useful, within limits of knowledge.

§ **Short explanation: Probability needs an event space:**

- **Specification of contingencies.**
- **Explicit, logically coherent.**
- **Cannot contain the unimaginable.**

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- **Explicit, logically coherent.**
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§ **Longer explanations:**

- **Shackle-Popper Indeterminism.**
- **Knightian uncertainty and info-gaps.**

3 *Principle of Indifference*

§ Question: Is ignorance probabilistic?

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The probabilistic domain of discourse
does not encompass all epistemic uncertainty.

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§ **The info-gap contention:**

The probabilistic domain of discourse
does not encompass all epistemic uncertainty.

§ **We will consider common misuses of probability.**

3.1 *Keynes' Example*

§ ρ = specific gravity [g/cm^3] is unknown:

$$1 \;\leq\; \rho \;\leq\; 3$$

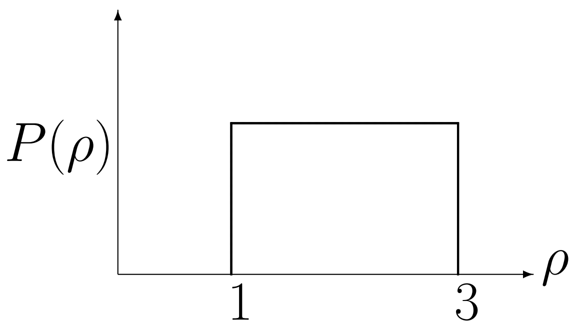
§

§ ρ = **specific gravity [g/cm^3] is unknown:**

$$1 \le \rho \le 3$$

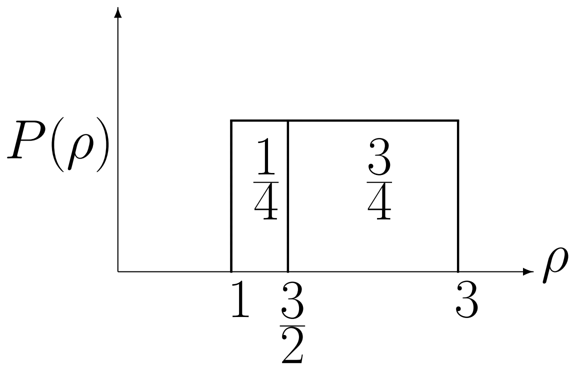
§ **Principle of indifference:**

Uniform distribution in $[1, \ 3]$, so:



§ Uniform distribution in $[1,\ 3]$, so:

$$\textbf{Prob}\left(\frac{3}{2} \ \le\ \rho\ \le\ 3\right) = \frac{3}{4}$$



§ **ϕ = specific volume [cm^3/g] is unknown:**

$$\frac{1}{3} \leq \phi \leq 1$$

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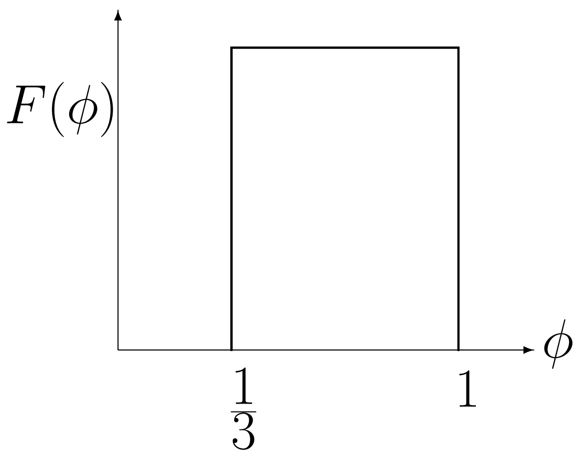
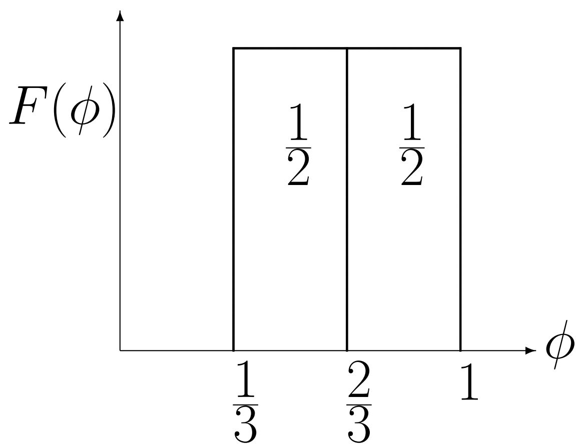


§ Principle of indifference:

Uniform distribution in $\left[\frac{1}{3},\ 1\right]$, so:

$$\textbf{Prob}\left(\frac{1}{3}\ \le\ \phi\ \le\ \frac{2}{3}\right) = \frac{1}{2}$$



§ These two events are identical:

$$\underbrace{\left(\frac{1}{3} \;\le\; \phi \;\le\; \frac{2}{3}\right)}_{\textbf{Specific volume}} \equiv \underbrace{\left(\frac{3}{2} \;\le\; \rho \;\le\; 3\right)}_{\textbf{Specific gravity}} \tag{1}$$

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§ Hence:

$$\frac{1}{2} = \frac{3}{4}$$

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§ **The Culprit: Principle of indifference.**

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§ **The Culprit: Principle of indifference.**

§ **Ignorance is not probabilistic. It's an info-gap.**

3.2 *2-Envelope Riddle*

§ The riddle:

- **You are presented with two envelopes.**
  - **Each contains a positive sum of money.**
  - **One contains twice the contents of the other.**
- **You choose an envelope, open it, and find \$ 50.**
- **Would you like to switch envelopes?**

§ **You reason as follows:**

- **Other envelope contains either $ 25 or $ 100.**
- **Principle of indifference:**
- **Assume equal probabilities.**

  The expected value upon switching is:

  E.V. $= \frac{1}{2}$ **$** $25 + \frac{1}{2}$ **$** $100 =$ **$** 62.50.

  $ $62.50 >$ **$** 50.

- **Yes! Let's switch, you say.**

§ The riddle, re-visited:

- **You are presented with two envelopes.**
  - **Each contains a positive sum of money.**
  - **One contains twice the contents of the other.**
- **You choose an envelope, but do not open it.**
- **Would you like to switch envelopes?**

§ You reason as follows:

- **This envelope contains $\$\, X > \ \$\, 0$.**
- **Other envelope contains either $\$\, 2X$ or $\$\, \frac{1}{2}X$.**
- **Principle of indifference:**
- **Assume equal probabilities.**

  The expected value upon switching is:

  E.V. $= \frac{1}{2}\ \$\, 2X + \ \frac{1}{2}\ \$\, \frac{1}{2}X = \$\, \left(1 + \frac{1}{4}\right)X \ > \ X$.
- **Yes! Let's switch, you say.**

§ **You reason as follows:**

- **This envelope contains $ $X > $ $0.**
- **Other envelope contains either $ $2X$ or $ $\frac{1}{2}X$.**
- **Principle of indifference:**
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- **Yes! Let's switch, you say.**

§ **You wanna switch again? And again? And again?**

§ We cannot know the extent of our ignorance,

And probability can't fill all the gaps.

•

§ We cannot know the extent of our ignorance,

And probability can't fill all the gaps.

- **Info-gap decision theory can help.**
- **Empiricism is essential.**

4 *Second Claim*

§ Simulation useful in exploring the known or possible.
Less useful in exploring the unknown or 'impossible'.

§ Computer simulation:

- **Predicts and explores implications of knowledge.**
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- **E.g. Simulation today cannot predict implications of tomorrow's discovery.**
- **Science vs. science fiction.**
  Induction vs. imagination. (Both important!)

5 *Third Claim*

- **Info-gap models of uncertainty for exploring the 'impossible'.**

- **Examples of info-gaps.**

Lewis Carroll's

∼∼*Transcendental Probability*∼∼

Figure 1: **Dodgson, 1832–1898.**

Figure 2: **Alice**

"A bag contains 2 counters, as to which nothing is known except that each is either black or white. Ascertain their colours without taking them out of the bag."

Lewis Carroll's

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Figure 3: **Dodgson, 1832–1898.**

Figure 4: **Alice**

"A bag contains 2 counters, as to which nothing is known except that each is either black or white. Ascertain their colours without taking them out of the bag."

Answer: "One is black, and the other white."

∼∼*Pascal's Wager*∼∼

Figure 5: **Blaise Pascal, 1623-1662.**

The wager is described in *Pensées* as:

"'God is, or He is not.' Reason can decide nothing here. ... Heads or tails will turn up. What will you wager? ...

"If you gain, you gain all; if you lose, you lose nothing. Wager, then, without hesitation that He is. ... Since there is an equal risk of gain and of loss, ..."

∼∼*Thames Flood Barrier*∼∼

Figure 6: **1953 barrier breach.**

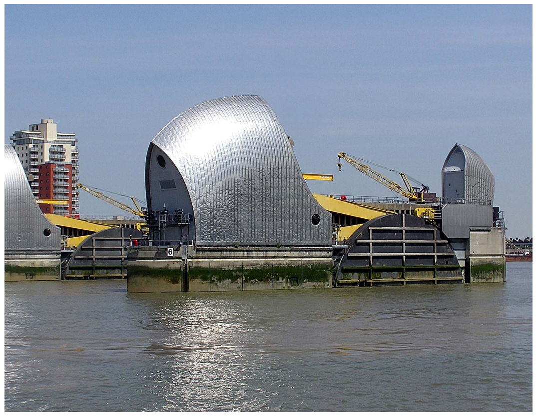


Figure 7: **Barrier element.**

§ Some facts:

- **1953: worst storm surge of century.**
- **Flood defences breached.**
- **307 dead. Thousands evacuated.**
- **Canvey Island in Estuary devastated.**
- **Current barrier opened May 1984.**

§ **Thames 2100:**

Major re-design of flood defences.

§ **Uncertainties:**

- **Statistics of surge height:**
  - **Fairly complete: most years since 1819.**
  - **Planning for 1000-year surge.**
- **Global warming: sea level rise.**
- **Tectonic settling of s. England.**
- **Damage vs flood depth.**
- **Human action: dredging, embanking.**
- **Urban development.**

§ **Severe Knightian uncertainties: Gaps in knowledge, understanding and goals.**

∼∼*Fukushima Nuclear Reactor*∼∼

Figure 8: **Sea wall breach.**

Figure 9: **Hydrogen explosion.**

§ Some facts:

- **11.3.2011: Richter-9 earthquake in NE Japan.**
- **Tsunami followed shortly.**
- **Sea wall breached: fig. 8.**‡
- **Hydrogen explosion several days later. Fig. 9.**‡
- **Slow disaster recovery.**

§ Info-gaps:

- **Sub-system interactions.**
- **Institutional constraints.**

‡ http://www.dailymail.co.uk/news/article-1388629/Japan-tsunami-destroyed-wall-designed-protect-Fukushima-nuclear-plant.html

∼∼*Interest rate after 9/11*∼∼

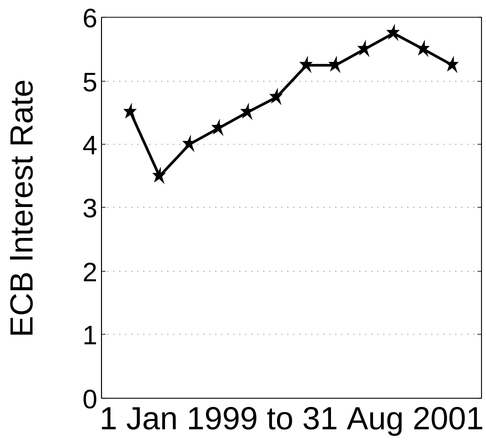


Figure 10: **ECB Interest Rates**

- **Rate fairly constant through Aug 2001**
-

∼∼*Interest rate after 9/11*∼∼

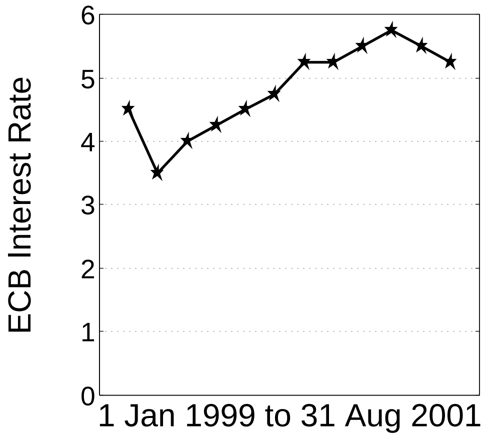


Figure 11: **ECB Interest Rates**

Figure 12: **11 Sept 2001.**

- **Rate fairly constant through Aug 2001**
- **After 9/11 ECB will reduce the rate.**
- **Info-gap:**
  - **Reduce by how much?**
  - **What is ECB decision model?**

∼∼*Phillips Curve*∼∼

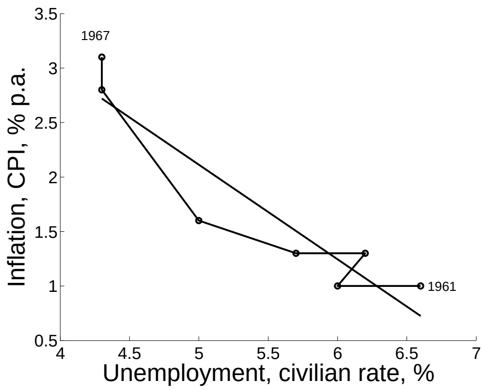


Figure 13: **Inflation vs. unemployment in the US, 1961–1967.**

- **Linear? Quadratic?**

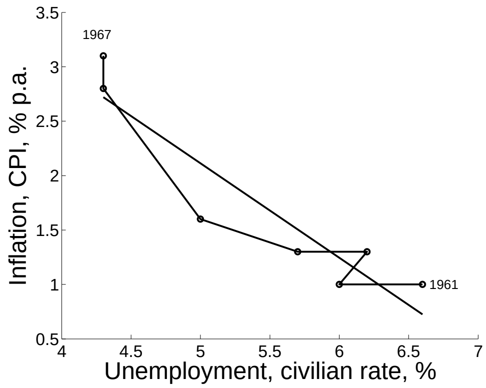


Figure 14: **Inflation vs. unemployment in the US, 1961–1967.**

Figure 15: **Inflation vs. unemployment in the US, 1961–1993.**

- **Linear? Quadratic?**
- **Info-gaps:**
  - **Uncertain data and process.**
  - **Unknown functional relation.**

∼∼*Climate Change*∼∼

§ The issue:

Sustained rise in green house gases

results in temperature rise

which results in adverse economic impact.

§ Models:

- **Temperature change:** $\Delta \mathrm{CO}_2 \Longrightarrow \Delta T$.
- **Economic impact:** $\Delta T \Longrightarrow \Delta \mathrm{GDP}$.

§ The problems:

- **Models highly uncertain.**
- **Data controversial.**

§ E.g., IPCC model for

Uncertainty in Equil'm Clim. Sensi'ty, S.

- **Likely range: 1.5^{o}C to 4.5^{o}C.**
- **Extreme values highly uncertain.**
- **95th quantile of S in 10 studies:**

  Mean: 7.1^{o}C. St. Dev: 2.8^{o}C.

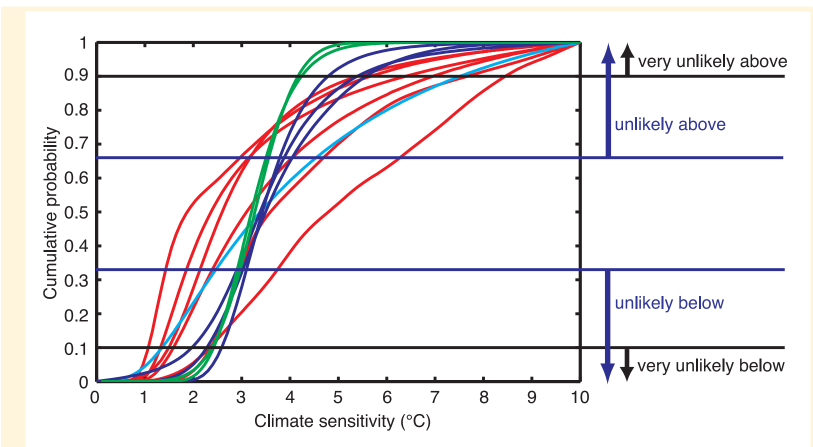


Figure 16: IPCC ch.10, p.799.

∼∼*Profiling Criminals*∼∼

Figure 17: **Profiling raises arrests.**

- **Profiling: focus policing resources.**
  - **Arrests rise in profiled group.**
  - **Crime rises in other groups.**
  - **Everybody happy?**
- **Info-gaps: Uncertain response functions.**

2 *Info-Gap Models of Uncertainty*

§ What are info-gaps?

- **Models are used to make decisions.**
- **Info-gap:**

  Disparity between what is known

  and what needs to be known

  in order to make a good decision.
- **Info-gap:**
  - **Non-probabilistic (Knightian) uncertainty.**
  - **Indeterminism, ignorance, surprise.**

§ Examples:

- **Contaminated field:**

  $f(x)$ = **uncertain spatial distribution.**

- **Parasite infestation:**

  $f(x)$ = **uncertain spatial distribution.**

  $P(n)$ = **uncertain prob. of n infestations.**

- **Seismic load:**

  $f(x,t)$ = **uncertain space/time variation.**

- **Strategic game:**

  π = **antagonist's uncertain preferences.**

- **Financial loss:**

  $p(\ell) =$ **uncertain probability of loss ℓ.**

- **Financial gain:**

  $\mu, \Sigma =$ **uncertain mean, covariance of returns.**

- **Medical treatment:**

  $u(x) =$ **uncertain dis-utility of side effect.**

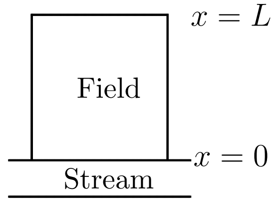
- **Mechanical friction:**

  $f(x, \dot{x}) =$ **uncertain force.**

- **Inflation-unemployment trade-off:**

  $u(x) =$ **uncertain Phillips curve.**

Spatial distribution of contaminate.



§ Mass-density functions:

- $\widetilde{f}(x)$ = **best estimate of distribution.**
- $f(x)$ = **unknown true distribution.**

§ Info-gap: **disparity between $f(x)$ and $\widetilde{f}(x)$.**

§ How to model the info-gap?

§ Fractional-error info-gap model.

$$\mathcal{U}(h, \widetilde{f}) = \left\{ f(x) : \ \left| f(x) - \widetilde{f}(x) \right| \le h \widetilde{f}(x) \right\}, \quad h \ge 0 \tag{1}$$

§ Two levels of uncertainty:

$f(x) =$ **unknown true realization.**

$h =$ **unknown horizon of uncertainty.**

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§ Fractional-error info-gap model.

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§ Two levels of uncertainty:

$f(x) =$ **unknown true realization.**

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§ Axioms of info-gap uncertainty:

- $\mathcal{U}(h, \widetilde{f})$ **is a set-valued function.**
- **Contraction:** $\mathcal{U}(0, \widetilde{f}) = \{\widetilde{f}\}$.
- **Nesting:** $h < h^{\bullet} \quad \Longrightarrow \quad \mathcal{U}(h, \widetilde{f}) \subseteq \mathcal{U}(h^{\bullet}, \widetilde{f})$

§ Fourier-ellipsoid info-gap model.

$$f(x) = \widetilde{f}(x) + \sum_{i=1}^{N} c_i \cos \frac{i\pi x}{L} \tag{3}$$

$$\mathcal{U}(h, \widetilde{f}) = \left\{ f(x) : \ c^T W c \le h^2 \right\}, \quad h \ge 0 \tag{4}$$

§ Two levels of uncertainty:

$f(x)$ = **unknown true realization.**

h = **unknown horizon of uncertainty.**

Parasite infestations

§ $P(n)$ = **probability of n attacks/season.**

- **Random, not independent.**
- **Poisson-like uncertain distribution $P(n)$.**
- $\mathcal{P}(h, \widetilde{p})$ = **info-gap model: uncertain $P(n)$.**

§ $u(x)$ = **intensity of attack at location x.**

- **Some areas more prone, some less.**
- **Some areas more variable, some less.**
- $\mathcal{U}(h, \widetilde{u})$ = **info-gap model: uncertain $u(x)$.**

∼∼***Summary***∼∼

§ **Deep Knightian uncertainties: Gaps in knowledge, understanding and goals.**

§

∼∼***Summary***∼∼

§ **Deep Knightian uncertainties:** **Gaps in knowledge, understanding and goals.**

§ **Info-Gap models of uncertainty:**

- **Disparity between what is known and what needs to be known for responsible decision.**
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∼∼*Summary*∼∼

§ **Deep Knightian uncertainties: Gaps in knowledge, understanding and goals.**

§ **Info-Gap models of uncertainty:**

- **Disparity between what is known and what needs to be known for responsible decision.**
- **Unbounded family of sets of events (points, functions or sets).**
- **No known worst case.**
- **No functions of probability, plausibility, likelihood, etc.**
- **Hybrid: info-gap model of probabilities.**

3 *Conclusion*

In Conclusion

§ **Info-gap uncertainty:**

innovation, discovery, ignorance, surprise.

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§ **Info-gap uncertainty is unbounded, non-probabilistic.**

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§ **Optimism:** **our knowledge gets better all the time.**

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§ **Optimism:** **our knowledge gets better all the time.**

§ **Realism:** **our knowledge is wrong now**
(and we don't know where or how much).

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In Conclusion

§ **Info-gap uncertainty:**
innovation, discovery, ignorance, surprise.

§ **Info-gap uncertainty is unbounded, non-probabilistic.**

§ **Optimism:** **our knowledge gets better all the time.**

§ **Realism:** **our knowledge is wrong now**
(and we don’t know where or how much).

§ **Responsible decision making:**

- **Specify your goals.**
- **Maximize your robustness to uncertainty.**
- **Study the trade offs.**
- **Exploit windfall opportunities.**